

AI-AUGMENTED ENGINEERING DYNAMICS: A CONVERSATIONAL APPROACH

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Author

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

by Taner Onsay

CHAPTER - 2

PART 4 · ≈ 85 MIN

Path Coordinates (n-t)

When the path is known, a frame that rides with the particle separates speed change from direction change. The wheel makes it measurable; the bent wire makes it local.

Observe

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Same bead, same circle. This time we want to measure the inward acceleration.

BICYCLE TIRE

STEADY-SPEED
120 FPS VIDEO



MEDIA PLACEHOLDER

The bead is 12in
from the center of
the spindle.



What would we need to put a number on the inward acceleration?

The radius r (measure it) and how fast the wheel turns (count it). Nothing else — if the model is right.



The path is known in advance

That is the situation this Part is built for: describe the motion **ALONG** the path and **ACROSS** it, instead of in x and y .

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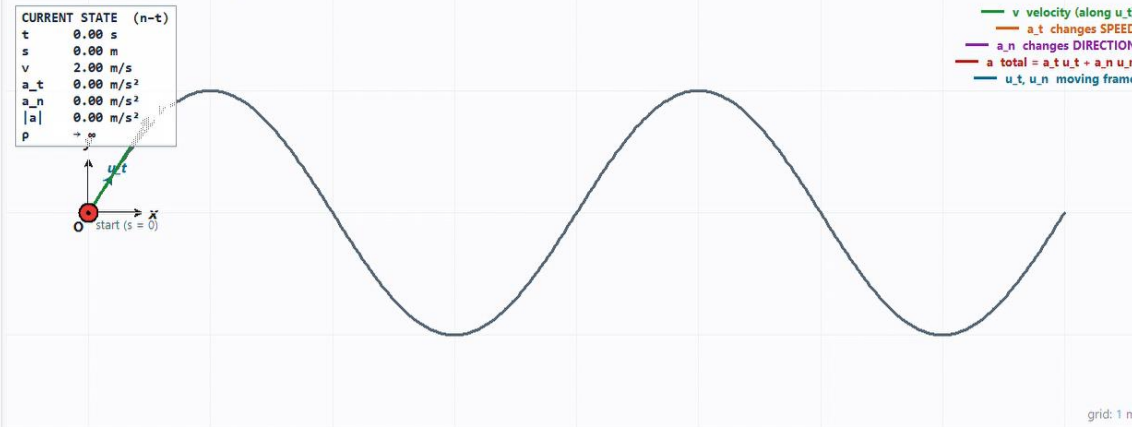
Report



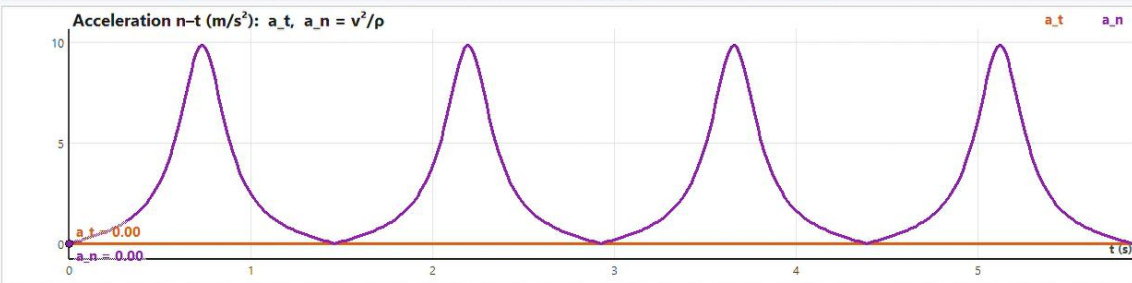
Two unit-vectors ride on the ball: one along the wire $\hat{u}_t$, one across it $\hat{u}_n$. Watch what they do.

Constant Speed Motion

Particle on the path — fixed x-y frame and moving t-n frame



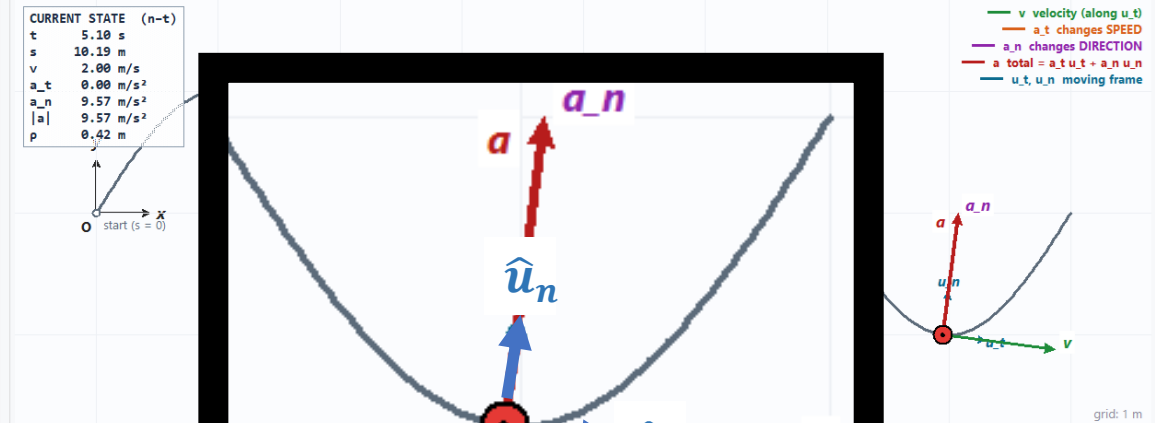
LOCALLY STRAIGHT: $p \rightarrow \infty$ so $a_n = 0$; only $a_t = 0.00$ m/s² remains ($a = a_t \hat{u}_t$ — Rectilinear Motion again)



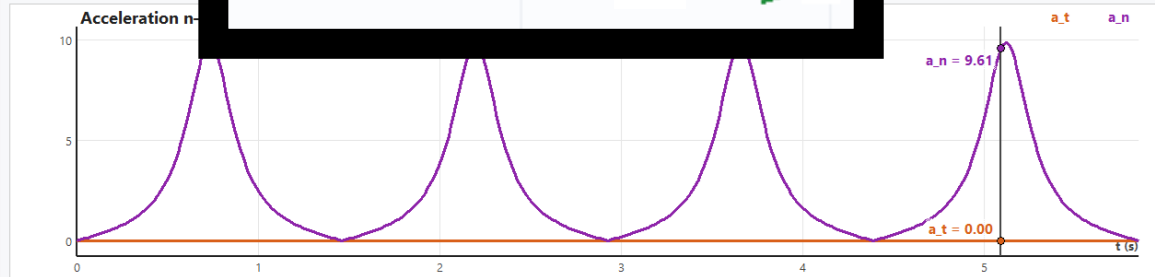
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SNAPSHOTS

Particle on the path — fixed x-y frame and moving t-n frame



CONSTANT SPEED ON



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Observe

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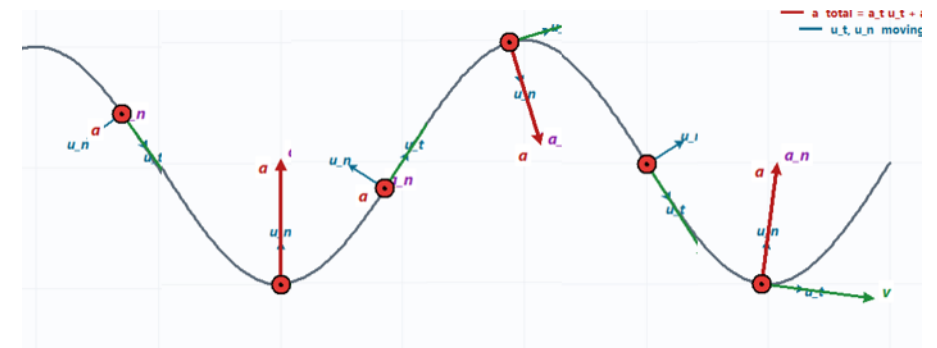
Validate

Report



PREDICT

The ball slides along the wire at constant speed.
Where is its acceleration largest?
Where is it zero?



Mark the wire. Also mark the DIRECTION of a at your "largest" point.

Vote by position: tight bend · gentle bend · straight-looking middle · nowhere (speed is constant!).

Observe

Idealize

Model

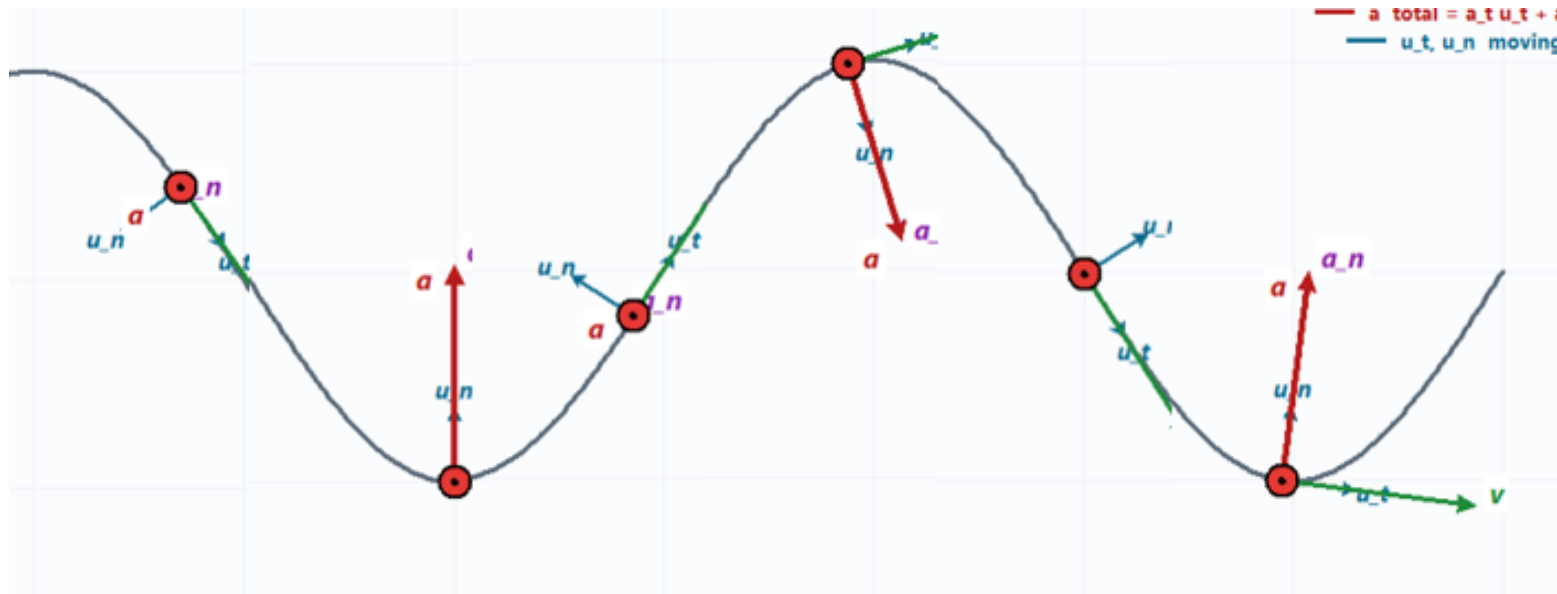
Solve / Analyze

Validate

Report



Perpendicular to the wire. Toward the concave side. Largest where the bend is tightest. Zero at the inflection.



Radius of curvature ρ

At each point, the wire is locally a circle of some radius ρ , centered on the concave side. Tight bend: small ρ . Straight: $\rho \rightarrow \infty$.



What constant speed leaves you with

No "along" acceleration. Only the "across" part — and it scales with how tight the bend is. We are about to quantify it.

At constant speed the acceleration is entirely about changing DIRECTION — and the path's local curvature decides how big it is.

Observe

Idealize

Model

Solve / Analyze

Validate

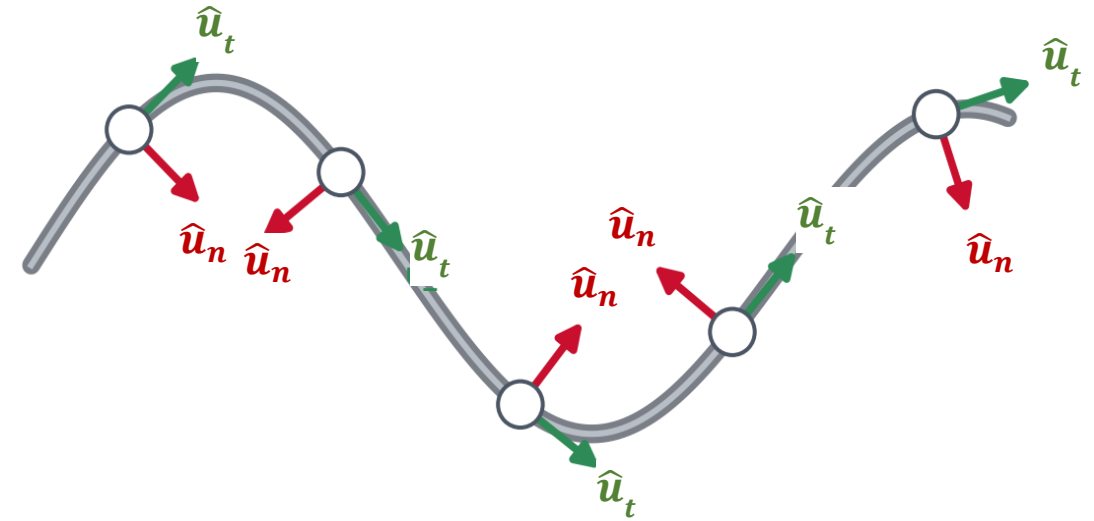
Report



t along the path, **n** toward the center of curvature. The origin is **ON** the particle and moves with it.

THE **n-t** DESCRIPTION

Particle	the bead, or the ball on the wire
Frame	t-n, attached to the particle; it MOVES and ROTATES with it
$\hat{u}_t$	tangent to the path, positive in the direction of motion
$\hat{u}_n$	perpendicular to $\hat{u}_t$, positive toward the center of curvature O'
ρ	radius of curvature: the circle that fits the path at this point; O' is on the concave side
Position	arc length s along the path from a fixed starting point
Contrast	Part 2: fixed origin, fixed axes. Part 5: fixed origin, rotating axes. Here: moving origin, rotating axes.



inflection: $\rho \rightarrow \infty$ — $\hat{u}_n$ has no side to choose



Two things students mix up

$\hat{u}_n$ is not "toward the origin of x-y" — there is no fixed origin here. And $\hat{u}_n$ never points outward: it is defined toward O' . Outward-pointing normals belong to Part 5 ($\hat{u}_r$).



The product rule gives two terms. In Part 2 the second one was zero. Here it is not.

VELOCITY, THEN THE PRODUCT RULE

$$\vec{v} = v_{sp} \hat{u}_t, \quad v_{sp} = |\vec{v}| \text{ speed}$$

speed (a scalar) times direction (a unit vector). Velocity is always tangent. Apply chain rule:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_{sp}}{dt} \cdot \hat{u}_t + v \cdot \frac{d\hat{u}_t}{dt}$$

first term: the SPEED changes. second term: the DIRECTION $\hat{u}_t$ changes.

In Part 2 the basis vectors were constant, so $d(\hat{i})/dt = 0$. Here $\hat{u}_t$ rotates as the particle moves — so $(d\hat{u}_t/dt) \neq 0$. Everything now depends on evaluating it.



Look at the golf ball again

The green pointer ($\hat{u}_t$) turned as the ball moved. A vector that turns has a nonzero derivative — even though its length (1) never changed.

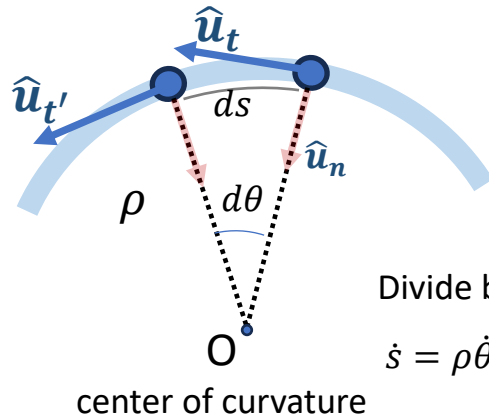


Predict the direction of $d\hat{u}_t/dt$

If a unit vector's length cannot change, which way can its derivative point? (Perpendicular to it. Toward the side it is turning to — the concave side.)



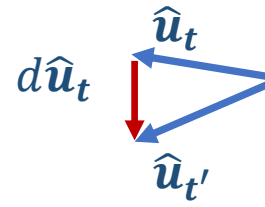
A small rotation $d\theta$ of $\hat{u}_t$ changes it by $|d\hat{u}_t| = d\theta$, in the $\hat{u}_n$ direction



$$ds = \rho d\theta$$

Divide by dt

$$\dot{s} = \rho \dot{\theta} \Rightarrow \dot{\theta} = \frac{\dot{s}}{\rho} = \frac{v}{\rho}$$



$$|d\hat{u}_t| = (1)d\theta$$

$d\hat{u}_t$ is perpendicular to $\hat{u}_t$

$$d\hat{u}_t = d\theta \cdot \hat{u}_n$$

Divide by dt

$$\frac{d\hat{u}_t}{dt} = \frac{d\theta}{dt} \cdot \hat{u}_n$$

$$\frac{d\hat{u}_t}{dt} = \left(\frac{v}{\rho}\right) \cdot \hat{u}_n$$

THE RESULT

$$\frac{d\hat{u}_t}{dt} = \left(\frac{v_{sp}}{\rho}\right) \hat{u}_n$$

rotation rate $\dot{\theta} = \dot{s}/\rho = v_{sp}/\rho$;
direction perpendicular to $\hat{u}_t$, toward the center of curvature.

Same idea returns in Part 5 for $\hat{u}_r$ and $\hat{u}_\theta$.

A unit vector cannot change its length — so its derivative is perpendicular to it, with magnitude equal to its rotation rate. Remember this one sentence.



Change in speed, change in direction, then combine

CHANGE IN SPEED

$$a_t = \frac{dv_{sp}}{dt}$$

along the path ($\hat{u}_t$). Positive: speeding up. Negative: slowing. Zero at constant speed. Also, $a_t ds = v dv$ (Chapter 1 along the path).

CHANGE IN DIRECTION

$$a_n = \frac{v_{sp}^2}{\rho}$$

toward the center of curvature ($\hat{u}_n$). A magnitude: never negative, never zero unless $v = 0$ or the path is straight. Direction carried by $\hat{u}_n$.

COMBINE — ONE ACCELERATION VECTOR, TWO PHYSICAL JOBS

$$\vec{a} = a_t \hat{u}_t + a_n \hat{u}_n = \underbrace{\left(\frac{dv_{sp}}{dt} \right)}_{a_t} \hat{u}_t + \underbrace{\left(\frac{v_{sp}^2}{\rho} \right)}_{a_n} \hat{u}_n$$

$$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

Not $a_t + a_n$; the components are perpendicular, so the magnitude is the root-sum-square. Direction: $\tan^{-1}(a_n/a_t)$ from the tangent.



What the equation says when one job is missing

1 Straight path: $\rho \rightarrow \infty$

$a_n = 0$, so $\vec{a} = a_t \hat{u}_t = \dot{v} \hat{u}_t$.
This is Chapter 1 exactly. Only speed can change.

2 Constant speed on a curve: $a_t = 0$

$\vec{a} = (v_{sp}^2 / \rho) \hat{u}_n$. The wheel at steady ω . The golf ball at constant speed. Direction change only — and $|\vec{a}| \propto 1/\rho$.

3 Constant a_t along the path

$s = s_0 + v_0 t + \frac{1}{2} a_t t^2$, $v = v_0 + a_t t$, $v^2 = v_0^2 + 2 a_t (s - s_0)$.
Chapter 1 with s instead of x — and a_n still acts alongside.

4 ρ for a path given as $y = f(x)$

$\rho = [1 + (y')^2]^{3/2} / |y''|$. Stated here; derivation in Appendix B.
Check: a straight line has $y'' = 0 \rightarrow \rho = \infty$.

Constant speed does NOT mean zero acceleration — case 2 says it with an equation: $a = (v^2 / \rho) \hat{u}_n \neq 0$ whenever the path bends.



$$f = \frac{N}{\Delta t}, \quad \omega = 2\pi f, \quad v_{sp} = \omega r, \quad a_n = v^2/r = \omega^2 r$$

In industry, usually N is given as *rpm* – revolutions per minute. Then, since $1 \text{ minute} = 60 \text{ s}$, $f = \left(\frac{N}{60}\right) / \Delta t$

MEDIA PLACEHOLDER

**Let's determine N – rev/min (RPM)
/ then we will derive ω – rotational
speed (rad/s)**

*Per RPM level: $N/60$ rotations per second,
 Δt (from frame count), f , ω , $v = \omega r$, $a_n = \omega^2 r$.
Check steadiness: same f in the first and
second halves?*



THE MODEL

Particle	the bead's center
Path	circle of radius r about the spindle (measured)
Assumption	ω approximately constant over the counting interval — test it
Knowns	r (tape), N , Δt (frame count)
Unknowns	ω , v_{sp} , a_n ; and $a_t \approx 0$ if steady

ILLUSTRATIVE NUMBERS — REPLACE WITH CLASS DATA

$r = 0.33 \text{ m};$
 $N = 10 \text{ rev in } 8.0 \text{ s} \rightarrow f = 1.25 \text{ Hz} \rightarrow \omega = 7.85 \text{ rad/s}$
 $v = \omega r = 2.59 \text{ m/s} \quad a_n = \omega^2 r = 20.3 \text{ m/s}^2 \approx 2.1 g$
 Note that: $1 \text{ Hz} = 1 \text{ rev/s}$ or more formally $1 \text{ Hz} = 1 \text{ cycle/s}$



2 g on a bicycle bead

Plausible? Where does the inward pull come from physically?
(Chapter 3 answers with a force; for now, say what would happen to an unglued bead.)



PREDICT



The wheel is spun twice as fast (2ω).

What happens to v_{sp} ?

What happens to a_n ?

Commit to both factors before we use the three RPM levels.

Vote: $v \times 2, a_n \times 2$ · $v \times 2, a_n \times 4$ · $v \times 4, a_n \times 4$ · $v \times 2, a_n$ unchanged (r did not change!)

Observe



Idealize



Model



Solve / Analyze



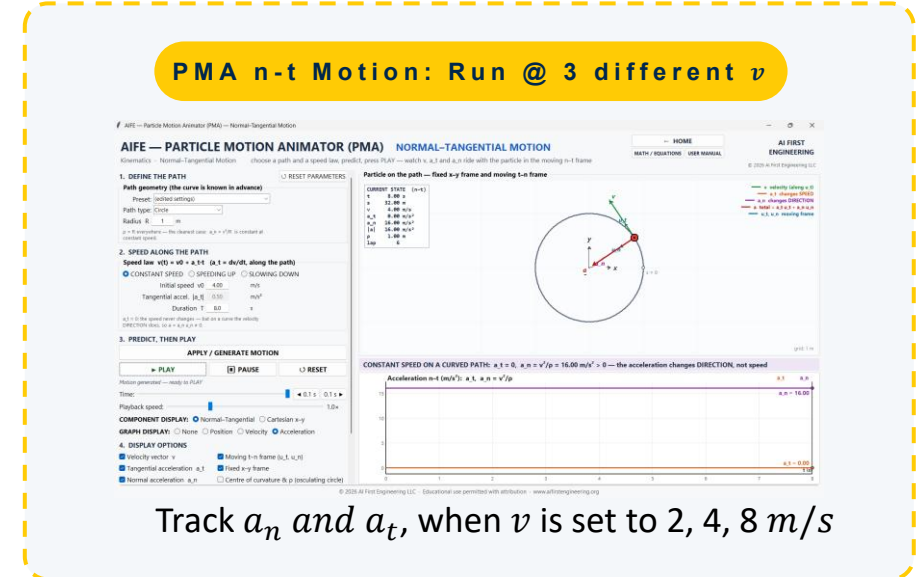
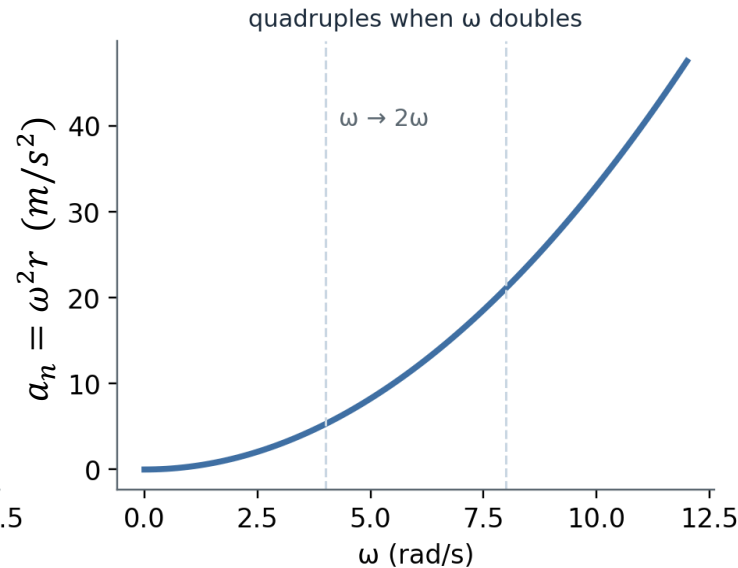
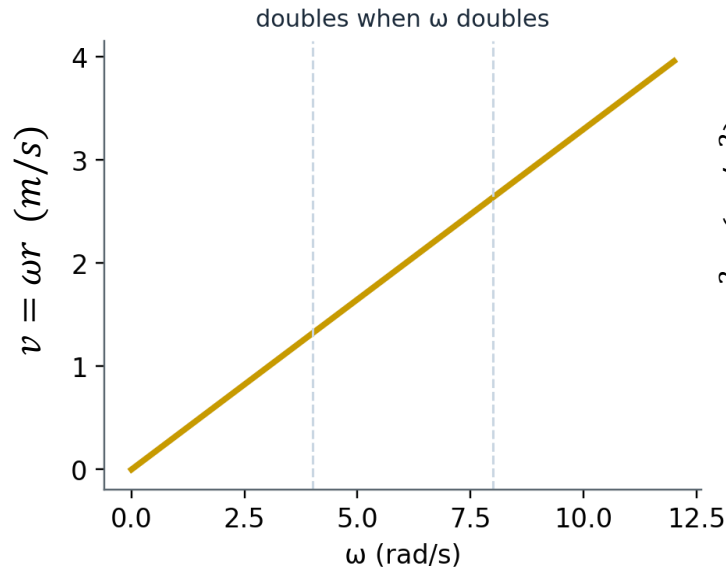
Validate



Report



v grows with ω ; a_n grows with ω^2 . Doubling the rate quadruples the inward acceleration.



Model says

$v \propto \omega$, $a_n \propto \omega^2$. Halving r at the same ω would halve both.



Data says

Plot the three measured points on the curves. Inside the frame-count band? Then the model holds here.



Why real points scatter

ω is not perfectly steady; Δt carries ± 1 frame; the bead center is a guess of ± 1 –2 mm on r .

Observe



Idealize



Model



Solve / Analyze

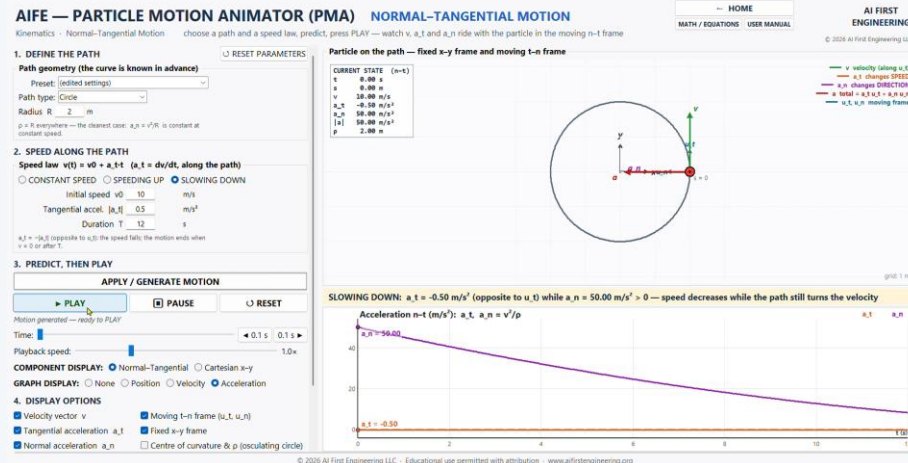

Validate


Report

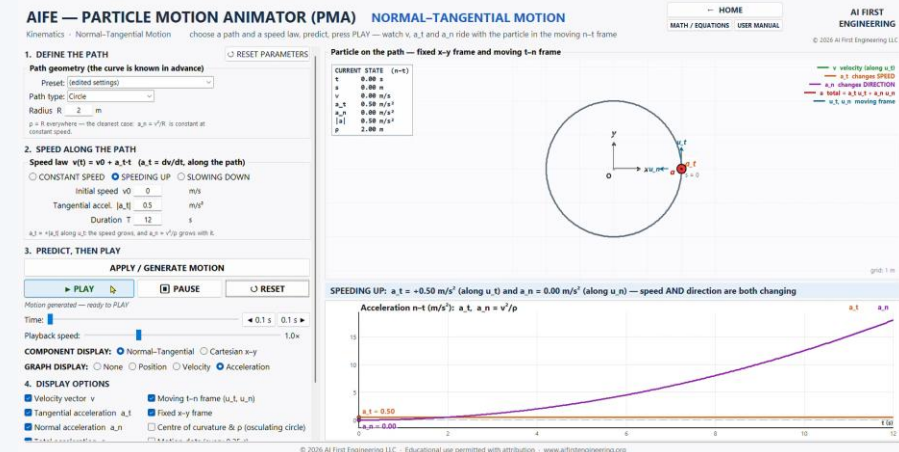


Now the speed changes too. Is the bead still accelerating toward the spindle? Is there another component?

BRAKING



SPEEDING UP



Vote 1 — Is the bead still accelerating toward the spindle while braking?

Yes / No / Only until it slows down. Say why.



Vote 2 — Is there now a second component? Which way does it point, and what does it change?

Along the path, backward; it changes the SPEED. The inward one still changes the DIRECTION.

Observe

Idealize

Model

Solve / Analyze

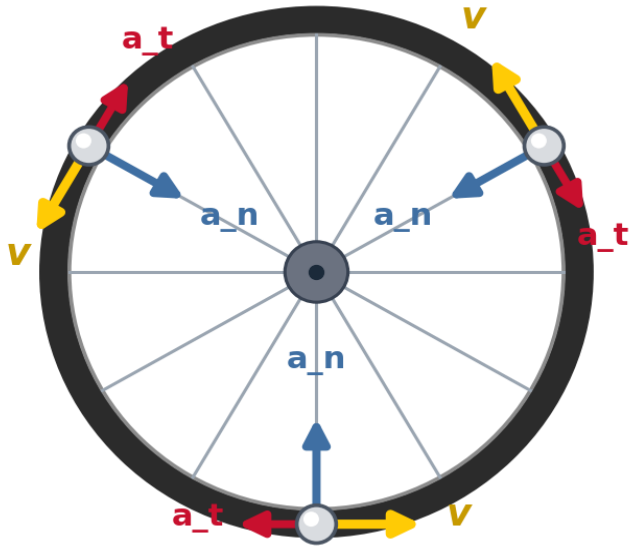
Validate

Report



$$a_t = r\alpha \quad (\text{backward while slowing}) \text{ and}$$

$$a_n = \omega^2 r \quad (\text{inward, as long as the wheel turns})$$



braking: $a_t = r\alpha$ (backward), $a_n = \omega^2 r$ (inward)

CIRCULAR PATH, RADIUS r , ANGULAR RATE $\omega(t)$

$$v_{sp} = \omega r$$

$$a_t = dv_{sp}/dt = r\alpha, \quad a_n = v_{sp}^2/r = \omega^2 r$$

$$\vec{a} = r\alpha \hat{u}_t + \omega^2 r \hat{u}_n \quad |\mathbf{a}| = r\sqrt{\alpha^2 + \omega^4}$$

$\alpha = d\omega/dt$. Braking: $\alpha < 0$ in the chosen positive sense, so a_t points backward along the path. a_n stays inward and shrinks as ω falls.



Which component changes the speed?

$a_t = r\alpha$ — set by how hard the brake acts.



Which changes the direction?

$a_n = \omega^2 r$ — set by how fast the wheel is turning right now, not by the brake.



Finite differences on the video: the derivative concept, done by hand

MEDIA PLACEHOLDER

[INSERT $\omega(t)$]

from successive rotation times, 120
fps

MEDIA PLACEHOLDER

[INSERT $\alpha(t)$]

$$\alpha \approx \Delta\omega / \Delta t$$

MEDIA PLACEHOLDER

[INSERT $a_t(t)$]

$$a_t = r\alpha$$

MEDIA PLACEHOLDER

[INSERT $a_n(t)$]

$$a_n = r\omega^2$$

FINITE DIFFERENCES — THE DERIVATIVE BY HAND

$$\omega_k \approx 2\pi / \Delta t_k \quad \alpha \approx (\omega_{k+1} - \omega_k) / \Delta t$$

$$a_t = r\alpha, \quad a_n = r\omega^2, \quad |a| = r\sqrt{\alpha^2 + \omega^4}$$

Illustrative: ω 7.85 $\rightarrow$ 3.9 rad/s in 6 s: $\alpha \approx -0.66$ rad/s², $a_t \approx -0.22$ m/s², while a_n falls from ≈ 20 to ≈ 5 m/s².



Read the curves, do not just make them

Why does a_n fall faster than ω ? (ω^2 .) If α is roughly flat, what does that say about the braking torque? If it is not flat — what changed: the pressure, the bearing friction, the temperature of the rubber?

Observe



Idealize



Model



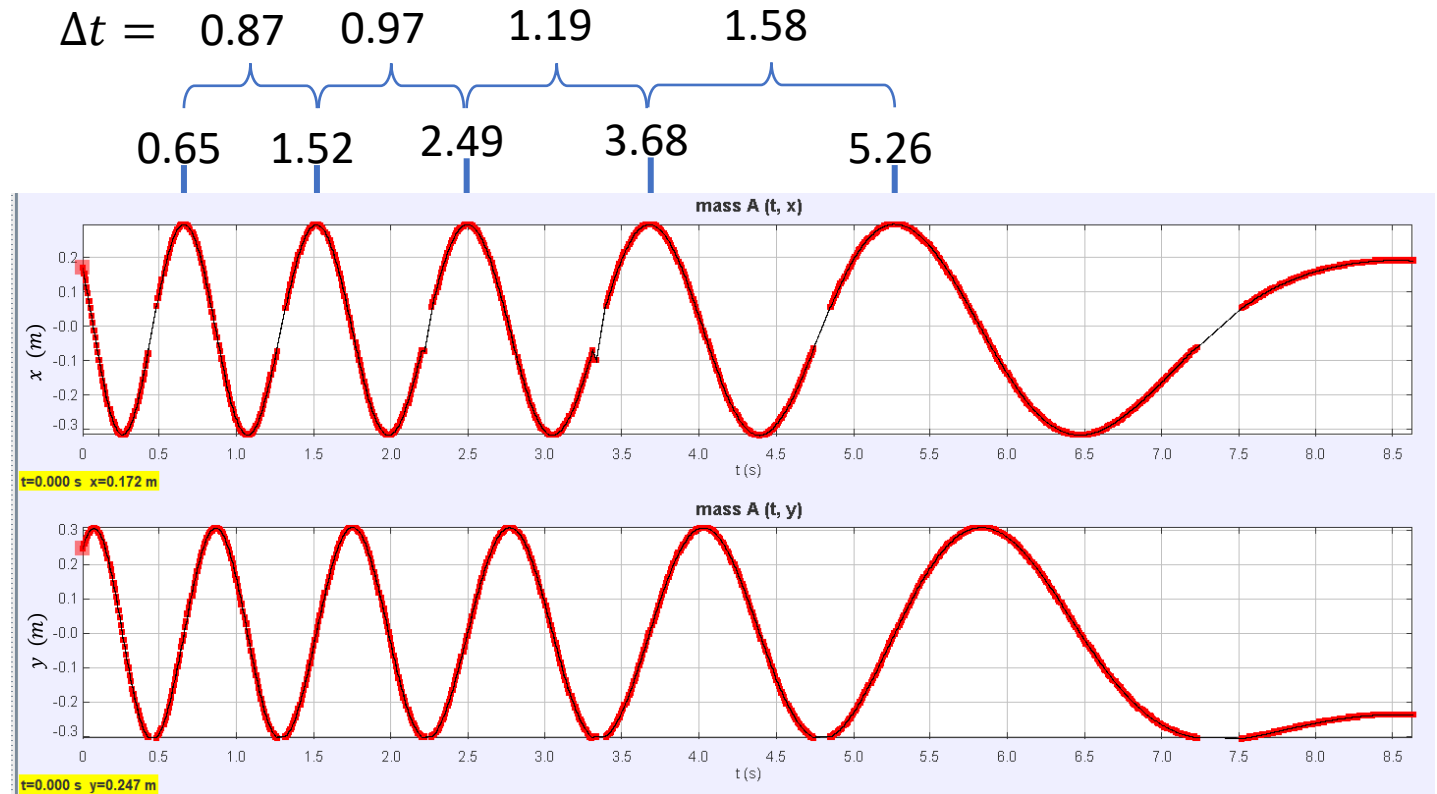
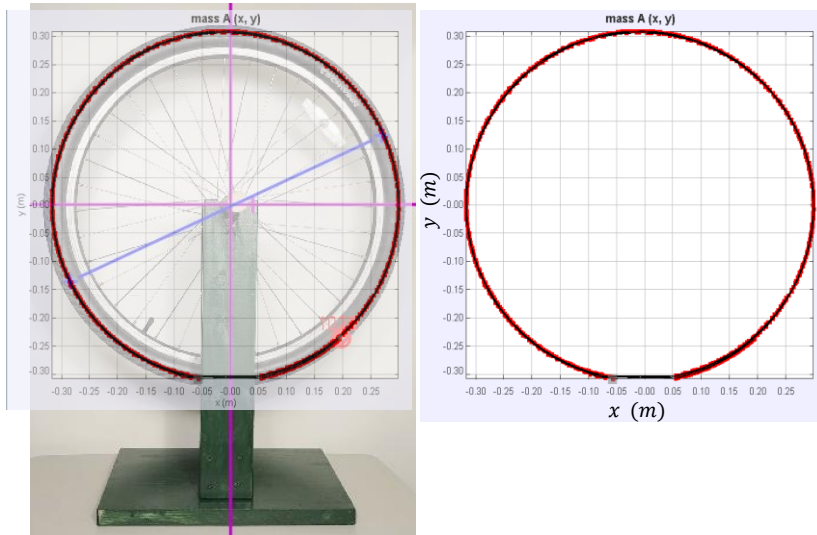
Solve / Analyze

**Validate**

Report



Tracker analysis of the braking video (120fps)



Same path, different motion. The circle does not change; the speed does. n-t separates the two.



Independent routes. Rotation count, Tracker, model. Agreement inside the ± 1 -frame band validates all three.



Uncertainties. Bead center (± 1 mm on r), radius, ± 1 frame per rotation, wobble, camera not perpendicular.

Observe



Idealize



Model



Solve / Analyze



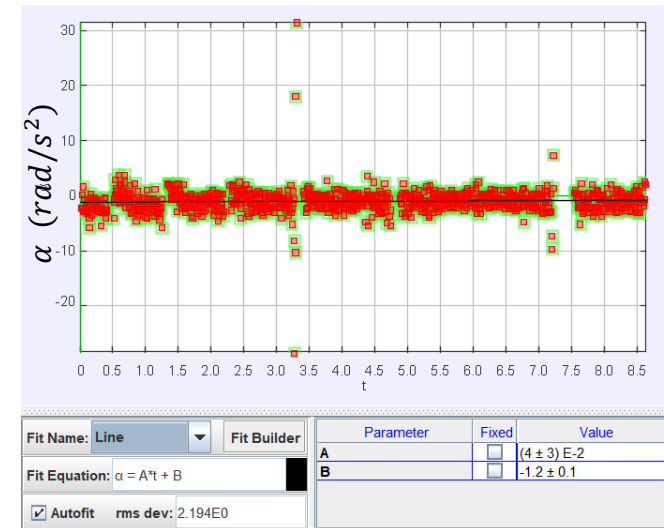
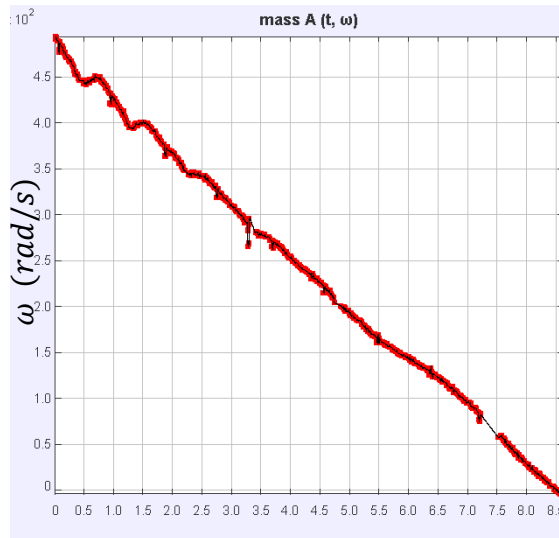
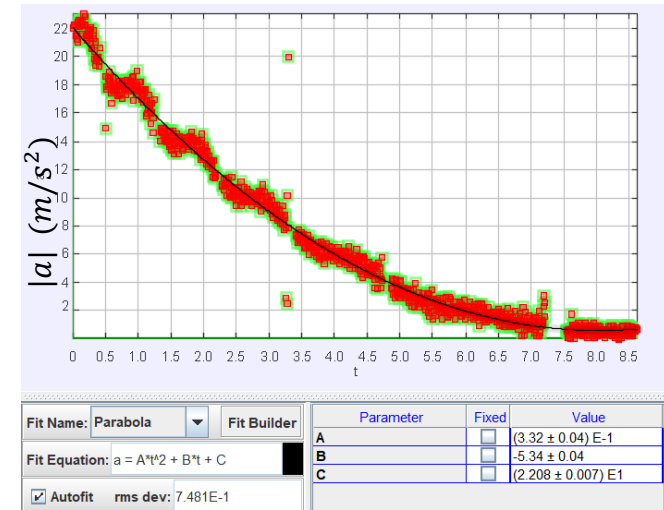
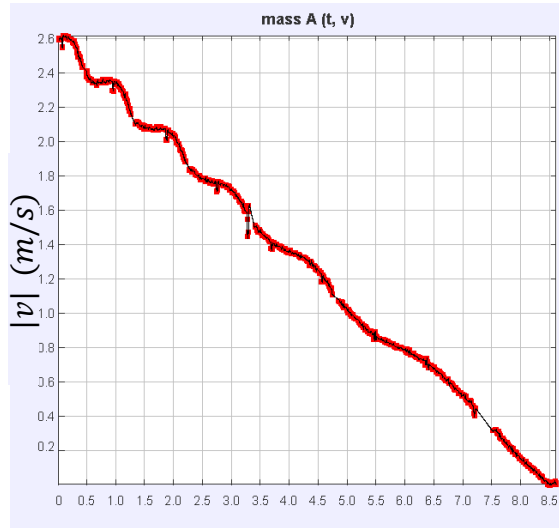
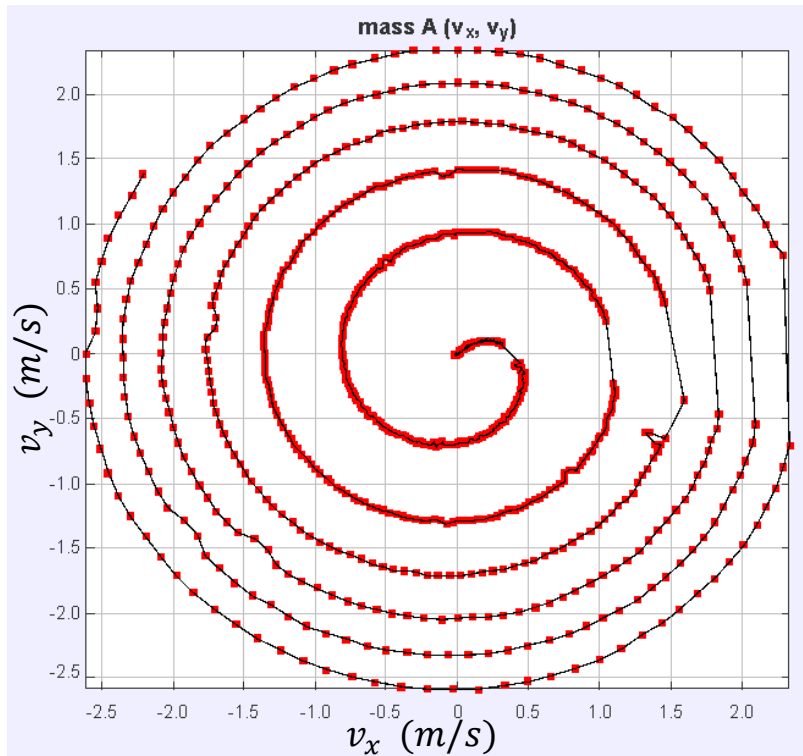
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Idealize



Model



Solve / Analyze



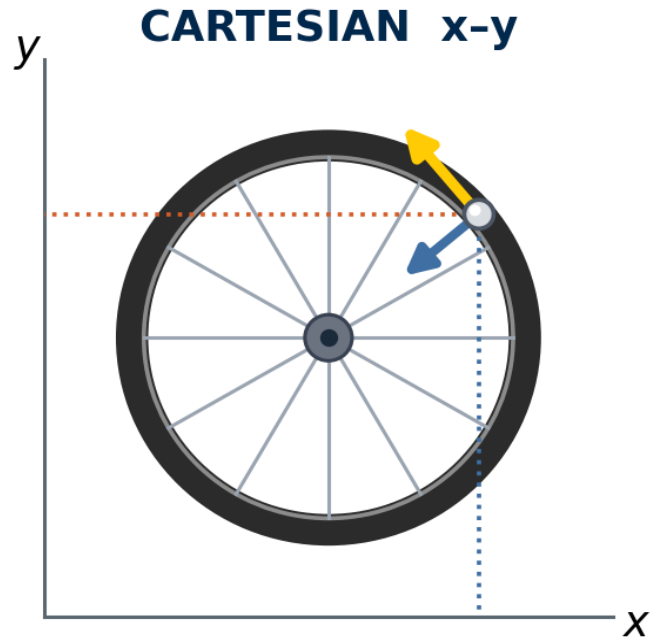
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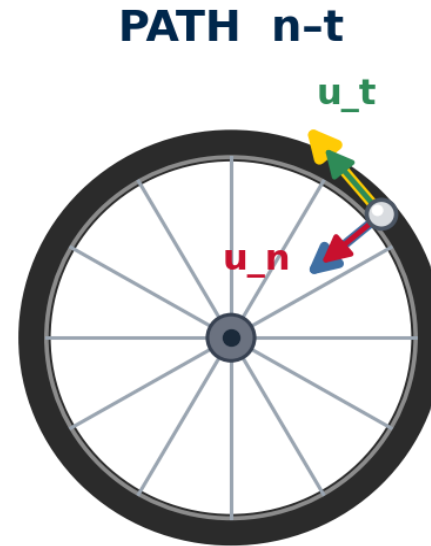
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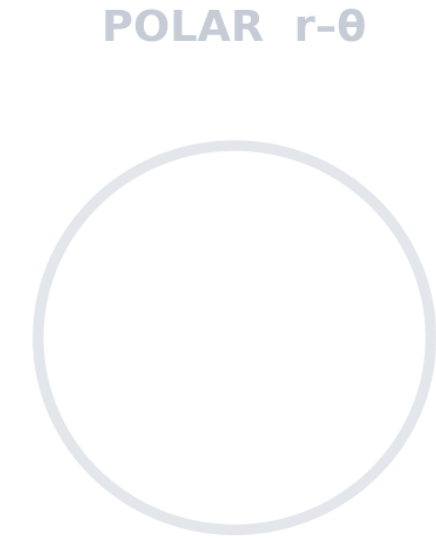
The bead in n-t language: the same v and a , now split into "speed" and "direction"



$$v = \dot{x} i + \dot{y} j \quad a = \ddot{x} i + \ddot{y} j$$



$$v = v u_t \quad a = a_t u_t + (v^2/\rho) u_n$$



Cartesian components hid the split. n-t hands it to you: a_t tells you about the brake, a_n about the turn. Same acceleration vector.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



ROLLER COASTER · SAME RIDER, CHANGING CURVATURE

Look at the
track—not the
equations.?

Where is the speed
high?

Where is the curvature
tight?



Observe

Idealize

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CLOVERLEAF HIGHWAY EXCHANGE

Why can't every part of this interchange be driven at the same speed?

What does the curvature
of the roadway have to
do with it?

A car driving 70 mph on the
straight highway, enters one of
these loops. Why does it slow
down?



Observe

Idealize

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Report



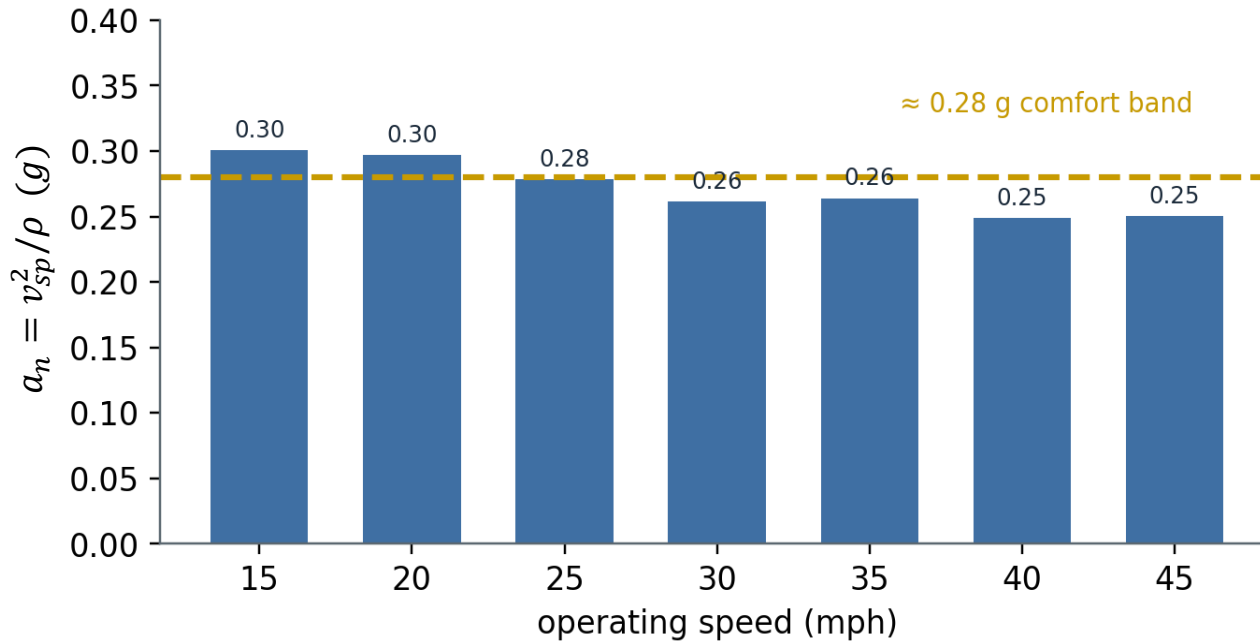
PREDICT

**Two highway curves:
25 mph on a 150 ft radius, or
45 mph on a 540 ft radius.
Which produces the larger sideways acceleration — and by
how much?**





Every entry lands near 0.25–0.30 g . The table was designed around the normal acceleration.



DESIGN RULE HIDDEN IN THE TABLE

$$\rho_{min} = \frac{v_{sp}^2}{a_{allow}}$$

with $a_{allow} \approx 0.25\text{--}0.30\text{ g}$ for comfort and grip. Double the speed $\rightarrow$ four times the radius. That is why freeway ramps are so much wider curvature than city corners.



Extend it

At 70 mph (103 ft/s) and 0.25 g , $\rho_{min} \approx 1300$ ft. Sketch that against a football field.

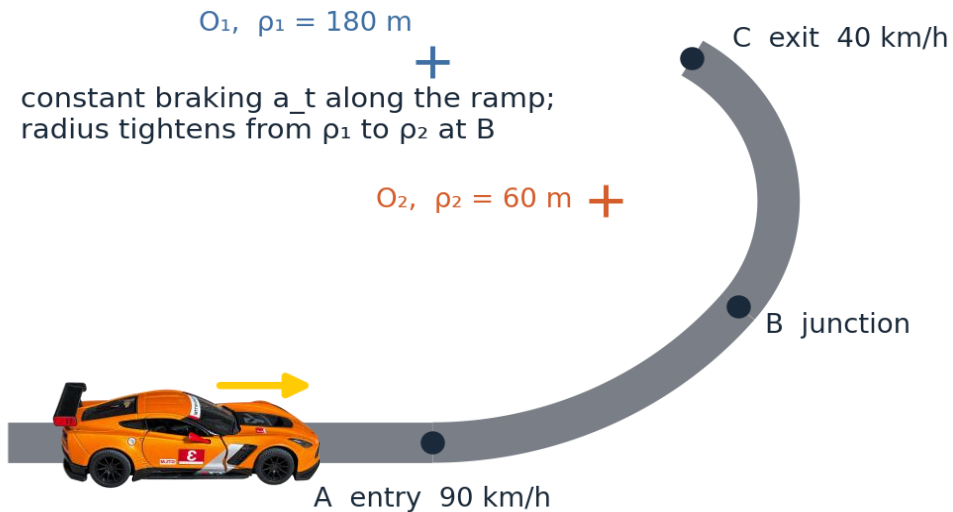


AI as Critic

"I claim the table keeps a_n constant. What else could set a minimum radius — sight distance? superelevation?"* Which of its points are kinematic?



Entry at 90 km/h, exit at 40 km/h, constant braking. The radius tightens from 180 m to 60 m. Is the ride within the comfort limit?



Modeling decision stated first: "the path is known, so I describe the motion along it and across it."

THE MODEL

Particle	the vehicle's mass center
Description	n-t: the path is known from the map; speed and curvature are what matter (Cartesian would need a curvature calculation at every point)
Path	arc AB: $\rho_1 = 180 \text{ m}$, 50° (157 m); arc BC: $\rho_2 = 60 \text{ m}$, 100° (105 m); $s_{\text{total}} = 262 \text{ m}$
Knowns	$v_{sp_A} = 90 \text{ km/h} = 25 \text{ m/s}$ $v_{sp_C} = 40 \text{ km/h} = 11.1 \text{ m/s}$ a_t constant (controller plan)
Assumptions	planar, level ramp; particle; the radius changes abruptly at B
Unknowns	a_t ; v_{sp_B} ; $a_n, a $ at A, B^- , B^+ , C; comfort verdict ($ a \leq 0.3 g \approx 2.9 \text{ m/s}^2$)



Constant braking fails at the junction: a_n jumps from 1.8 to 5.4 m/s² when the radius tightens

SOLVE

$$a_t = (v_{spC}^2 - v_{spA}^2)/(2s) = (123 - 625)/524 = -0.96 \text{ m/s}^2$$

$$v_{spB}^2 = v_{spA}^2 + 2a_t s_{AB} = 625 - 301 \rightarrow v_{spB} = 18.0 \text{ m/s}$$

$$a_n = \frac{v_{sp}^2}{\rho} \Rightarrow @A \ 3.5 \quad @B^- \ 1.8 \quad @B^+ \ 5.4 \quad @C \ 2.1 \quad (\text{m/s}^2)$$

$$|\vec{a}|_{B^+} = \sqrt{5.4^2 + 0.96^2} = 5.5 \text{ m/s}^2 \approx 0.56 g$$

Verdict: the plan violates the 0.3 g limit at B⁺ (and marginally at A). The tangential part is tiny; the normal part dominates and JUMPS where ρ changes.

$$|a|_{B^+} = \sqrt{(5.4^2 + 0.96^2)} = 5.5 \text{ m/s}^2 \approx 0.56 g$$



Interpret — a controller decision

Shed speed on the wide arc: v_B must be $\leq \sqrt{a_{\text{allow}} \rho_2} = \sqrt{(2.9 \times 60)} = 13.2 \text{ m/s}$ before entering the tight arc. Constant braking is the wrong plan; front-loaded braking is right.



Validate

Units: $\text{m}^2/\text{s}^2 \div \text{m} = \text{m/s}^2$ ✓. Limit: $\rho \rightarrow \infty$ kills a_n , leaving Chapter 1 ✓. Sign: $a_t < 0$ while slowing ✓. Alternate description: Part 6 re-solves the instant at B in x-y and must get the same $|a|$.



Is a number believable? And what must supply a_n ?



Centrifugal launcher — 8,000 g?

A proposed launcher spins a payload on $\rho \approx 50 \text{ m}$ arm to a tip speed of $\approx 2 \text{ km/s}$, then releases it.

$$a_n = v_{sp}^2 / \rho = (2000)^2 / 50 = 80\,000 \text{ m/s}^2 \approx 8\,000 \text{ g}$$

Order-of-magnitude check: is that believable? What does it mean for what can be launched? (Numbers to be re-verified against the current public specification.)

AI as Validator: "Check my a_n for a 50 m arm at 2 km/s and state the limiting case if the arm were 500 m."

<https://www.youtube.com/watch?v=TGO4LtCctTk>



Bridge to Chapter 3 — what supplies a_n ?

MEDIA PLACEHOLDER

[INSERT ORIGINAL FIGURE — MOTORCYCLE CORNERING, KINEMATICS ONLY]

v tangent, a_n toward the inside of the turn, ρ marked. No force arrows yet.

A motorcycle leaning through a corner and a trapeze artist at the bottom of a swing both have $a_n = v^2/\rho$ toward the center.

We can now compute it. Chapter 3 asks what force supplies it — tire friction, rope tension — and what happens when it cannot.



Part 4 — what you can now explain and do



You can now EXPLAIN

- Why a unit vector that rotates has a derivative, and why that derivative is $(v/\rho) \hat{u}_n$
- What a_t and a_n each do physically — speed change vs direction change — and why a_n is a magnitude with direction $\hat{u}_n$
- Why constant speed on a curve still requires acceleration, with three kinds of evidence: the golf ball, the wheel data, and the equation
- Why n–t is a LOCAL description (the bent wire) and when it is the natural choice (path known; speed and curvature matter)



You can now DO

- Compute a_t , a_n , $|\vec{a}|$ and its direction for circular and non-circular paths, from $v(t)$, $v(s)$, or constant a_t
- Measure ω from video by counting rotations, form α by finite differences, and turn both into a_t and a_n with uncertainties
- Read a design table through $a_n = v^2/\rho$ and size a minimum radius for an allowable acceleration
- Decide where a vehicle must brake on a path whose radius changes — and validate by units, limits, and an alternate description

Next — Part 5: when motion is naturally seen from a POINT (range and bearing), a third description is simplest. The wheel gives its limiting case in one line.



Curvature of a graph in three lines

FOR A PATH $y = f(x)$, PARAMETRIZED BY x

$$\tan \theta = y' \rightarrow \sec^2 \theta d\theta = y'' dx \rightarrow d\theta/dx = y''/(1 + y'^2)$$

$$ds = \sqrt{1 + y'^2} dx$$

$$1/\rho = d\theta/ds = (d\theta/dx)/(ds/dx) = y'' / (1 + y'^2)^{3/2}$$

$$\rho = (1 + y'^2)^{3/2} / |y''|$$

Checks:

Straight line $(y'' = 0) \rightarrow \rho = \infty$.

Circle $x^2 + y^2 = R^2$ at the top: $y' = 0, y'' = -1/R \rightarrow \rho = R$ ✓.



Use it once, then trust it

Homework will ask for ρ at one point on a known parabola. Compute y' and y'' by hand; let the algebra be checked by an AI validator.



Space curves, the osculating plane, and the binormal



Osculating plane

The plane containing $\hat{u}_t$ and $\hat{u}_n$ at a point of a space curve. The path's local circle (defining the curvature) lies in it.



Binormal $\hat{u}_b = \hat{u}_t \times \hat{u}_n$

Completes a right-handed t-n-b frame. Nothing moves along it: v and a both lie in the osculating plane.



What changes in 3-D

The osculating plane itself rotates as you move along the curve (a helix: constantly). The equations for a_t and a_n are unchanged.

GOLF BALL SLIDING ON A WIRE

Consider the ball sliding down the wire at a constant speed. Would the direction of the binormal $\hat{u}_b$ change; as it moves from A to B to C? How?

