

AI-AUGMENTED ENGINEERING DYNAMICS: A CONVERSATIONAL APPROACH

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August 29, 2026 · Revision 1.0

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

By Taner Onsay, PhD

CHAPTER - 2

PART 2 · ≈ 45 MIN

Cartesian Kinematics

How do we describe curved motion with fixed x–y axes — and what does "independent components" really mean when there is only one moving object?

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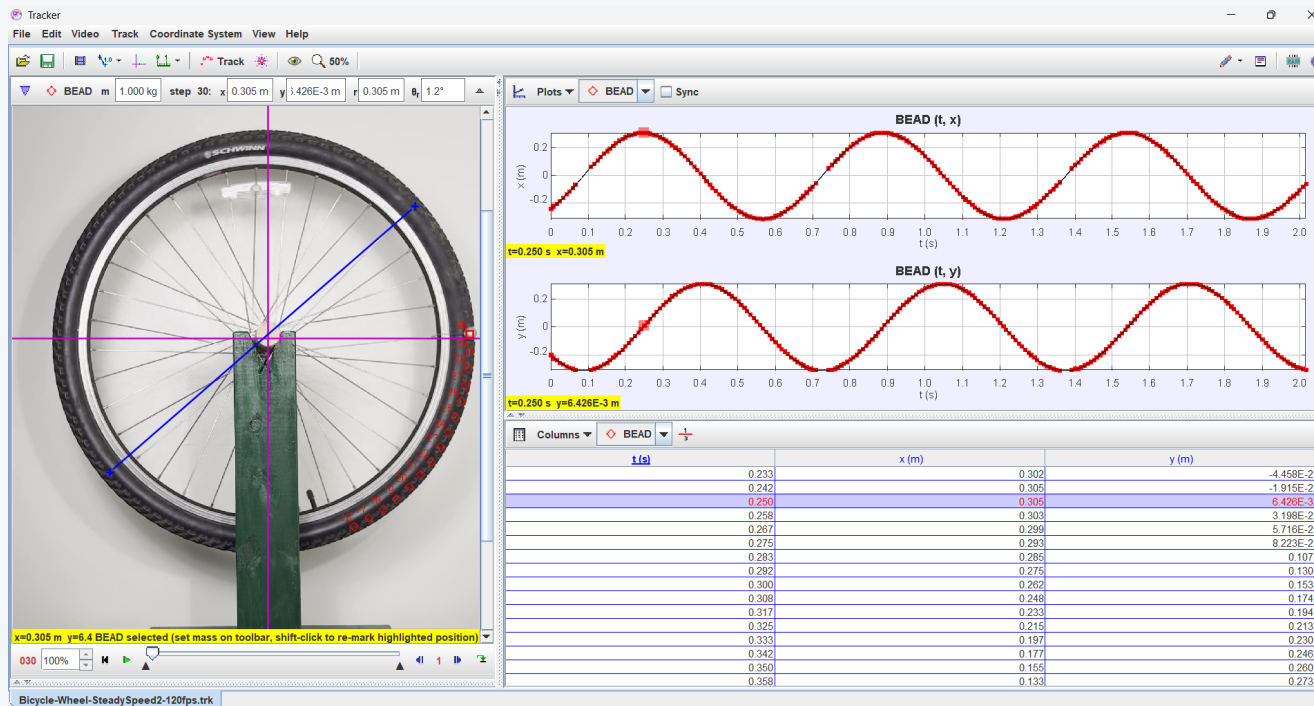
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Tracker hands you a table of (t, x, y). What does it mean?

TRACKER MOTION ANALYSIS



The data are components

Two columns, $x(t)$ and $y(t)$, every $1/120$ s. Nothing in the table says "circle" — that appears only when you plot one against the other.



Velocity comes from slopes

$v_x \approx \Delta x / \Delta t$ and $v_y \approx \Delta y / \Delta t$ — the Chapter 1 slope idea applied to each column separately. Tracker does exactly this.



The question of this Part

We can analyze x - and y -components separately—but does that mean the physical motions are independent? There is only one bead. No. "Separate components" does not necessarily mean independent motion.

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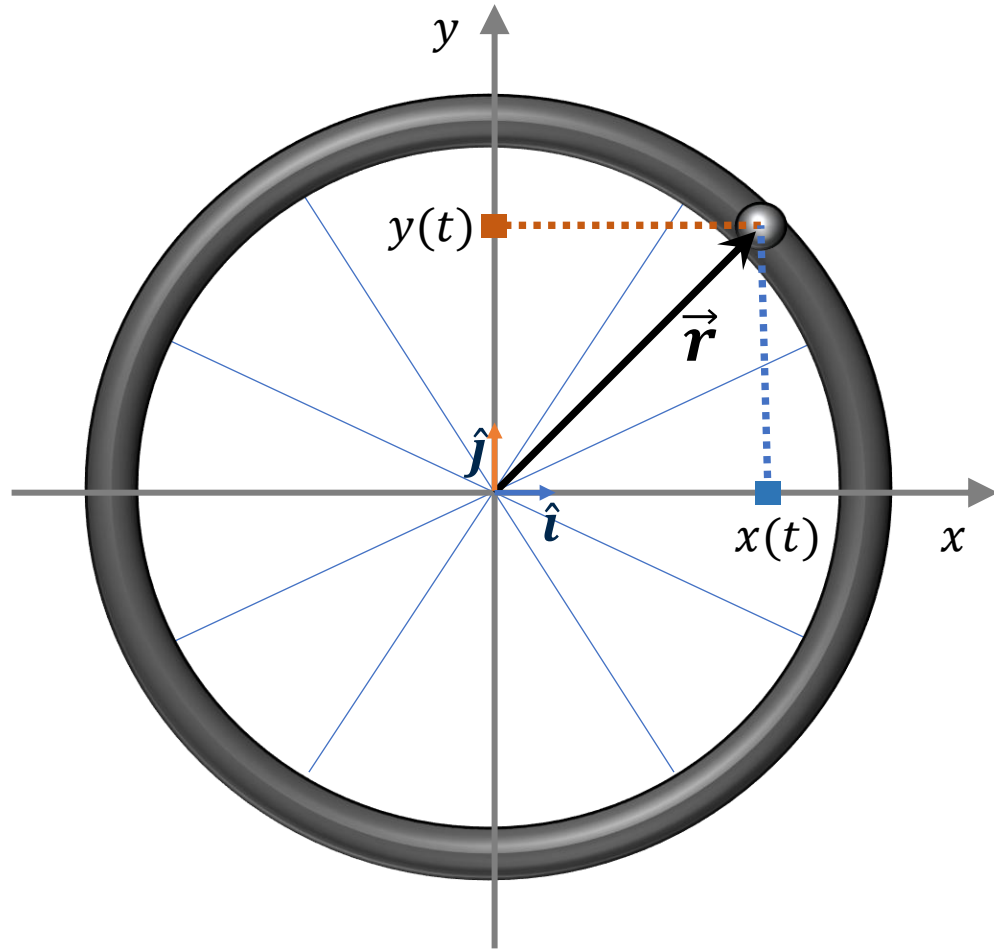
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$\vec{r} = x \hat{i} + y \hat{j}$ — and $\hat{i}, \hat{j}$ (unit vectors) **never change**



THE MODEL

Particle	the bead (or any tracked point)
Coordinates	fixed x–y axes; you choose their directions
Origin	your choice — here, the spindle center
Basis vectors	$\hat{i}, \hat{j}$: constant in magnitude AND direction
Position	$\vec{r} = x \hat{i} + y \hat{j}$

THE ONE FACT THAT MAKES THIS PART EASY

$$d\hat{i}/dt = 0, \quad d\hat{j}/dt = 0$$

The axes do not move, so differentiating r only differentiates the components. Remember this — in Parts 4 and 5 the basis vectors will rotate, and a new term will appear.

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One vector equation, two scalar equations

VELOCITY AND ACCELERATION

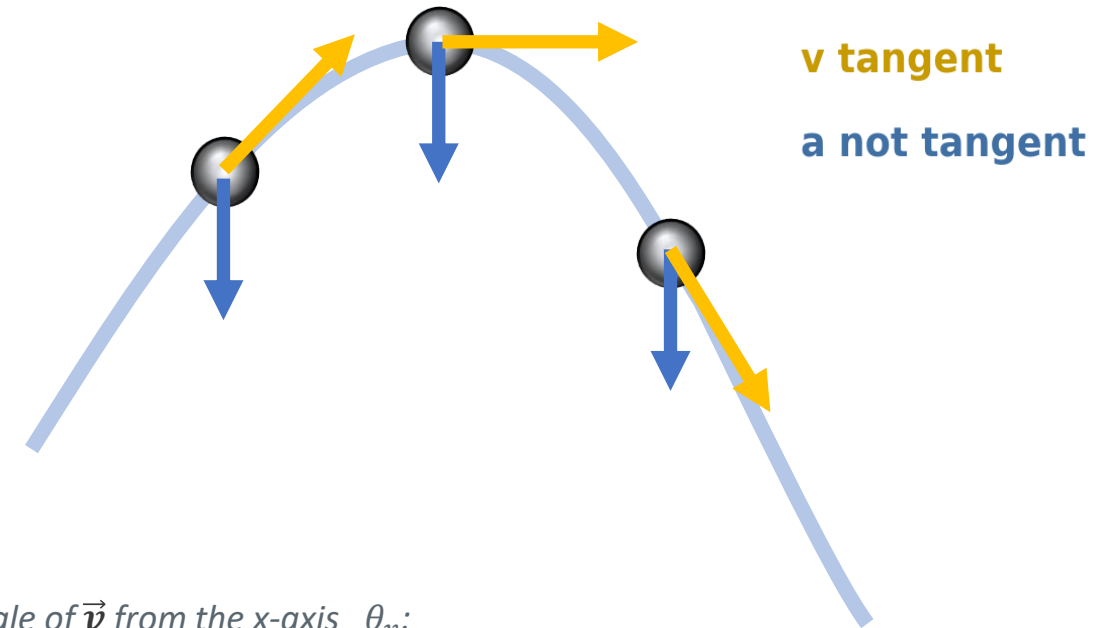
$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x} \hat{i} + \dot{y} \hat{j}$$

$v_x = \dot{x}$, $v_y = \dot{y}$, magnitude: $|v| = \sqrt{\dot{x}^2 + \dot{y}^2}$,
direction: tangent to the path

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x} \hat{i} + \ddot{y} \hat{j}$$

$a_x = \ddot{x}$, $a_y = \ddot{y}$, magnitude: $|a| = \sqrt{\ddot{x}^2 + \ddot{y}^2}$,
direction: generally, NOT tangent

Two scalar equations per vector — the mathematics separates. The physics does not: the components describe ONE motion.



Angle of $\vec{v}$ from the x-axis θ_v :

Calculator: $\tan^{-1}(v_y/v_x)$ — check the quadrant against the sketch

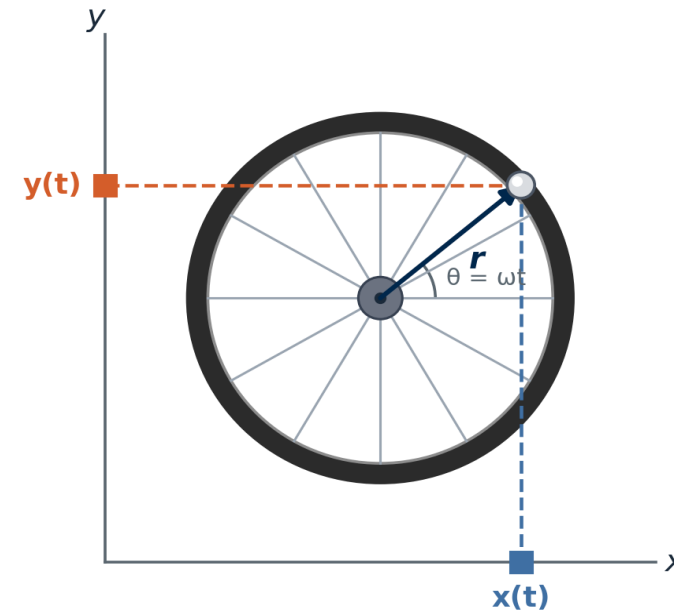
In Code: use special function $\text{atan2}(v_y, v_x)$

2 nd Quad (-x,+y)	1 st Quad (+x,+y)
3 rd Quad (-x,-y)	4 th Quad (+x,-y)



PREDICT

The bead circles at steady speed.
Sketch $x(t)$ and $y(t)$.
Are they "independent"?



Draw both curves before the reveal. Then decide: can one bead be two independent motions?

Vote: independent · not independent · the word is being used loosely.

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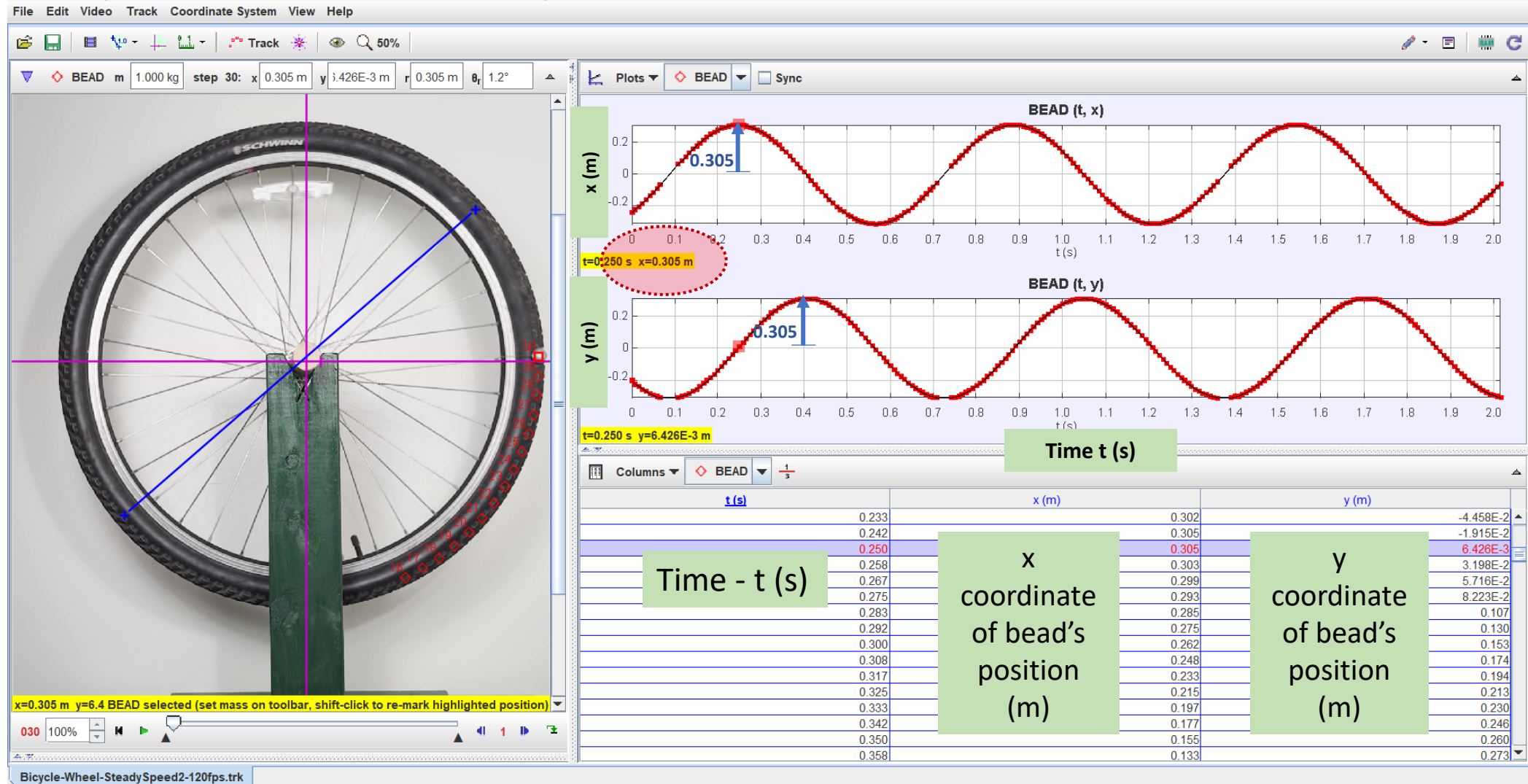
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Tracker Analysis: Bead's path is 12" from the spindle center, i.e. $r = 0.3048 \text{ m}$. Validation check: the amplitudes of $|x(t)|$ and $|y(t)|$ should each be approximately equal to the measured bead radius r . We observe $x_{\max} = 0.305$, and $y_{\max} = 0.305$!



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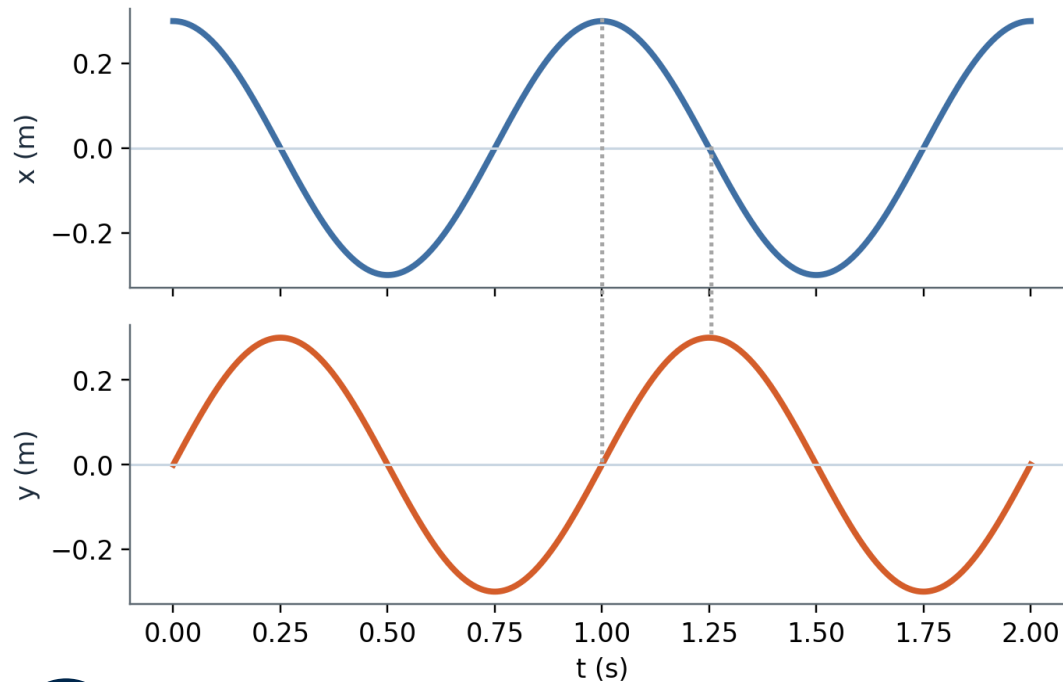
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$x(t)$ and $y(t)$ are the bead's shadows on the axes. Each shadow oscillates; the bead just goes around.



STEADY BEAD, RADIUS R , ANGULAR RATE ω

$$\begin{aligned} x &= R \cos \omega t, & y &= R \sin \omega t \\ \dot{x} &= -R\omega \sin \omega t, & \dot{y} &= R\omega \cos \omega t \\ \ddot{x} &= -R\omega^2 \cos \omega t, & \ddot{y} &= -R\omega^2 \sin \omega t \end{aligned}$$

$$\vec{r} = R \cos \omega t \hat{i} + R \sin \omega t \hat{j} \quad |\vec{r}| = R$$

$$\vec{v} = -R\omega \sin \omega t \hat{i} + R\omega \cos \omega t \hat{j} \quad |v| = R\omega$$

$$\vec{a} = -R\omega^2 \cos \omega t \hat{i} - R\omega^2 \sin \omega t \hat{j} \quad \vec{a} = -\omega^2 \vec{r}$$

$\vec{a}$ points to the center with magnitude $\omega^2 R$ — the Part 1 result, now with numbers. But you had to recognize it; the components do not label it.



"Independent" means the equations separate

Fixed Cartesian basis $\rightarrow$ differentiation is component-wise. $\vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j}$, $\vec{a} = \ddot{x} \hat{i} + \ddot{y} \hat{j}$

But the components need not be independent. For this wheel, $x^2 + y^2 = R^2$: one physical motion constrains both components.



It does not mean two objects or two motions

One bead, one velocity vector, one acceleration vector. The components are its shadows.

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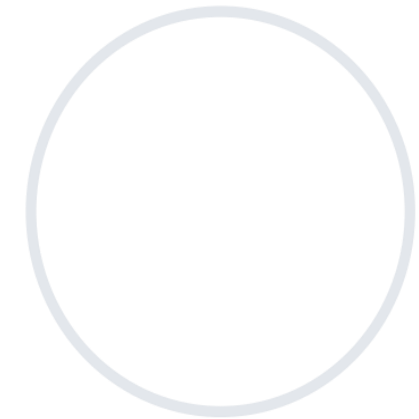
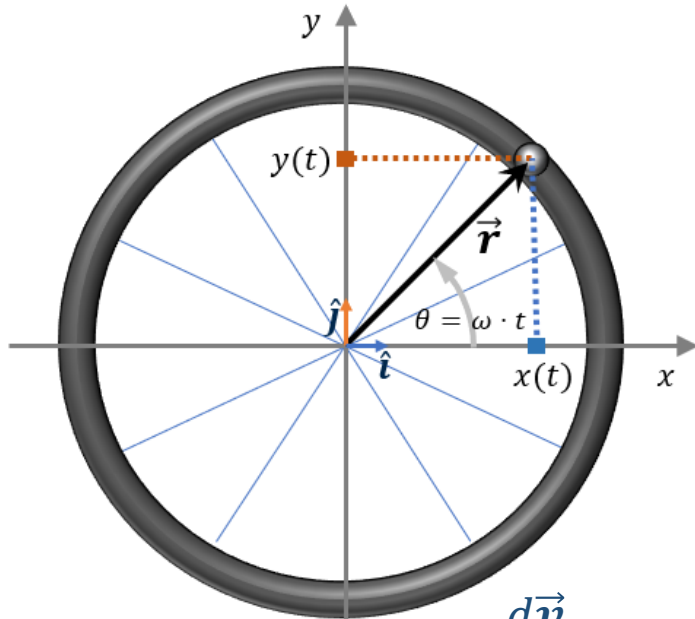


The bead in Cartesian language. Two more languages are coming.

CARTESIAN x-y

PATH n-t

POLAR r-θ



$$\vec{v} = \dot{x} \hat{i} + \dot{y} \hat{j} \quad \vec{a} = \frac{d\vec{v}}{dt} = \ddot{x} \hat{i} + \ddot{y} \hat{j}$$

Cartesian components describe the bead completely — but they hide the split between "speed changing" and "direction changing" that Part 1 made visible. Keep that in mind.

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Differentiate with respect to time, not with respect to x

CHAIN RULE, THEN PRODUCT RULE

$$y = f(x), \quad x = x(t)$$

$$\dot{y} = f'(x) \cdot \dot{x}$$

chain rule: $\left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)$. The slope of the path times the x -speed.

$$\ddot{y} = f''(x) \cdot \dot{x}^2 + f'(x) \cdot \ddot{x}$$

product rule on $f'(x) \cdot \dot{x}$: the first term comes from f' changing (chain rule again $\rightarrow \dot{x}^2$); the second from $\dot{x}$ changing.

Check the dimensions: f' is dimensionless, f'' has units $1/\text{length}$ — both terms are $\text{length}/\text{time}^2$.



The trap

Writing $\dot{y} = f'(x)$. That is dy/dx , a slope — not a velocity. Time must enter through $\dot{x}$.



AI as Validator

"Here is my expression for $\ddot{y}$ from $y = f(x)$. Check the chain-rule and product-rule steps, and tell me which term vanishes if $\ddot{x} = 0$."



CONCEPT CHECK



**A point moves on $y = 0.5 x^2$ (metres).
At $x = 2 \text{ m}$ its x-velocity is 3 m/s .
What is its y-velocity?**

Answer in 30 seconds. No calculator needed.

A) 2 m/s B) 3 m/s C) 6 m/s D) 1.5 m/s E) cannot tell without $\ddot{x}$

Observe



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C — 6 m/s. The slope (2) is not a velocity until you multiply by $\dot{x}$.

THREE STEPS

$dy/dx = x = 2$ (dimensionless slope)

$$\dot{y} = (dy/dx) \dot{x} = 2 \times 3 = 6 \text{ m/s}$$

$$\vec{v} = 3\hat{i} + 6\hat{j} \frac{m}{s}, \quad |\vec{v}| = 6.7 \text{ m/s at } 63^\circ \text{ to } x$$

Direction check: $\tan 63^\circ = 2 = \text{the slope of the path. } v \text{ is tangent — as it must be.}$



Extension

Now suppose $\ddot{x} = -1 \text{ m/s}^2$. Which term of $\ddot{y} = f''\dot{x}^2 + f'\ddot{x}$ needs it? Compute $\ddot{y}$. ($f'' = 1$, so $\ddot{y} = 9 - 2 = 7 \text{ m/s}^2$.)



Read the sign

$\ddot{y} > 0$ while $\ddot{x} < 0$: the point slows in x but its y -velocity still grows. Components can disagree in sign — the vector $\vec{a}$ tells the real story.

Habit for the whole chapter: after computing components, draw the vector on the path and ask whether its direction makes physical sense.

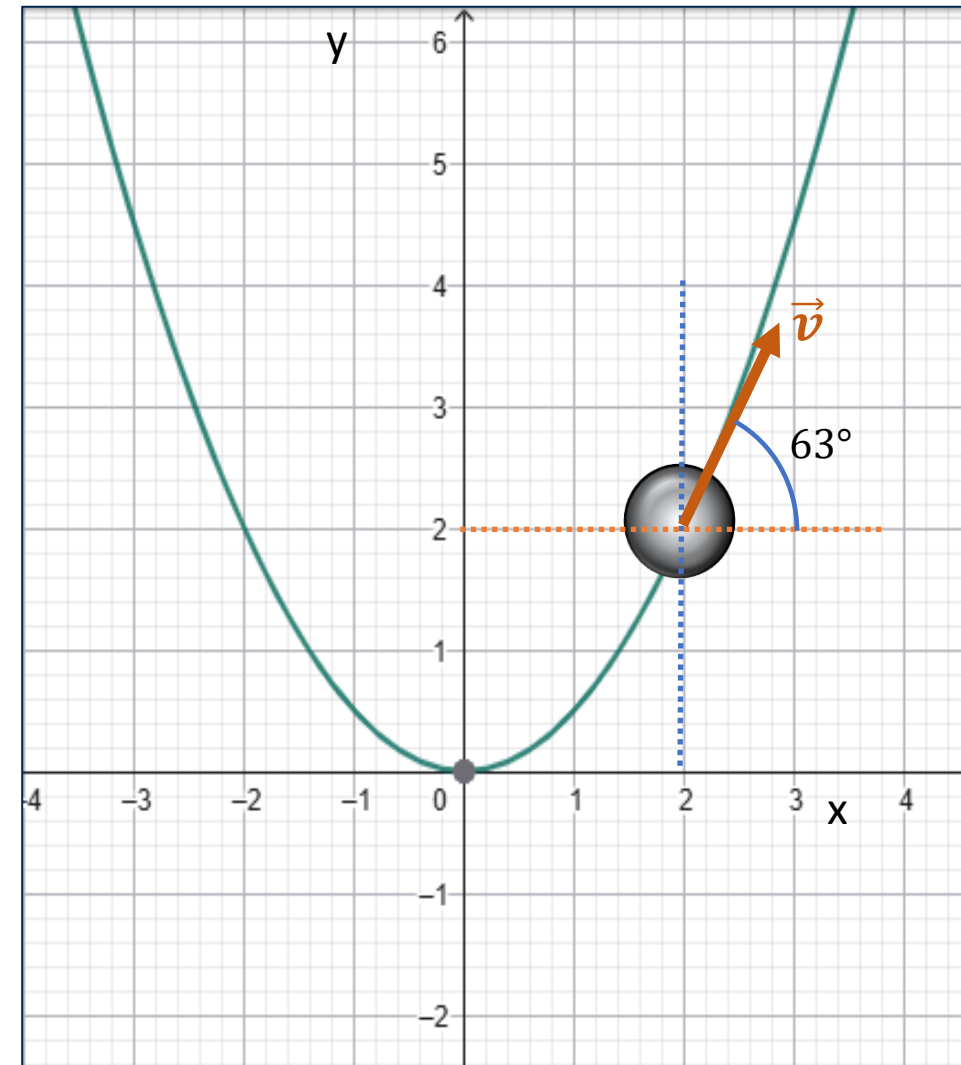
Check: Velocity is always tangent to the path.

GEOGEBRA: Graphing the motion

$$y = 0.5x^2$$

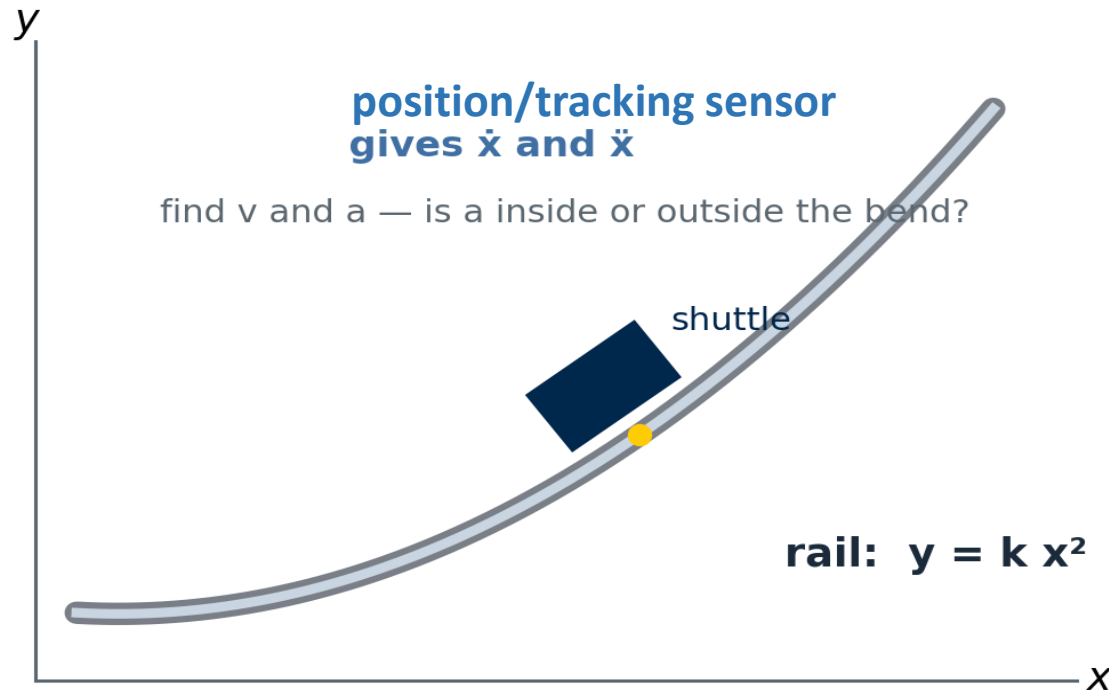
$$\vec{v} = 3\hat{i} + 6\hat{j} \frac{m}{s}, \quad |\vec{v}| = 6.7 \text{ m/s at } 63^\circ \text{ to } x$$

Direction check: $\tan 63^\circ = 2 = \text{the slope of the path}$. v is tangent — as it must be. Graphical analysis helps understand the motion better.





A shuttle follows a curved guide rail. Its position/tracking sensor reports along-x motion only.



Modeling decision: the position/tracking sensor does not give path speed. What it gives is $\dot{x}$. Say so before writing an equation.

THE MODEL

Particle	the shuttle's tracked reference point
Coordinates	fixed x (along the aisle), y (across); origin at the rail's lowest point
Constraint	the shuttle stays on the rail: $y = kx^2$, $k = 0.08 \text{ m}^{-1}$
Knowns	$x = 2.5 \text{ m}$, $\dot{x} = 1.2 \text{ m/s}$, $\ddot{x} = -0.40 \text{ m/s}^2$ (position/tracking sensor)
Unknowns	v (magnitude, direction), a (magnitude, direction), inside/outside?
Assumptions	rigid rail, no wheel slip, planar motion, position/tracking sensor calibrated



Differentiate the constraint, then reassemble the vectors

SOLVE

$$\dot{y} = 2k x \dot{x} = 2(0.08)(2.5)(1.2) = 0.48 \text{ m/s}$$

$$\ddot{y} = 2k(\dot{x}^2 + x \ddot{x}) = 0.16(1.44 - 1.00) = 0.070 \text{ m/s}^2$$

$$\vec{v} = 1.20 \hat{i} + 0.48 \hat{j} \text{ m/s} \rightarrow 1.29 \text{ m/s at } 21.8^\circ$$

$$\vec{a} = -0.40 \hat{i} + 0.070 \hat{j} \text{ m/s}^2 \rightarrow |\vec{a}| = 0.41 \text{ m/s}^2$$

Project $\vec{a}$ onto the inward normal $\hat{n} = (-0.371, 0.928)$:
 $\vec{a} \cdot \hat{n} = +0.21 \text{ m/s}^2 > 0 \rightarrow \vec{a}$ points *INSIDE* the bend. Along
 the tangent: $\vec{a} \cdot \hat{t} = -0.35 \text{ m/s}^2 \rightarrow$ the shuttle is slowing.



Interpret

The acceleration has an inward part (the rail is turning the shuttle) and a backward part (the shuttle is braking). Part 1's "both change" case — hidden inside two Cartesian numbers.



Where Part 4 is heading

We just computed 0.21 m/s^2 inward and 0.35 m/s^2 backward by hand-projection. n-t coordinates will hand us those two numbers directly.



Would you sign this? Three independent checks



Tangency. $\vec{v}$ at 21.8° ; rail slope $\tan^{-1}(2kx) = 21.8^\circ$. Match.



Curvature Check. The normal component of $\vec{a}$ must point toward the local center of curvature—the concave side of the path. Match.



Dimensions. $f''\dot{x}^2$ and $f'\ddot{x}$ are both m/s^2 . Match.



Limiting case

Set $\ddot{x} = 0$ (constant x -speed). Then $\ddot{y} = 2k\dot{x}^2 = 0.23 \text{ m/s}^2$ — still nonzero. Even with a steady $\dot{x}$ reading, the bend forces an acceleration. Does that agree with Part 1? (Yes: direction change.)



AI as Validator

*"I solved a rail-constrained shuttle in x - y and found $a = (-0.40, 0.070) \text{ m/s}^2$. Check my chain-rule steps and confirm whether a points toward the concave side of $y = 0.08x^2$."** Then judge the AI's answer against your own checks.



AI as Coding Companion

*"Write a short Python script that plots $y = kx^2$, animates a point with given $\dot{x}(t)$, and draws v and a at each frame."** Run it. Does the a -arrow tilt inward and backward, as we found?

MEDIA PLACEHOLDER

[INSERT PYTHON ANIMATION — path with v and a vectors at successive instants]

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Part 2 — what you can now explain and do



You can now EXPLAIN

- Why Cartesian differentiation is componentwise: the basis vectors $\hat{i}, \hat{j}$ remain fixed.
- Why component-wise does not necessarily mean independent: geometry or constraints may couple x and y
- What a tracking instrument gives you (a table of x, y) and what you must add (differentiation, then reassembly into one vector)
- What Cartesian components hide: the split between speed change and direction change



You can now DO

- Compute $\vec{v}$, $\vec{a}$ magnitudes and directions from $x(t), y(t)$
- Time-differentiate a path constraint $y = f(x)$ correctly (chain rule, then product rule) and check dimensions
- Reassemble components into one vector and check tangency and inward sense on the sketch
- State what an instrument actually measures before writing any equation

Next — Part 3: the projectile model is Cartesian kinematics with $a_x = 0$ and $a_y = -g$. The four balls return, and this time we will measure them.

EXPLORE WITH AI — Go One Step Deeper



What we established

For a particle following a known path $y = f(x)$ the slope, $dy/dx = f'(x)$ tells us the local direction of the path.

- The velocity vector is always tangent to the path.
- The acceleration vector is not necessarily tangent to the path.

But how do we mathematically construct the tangent and normal directions?



AI EXPLORATION PROMPT.

“I am studying Cartesian curvilinear motion in a 300-level Dynamics course. A particle follows a path $y = f(x)$. Teach me how to construct a unit tangent vector and a unit normal vector to the path using $f'(x)$. Start from the slope and explain the geometry step by step. Then give me one simple example and ask me to calculate the tangent and normal vectors myself before revealing the answer.”



ENGINEER'S CHECK BOX:

- ☐ Don't just accept the answer. **Verify it.**
- ☐ Are both vectors unit vectors? $|\hat{\mathbf{u}}_t| = 1, |\hat{\mathbf{u}}_n| = 1$
- ☐ Does $\hat{\mathbf{u}}_t$ point in the direction of motion?
- ☐ Does the chosen $\hat{\mathbf{u}}_n$ point toward the local center of curvature?
- ☐ Are the tangent and normal perpendicular?

$$\hat{\mathbf{u}}_t \cdot \hat{\mathbf{u}}_n = 0$$