

AI-AUGMENTED ENGINEERING DYNAMICS: A CONVERSATIONAL APPROACH

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

by Taner Onsay

CHAPTER - 2

PART 5 · ≈ 70 MIN

Polar Coordinates

When motion is naturally seen from a point — range and bearing — a third description is simplest. Its extra terms have physical meanings, and the wheel gives its limiting case.

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As the vehicle moves, how must r and θ change for the robot to track the weld point?



Radial: how far from the robot base — r

Distance from the fixed robot base O to the current target point P.



Transverse: in which direction — θ

Angular position of the target line from a fixed reference.

As the bucket moves outward while the boom rotates, how do r and θ describe its motion?



Radial motion — $r(t)$

The bucket moves toward or away from the boom pivot.



Angular motion: — $\theta(t)$

The boom changes the direction of the bucket.

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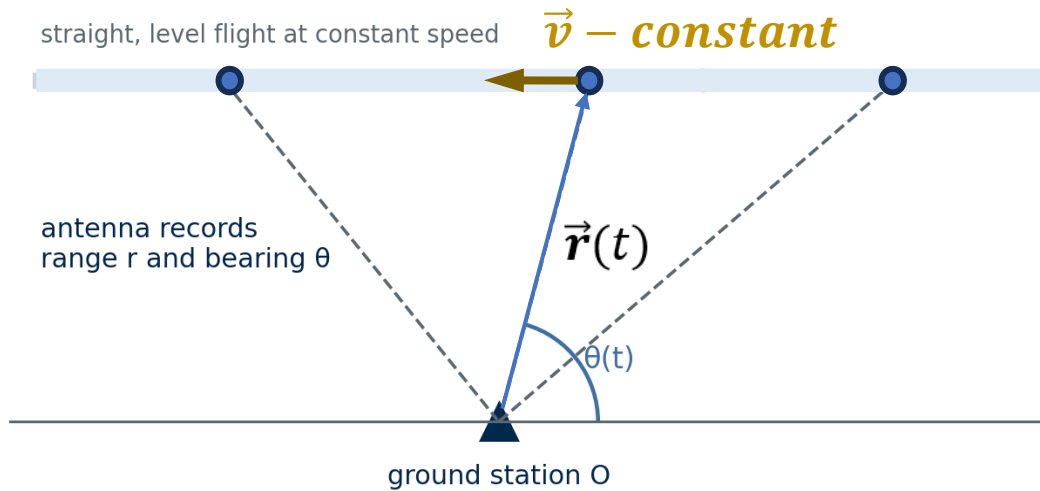
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Range and bearing. Not x and y — "how far" and "in which direction" from a fixed point.



Radial: how far from the point — r

Measured along the line of sight, outward from the fixed origin O.



Transverse: which direction — θ

The bearing angle of that line, measured from a fixed reference direction.

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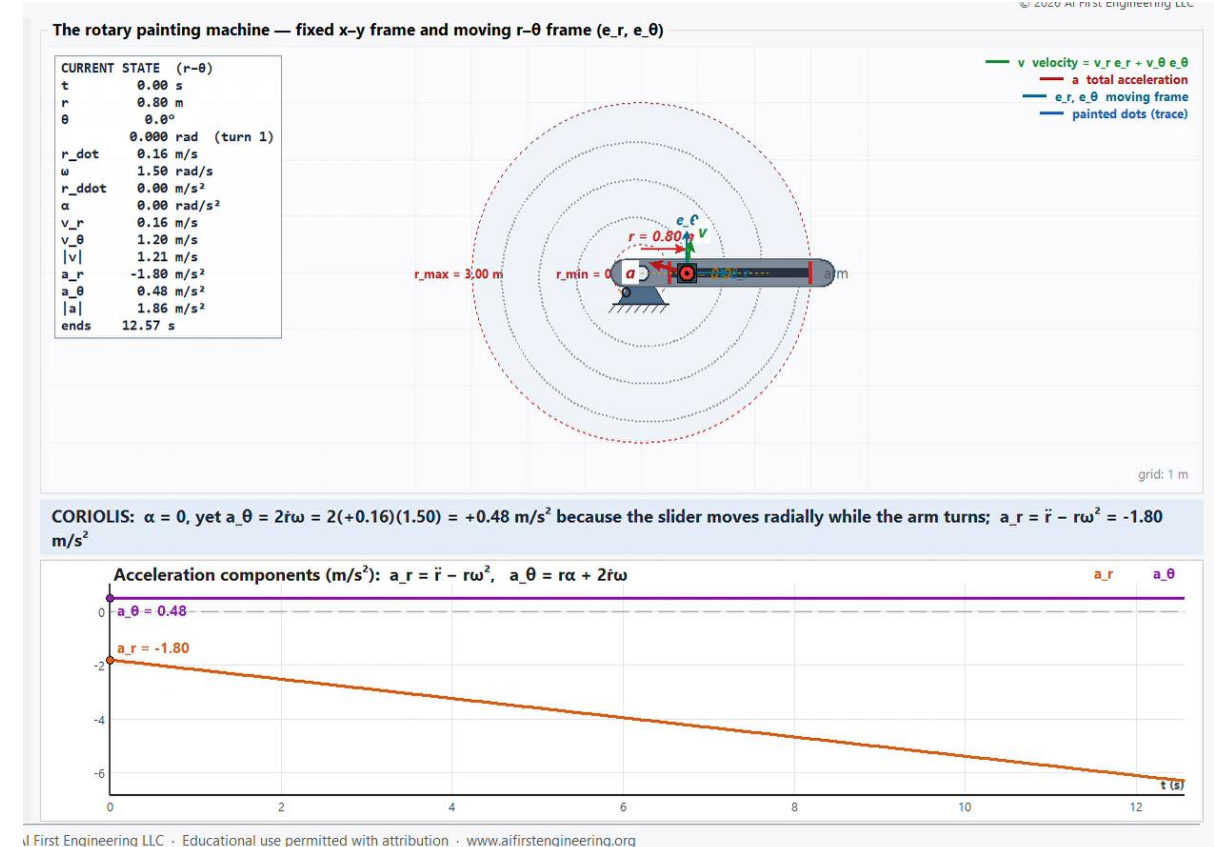


$\hat{u}_r$ points away from O; $\hat{u}_\theta$ is perpendicular: r and/or θ may vary in time:
Slotted arm with a moving peg painting dots:

CIRCULAR PATH (Video)



SPIRAL PATH (Video)



Observe



Idealize



Model



Solve / Analyze



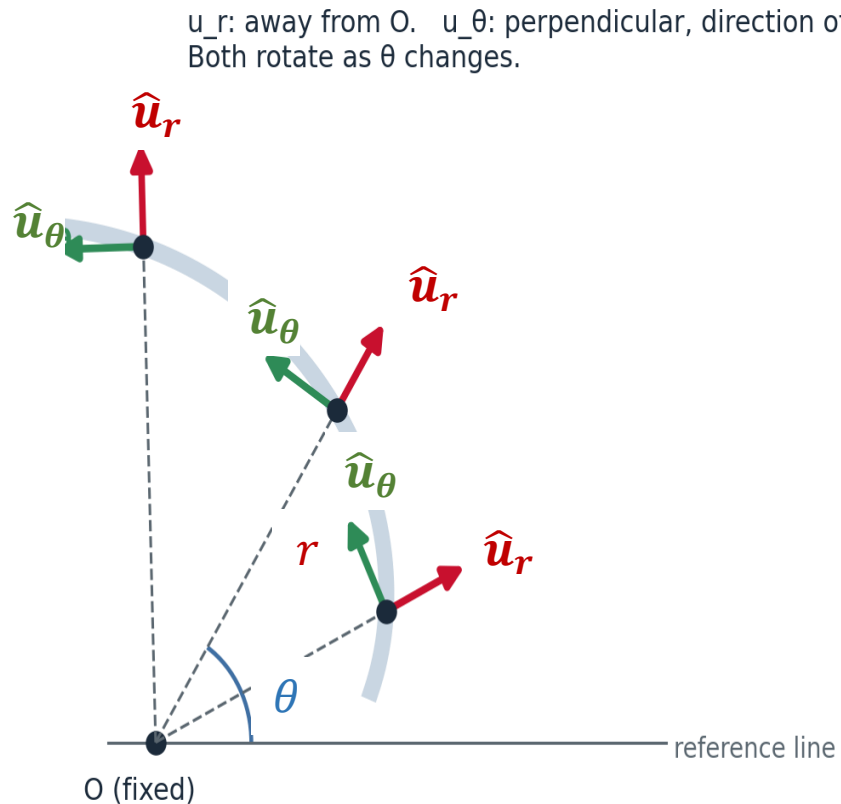
Validate



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$\hat{u}_r$ points away from O; $\hat{u}_\theta$ is perpendicular, in the direction of increasing θ . Both turn as θ changes.



THE POLAR DESCRIPTION

- Origin** fixed point O (the antenna, the spindle, the pivot)
- r** distance from O to the particle, along the line of sight
- θ** angle of that line from a fixed reference direction
- $\hat{u}_r$** unit vector from O toward the particle — depends on θ
- $\hat{u}_\theta$** unit vector $\perp \hat{u}_r$, in the direction of increasing θ — depends on θ

Position $\vec{r} = r \hat{u}_r$ (one term — $\hat{u}_r$ already aims at the particle)



Three descriptions, three kinds of axes

Cartesian: fixed origin, fixed axes.

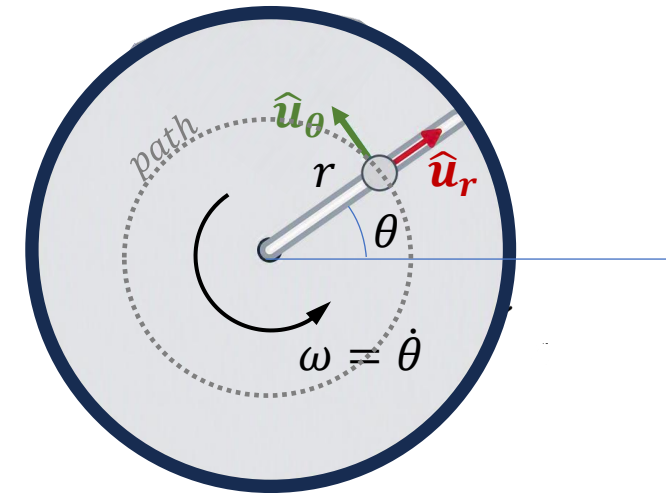
n-t: moving origin, axes follow the path.

Polar: fixed origin, axes follow the line of sight.



PREDICT

The turntable spins at a steady rate. A ball sits **still** in the groove **at a fixed radius**.
Is the ball accelerating? Which way?



You have seen this question before — in Part 1 and Part 4. Answer it in polar words this time.

Vote: no · yes, outward (it wants to fly off) · yes, inward · yes, along the groove's motion.

Observe

Idealize

Model

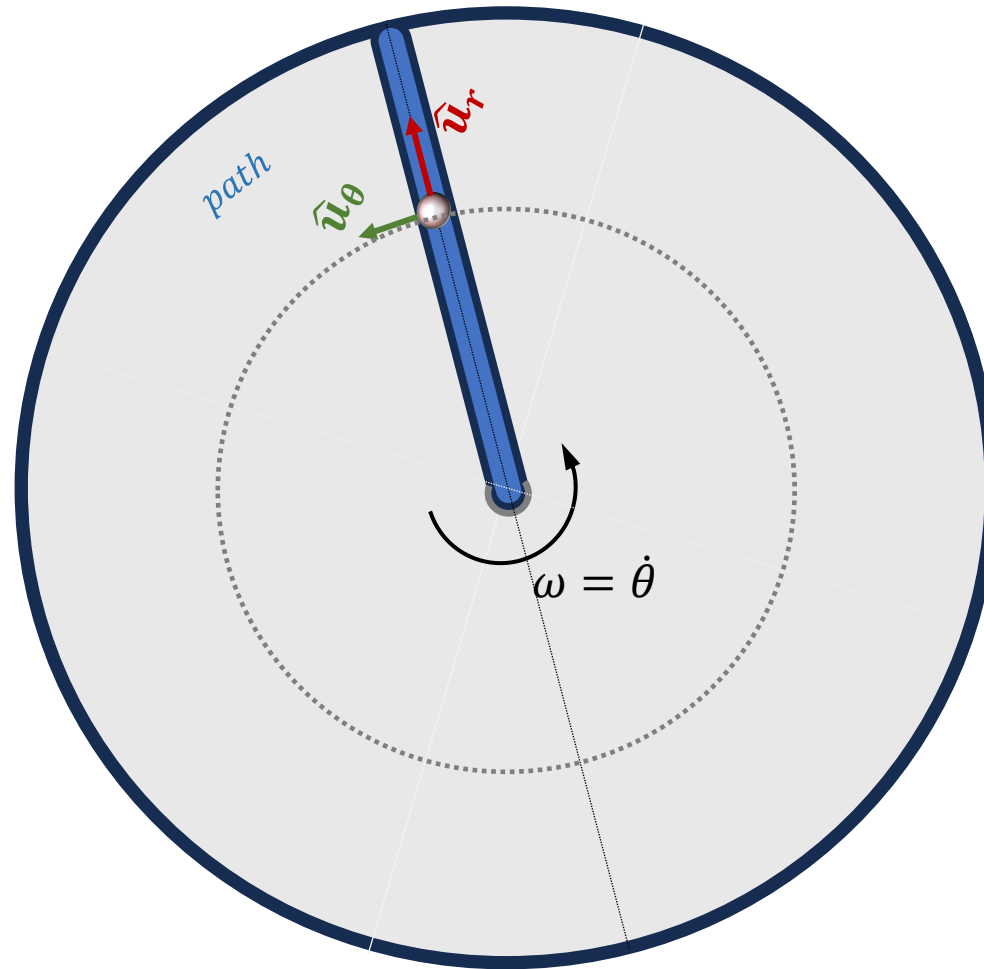
Solve / Analyze

Validate

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PREDICT



Observe

Idealize

Model

Solve / Analyze

Validate

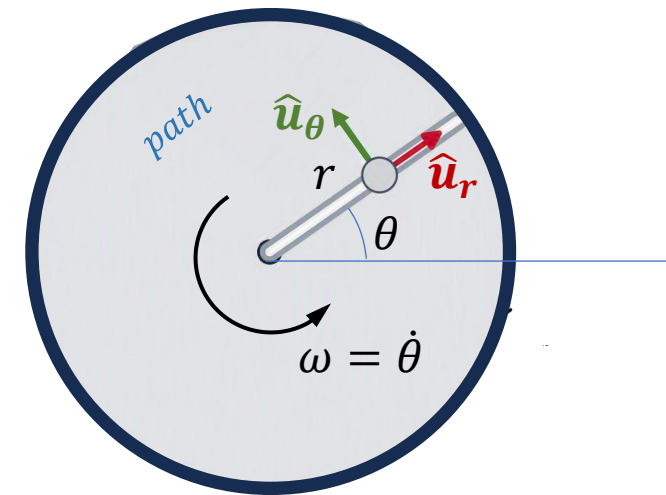
Report



PREDICT



Now the ball slides **OUTWARD** along the groove at a steady rate while the table keeps its steady spin.
Is there any acceleration **ALONG** the groove's direction of motion?

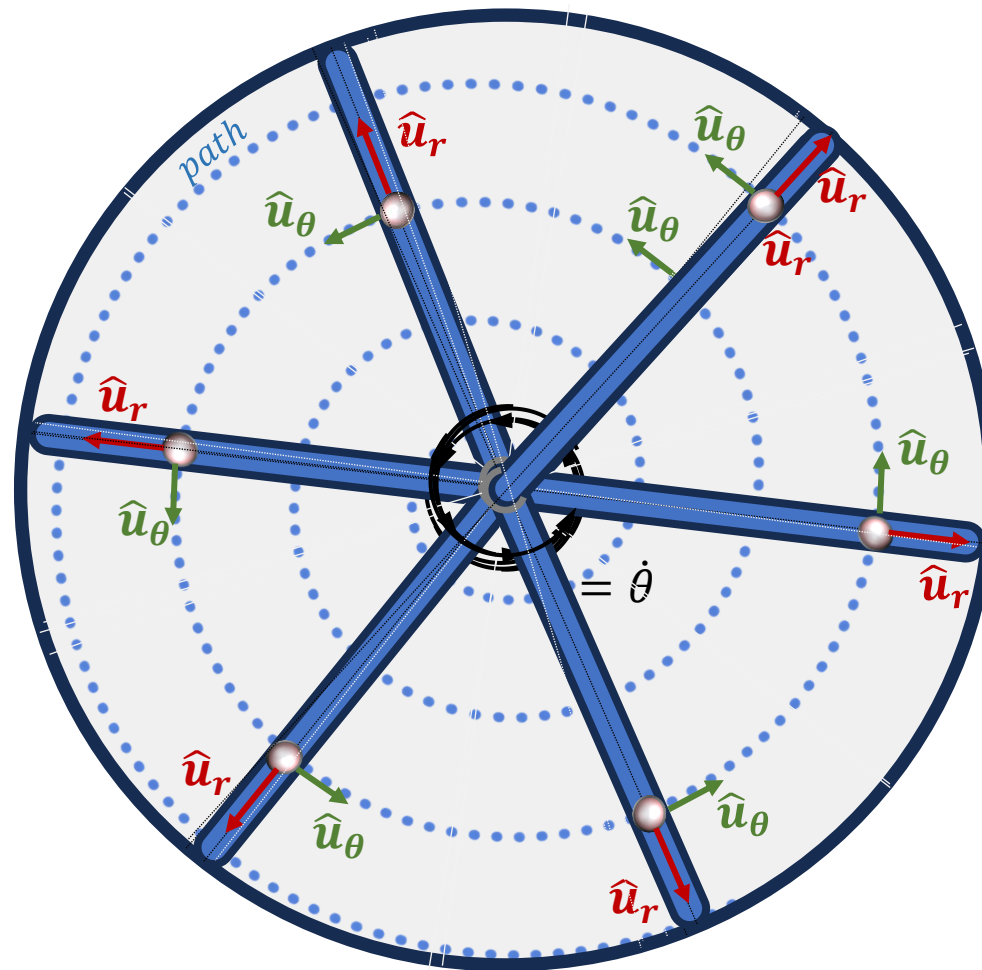


$\dot{\theta}$ is constant. Commit before the derivation.

Vote: no — $\dot{\theta}$ is constant · yes, forward · yes, backward · only if $\dot{r}$ changes.



ANIMATION



Observe

Idealize

Model

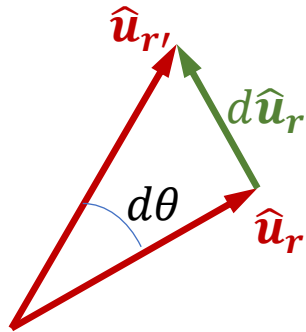
Solve / Analyze

Validate

Report

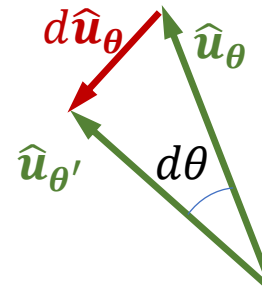


$\dot{\hat{u}}_r = \dot{\theta} \hat{u}_\theta$ and $\dot{\hat{u}}_\theta = -\dot{\theta} \hat{u}_r$. Same picture as Part 4.



$$|d\hat{u}_r| = (1)d\theta, \quad \text{direction } \hat{u}_\theta$$

$$\frac{d\hat{u}_r}{dt} = (1)\frac{d\theta}{dt} \hat{u}_\theta \rightarrow \dot{\hat{u}}_r = \dot{\theta} \hat{u}_\theta$$



$$d\hat{u}_\theta \text{ points along } -\hat{u}_r$$

$$\frac{d\hat{u}_\theta}{dt} = \frac{d\theta}{dt} \hat{u}_r \rightarrow \dot{\hat{u}}_\theta = -\dot{\theta} \hat{u}_r$$

THE TWO FACTS

$$\dot{\hat{u}}_r = \dot{\theta} \hat{u}_\theta$$

$$\dot{\hat{u}}_\theta = -\dot{\theta} \hat{u}_r$$

Both rotate at the rate $\dot{\theta}$. Neither changes length.

Part 4, one sentence: a unit vector cannot change length, so its derivative is perpendicular to it, with magnitude equal to its rotation rate. Here the rotation rate is $\dot{\theta}$.



$\vec{v} = \dot{r} \hat{u}_r + r\dot{\theta} \hat{u}_\theta$ — two terms, two physical meanings

DIFFERENTIATE $\vec{r} = r \hat{u}_r$

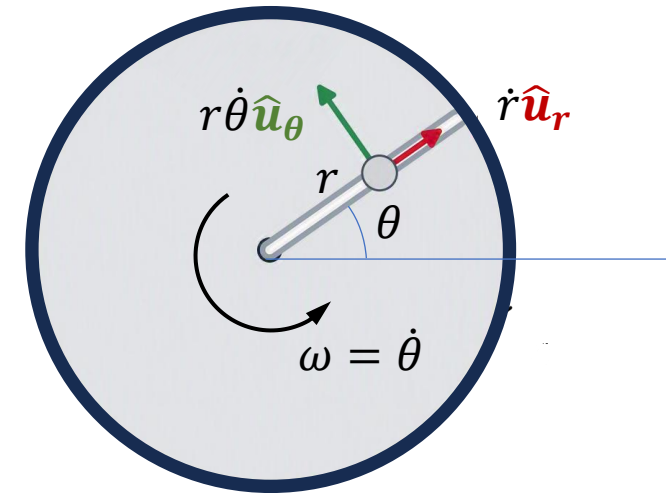
$$\vec{r} = r \hat{u}_r$$

$$\vec{v} = \dot{r} \hat{u}_r + r\dot{\theta} \hat{u}_\theta$$

$v_r = \dot{r}$: rate at which the range changes (moving toward or away from O).

$v_\theta = r\dot{\theta}$: the line of sight sweeping around, carrying the particle sideways.
Faster far out ($\propto r$).

$|\vec{v}| = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$. The velocity is still tangent to the path — the path is just not along $\hat{u}_r$ or $\hat{u}_\theta$.



Read the turntable

Slide outward: $\dot{r} \hat{u}_r$. Table sweeping: $r\dot{\theta} \hat{u}_\theta$.
The ball's velocity is the vector sum —
diagonal across the groove.



Which term vanishes...

...if the ball sits still in the groove? ($\dot{r} = 0$)
...if the table is stopped? ($\dot{\theta} = 0$)



Differentiate each velocity term; group radial and transverse; name every piece

$$\vec{v} = \dot{r} \hat{u}_r + r \dot{\theta} \hat{u}_\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \underbrace{(\ddot{r} - r \dot{\theta}^2)}_{\text{Radial Component}} \hat{u}_r + \underbrace{(r \ddot{\theta} + 2 \dot{r} \dot{\theta})}_{\text{Transverse Component}} \hat{u}_\theta$$

RADIAL COMPONENT — a_r

$$a_r = \ddot{r} - r \dot{\theta}^2$$

$(\ddot{r})$ — the range itself speeding up or slowing (moving in or out faster)

$(-r \dot{\theta}^2)$ — direction change seen from O: the inward (**centripetal**) term. Present even at constant r and constant $\dot{\theta}$.

a_θ — TRANSVERSE COMPONENT

$$a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta}$$

$(r \ddot{\theta})$ — the rotation rate changing (the turntable speeding up or braking)

$(2 \dot{r} \dot{\theta})$ — moving outward WHILE rotating: two equal contributions of $(\dot{r} \dot{\theta})$

COMBINE

$$\vec{a} = a_r \hat{u}_r + a_\theta \hat{u}_\theta \quad |\mathbf{a}| = \sqrt{a_r^2 + a_\theta^2}$$

Four terms, four physical situations. Do not memorize the line — rebuild it from $\dot{\hat{u}}_r = \dot{\theta} \hat{u}_\theta$ and $\dot{\hat{u}}_\theta = -\dot{\theta} \hat{u}_r$ in two minutes whenever you need it (Appendix D shows every step).



$2 \dot{r} \dot{\theta}$ —two ways to see the same term

1

The transverse speed grows

$v_{\theta} = r\dot{\theta}$. Sliding outward makes r bigger, so v_{θ} gets bigger even though $\dot{\theta}$ does not. Rate of growth: $\dot{r} \dot{\theta}$.

2

The radial velocity rotates

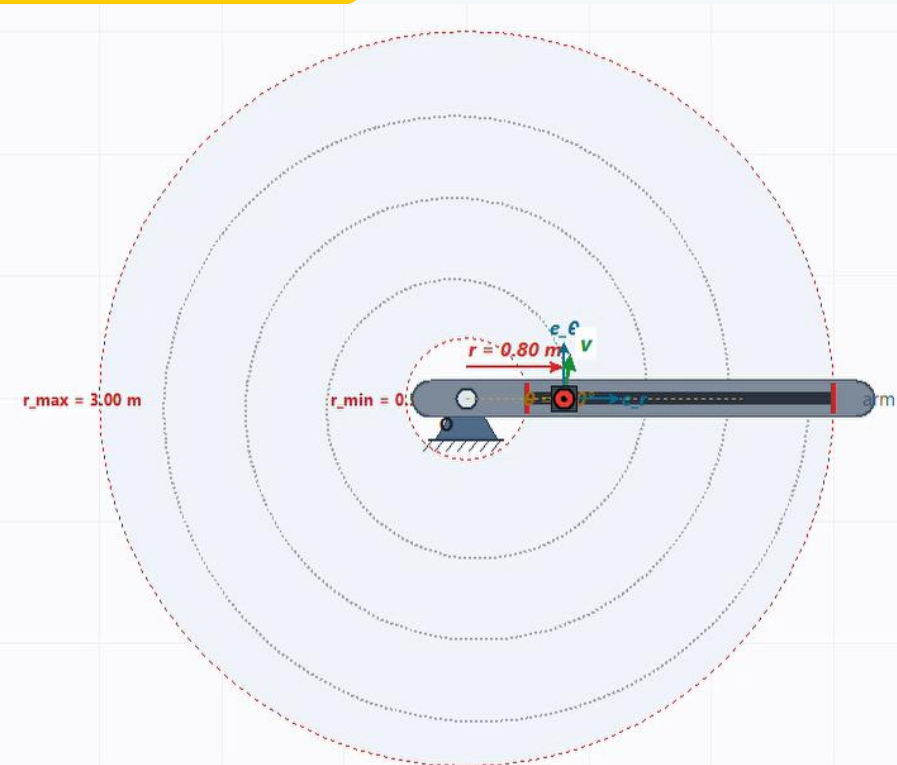
$\dot{r}\hat{u}_r$ is a vector of fixed length turnings with the table; with a derivative: $\dot{r} \dot{\theta} \hat{u}_{\theta}$. Same size again: $\dot{r} \dot{\theta}$.

ADD THEM

$$a_{\theta} = \dot{r} \dot{\theta} + \dot{r} \dot{\theta} = 2\dot{r} \dot{\theta} \quad (\text{with } \ddot{\theta} = \alpha = 0)$$

Forward, in the direction of rotation, for outward sliding; backward for inward sliding. Zero if either $\dot{r}$ or $\dot{\theta}$ is zero.

POLAR PAINTER:



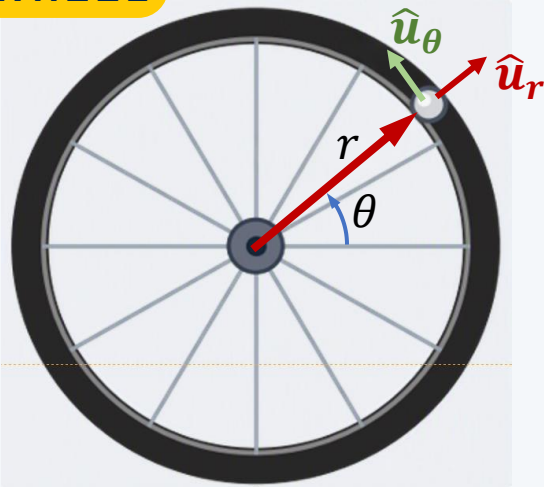


Origin at the spindle: $\dot{r} = 0, \ddot{r} = 0$ The four terms collapse to two — and they are Part 4's a_n and a_t .

BICYCLE WHEEL

$$\dot{\theta} = \omega$$

$$\ddot{\theta} = \alpha$$



$$r = \text{CONSTANT}, \quad \dot{\theta} = \omega, \quad \ddot{\theta} = \alpha$$

Recall:

ω : angular velocity
 α : angular acceleration

$$a_r = 0 - r\omega^2 = -r\omega^2 \quad a_\theta = r\alpha + 0 = r\alpha$$

Part 4 said $a_n = +r\omega^2$ toward the center and $a_t = r\alpha$ along the path. Here $a_r = -r\omega^2$ because u_r points outward: on a circle about O , $\hat{u}_n = -\hat{u}_r$ and $\hat{u}_\theta = \hat{u}_t$

SAME PHYSICS. DIFFERENT BASIS, DIFFERENT SIGN CONVENTION. The acceleration vector of the braking bead is identical in n-t and polar — only the labels changed.



Steady wheel

$\ddot{\theta} = 0 \rightarrow \vec{a} = -r\omega^2 \hat{u}_r$. Inward only.
Constant speed, nonzero acceleration — third time, third description.



Braking wheel

$\ddot{\theta} = \alpha < 0 \rightarrow a_\theta = r\alpha$ backward. The transverse polar term IS the tangential n-t term.



When would they differ?

If r changed (a bead sliding along a spoke): polar gains $\dot{r}$ and $2\dot{r}\dot{\theta}$; n-t would need the spiral's ρ .

Observe



Idealize



Model



Solve / Analyze



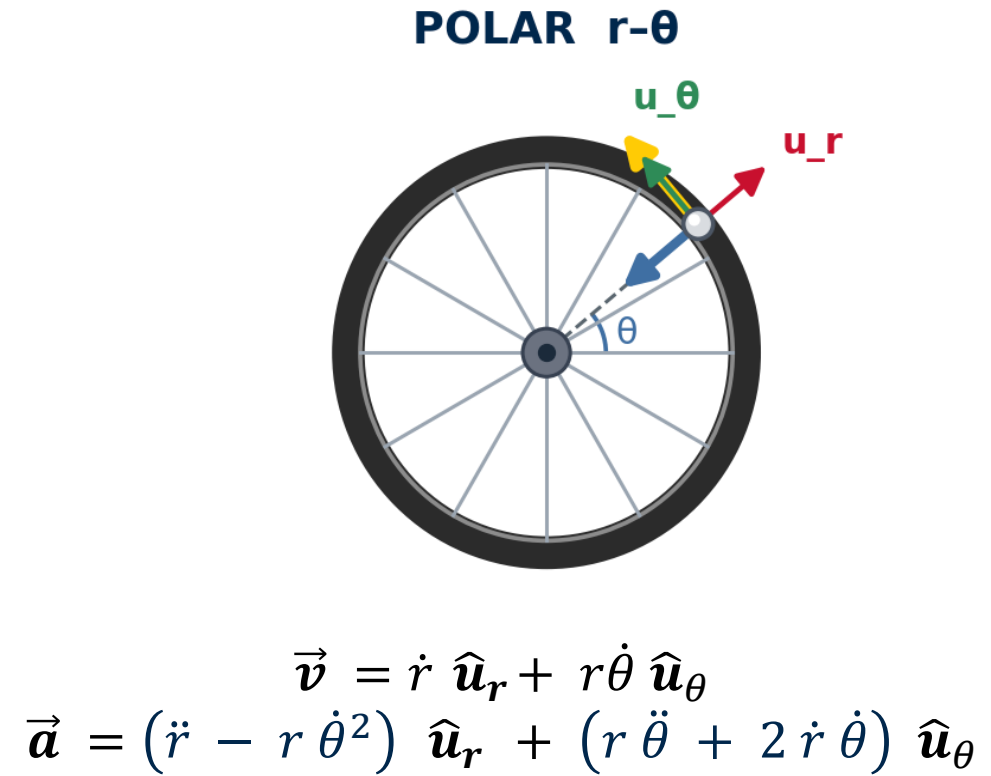
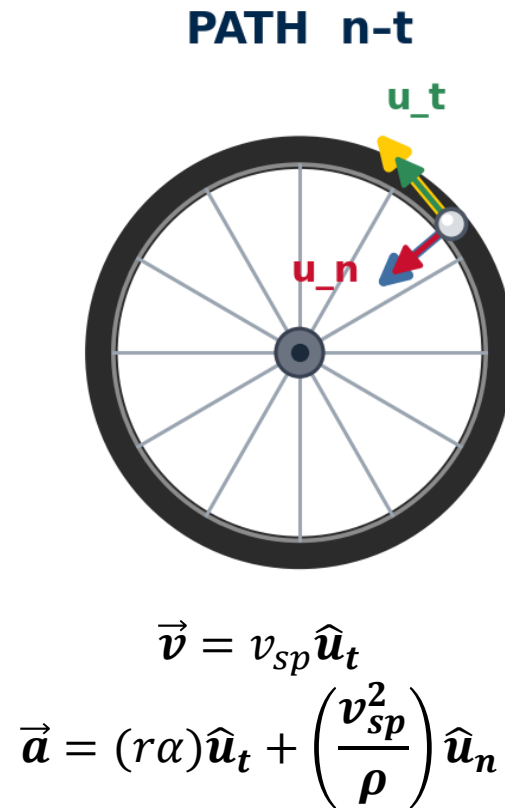
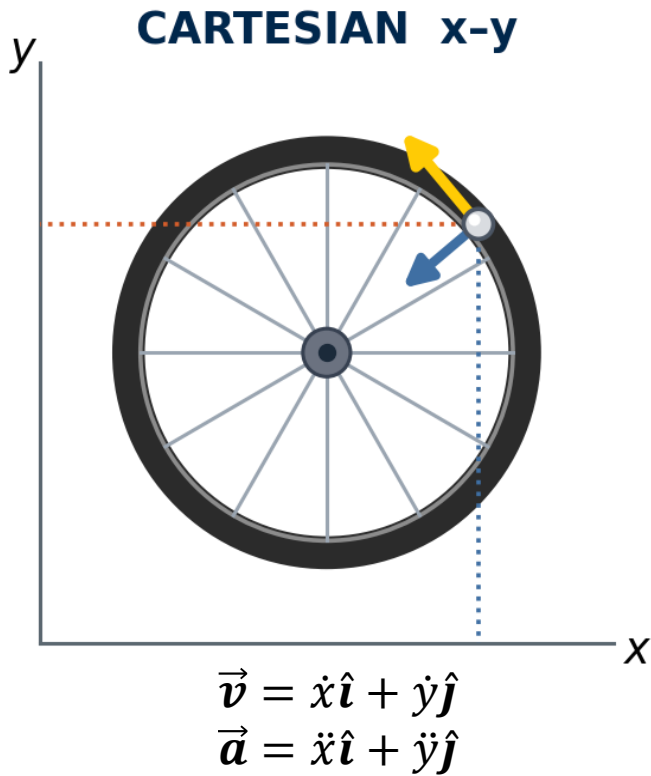
Validate



Report



The bead in polar language. Three descriptions, one velocity vector, one acceleration vector.



The motion never changed. Cartesian shows shadows; n-t shows speed vs direction; polar shows range vs bearing. Part 6 asks: which do you choose, and why?



Six numbers at the instant of interest: $r, \dot{r}, \ddot{r}, \theta, \dot{\theta}, \ddot{\theta}$. Then assemble.

1 Write what is given

$r(t)$ and $\theta(t)$ — or r as a function of θ with $\theta(t)$ given
(then $\dot{r} = \left(\frac{dr}{d\theta}\right) \dot{\theta}$: chain rule, Part 2).

2 Differentiate twice each

$\dot{r}, \ddot{r}, \dot{\theta}, \ddot{\theta}$. Keep θ in radians.

Units: r [m], $\dot{r}$ [m/s], $\ddot{r}$ [m/s²], $\dot{\theta}$ [rad/s], $\ddot{\theta}$ [rad/s²].

3 Evaluate at the instant

Six numbers. Write them in a row. This is the moment errors are easiest to catch.

4 Assemble

$$\vec{v} = \dot{r} \hat{u}_r + r \dot{\theta} \hat{u}_\theta$$

$$\vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{u}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \hat{u}_\theta$$

Then magnitudes and directions. Then a limiting-case check.



AI AS TUTOR

"Explain the $2\dot{r}\dot{\theta}$ term to me two different ways, using the turntable. Then ask me which way the ball deflects if it slides inward instead of outward." Decide whether the AI's second explanation was actually different from its first.

Observe



Idealize



Model



Solve / Analyze



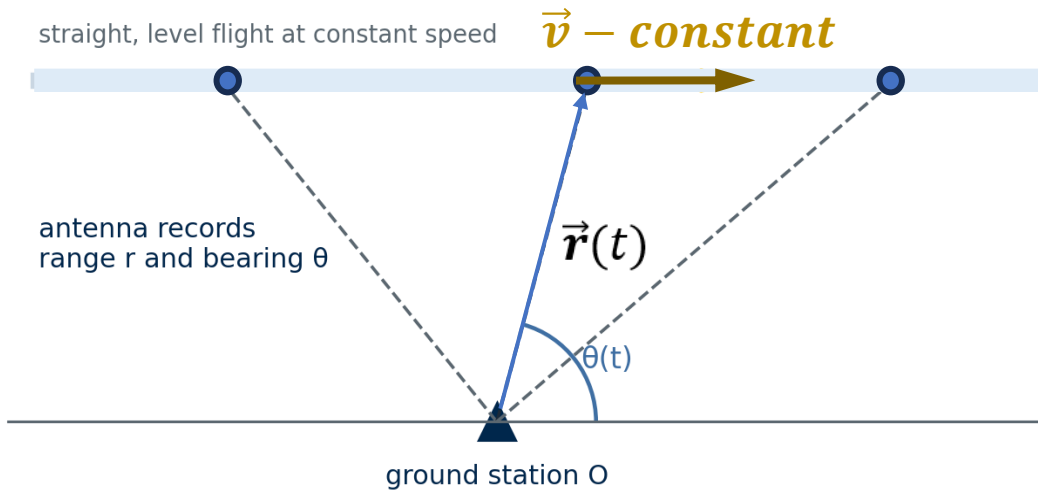
Validate



Report



The station records range and bearing; the flight is straight at constant speed. What must the polar components add up to?



Modeling decision: describe in the instrument's language (polar); validate in the flight's language (Cartesian).

THE MODEL

Particle Description	the quadcopter polar about the station O — because that is what the antenna measures
Geometry	straight, level line at perpendicular distance $d = 60 \text{ m}$ from O; constant speed $v_{sp} = 8 \text{ m/s}$
Given functions	$r(t) = \sqrt{d^2 + (v_{sp}t)^2},$ $\theta(t) = \tan^{-1}(d / (v_{sp}t))$
Instant	$x = v_{sp}t = 45 \text{ m} \rightarrow r = 75 \text{ m} \quad \theta = 53.1^\circ$
Unknowns	$v_r, v_\theta, a_r, a_\theta$ — and the check: $ a $ must be 0



The six-number table, then the components

$$\vec{v} = \dot{r} \hat{u}_r + r\dot{\theta} \hat{u}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{u}_\theta$$

SIX NUMBERS AT $x = 45$ m

the quadcopter flies horizontally at constant speed v_{sp} , so $x = v_{sp}t$.

the range measured by the station is $r = \sqrt{d^2 + x^2} \rightarrow r^2 = d^2 + x^2$

Take time derivative of both sides $2r\dot{r} = 2x\dot{x}$, since $\dot{x} = v_{sp}$ and $r = 75$ m

$$\dot{r} = v_{sp} \frac{x}{r} = 4.8 \frac{m}{s}, \quad \text{note that } \dot{r} = v_{sp} \cos(\theta)$$

Take time derivative of both sides again $2r\ddot{r} = 2x\ddot{x}$

$$\ddot{r} = (v_{sp}^2 - \dot{r}^2)/r = 0.546 \text{ m/s}^2$$

SEE APPENDIX E for $\dot{\theta} = -v_{sp} \frac{d}{r^2}$

$$\theta = 53.1^\circ, \quad \dot{\theta} = -0.0853 \text{ rad/s}, \quad \ddot{\theta} = 2v_{sp}^2 x d / r^4 = 0.0109 \text{ rad/s}^2$$

$$v_r = 4.8, \quad v_\theta = r\dot{\theta} = -6.4 \rightarrow |v| = 8.0 \text{ m/s} \checkmark$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.546 - 0.546 = 0$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.819 - 0.819 = 0$$

Every polar term is nonzero. Their sums are exactly zero — as they must be for straight, constant-speed flight.



Interpret

The antenna sees the range rate and the bearing rate both changing. None of that is acceleration of the aircraft — it is the rotation of the line of sight. $-r\dot{\theta}^2$ and $\ddot{r}$ cancel; $2\dot{r}\dot{\theta}$ and $r\ddot{\theta}$ cancel.



Why this example

A nonzero answer would expose an algebra or sign error immediately. Validation is built into the physics.



Zero total acceleration is the check. Then the limits.



Speed. $v(4.8^2 + 6.4^2) = 8.0 \text{ m/s}$ — the given speed.



Total acceleration. $a_r = a_\theta = 0 \rightarrow a = 0$. Straight, constant-speed flight. ✓



Limits. Far away: $\dot{r} \rightarrow v, \dot{\theta} \rightarrow 0$.
Closest approach: $\dot{r} = 0, \dot{\theta} = -\frac{v_{sp}}{d}$
 $\ddot{r} = r\dot{\theta}^2$. Both sensible.



Misconception caught here

" $a_r = \ddot{r}$." At closest approach $\ddot{r} = 0.85 \text{ m/s}^2$ but $a_r = 0$. The range can accelerate while the aircraft does not — the $-r\dot{\theta}^2$ term is not optional.



Second misconception

" θ constant $\Rightarrow a_\theta = 0$." Here $\theta \neq 0$ anyway, but even with $\dot{\theta} = 0$ the $2\dot{r}\dot{\theta}$ term survives whenever $\dot{r} \neq 0$.



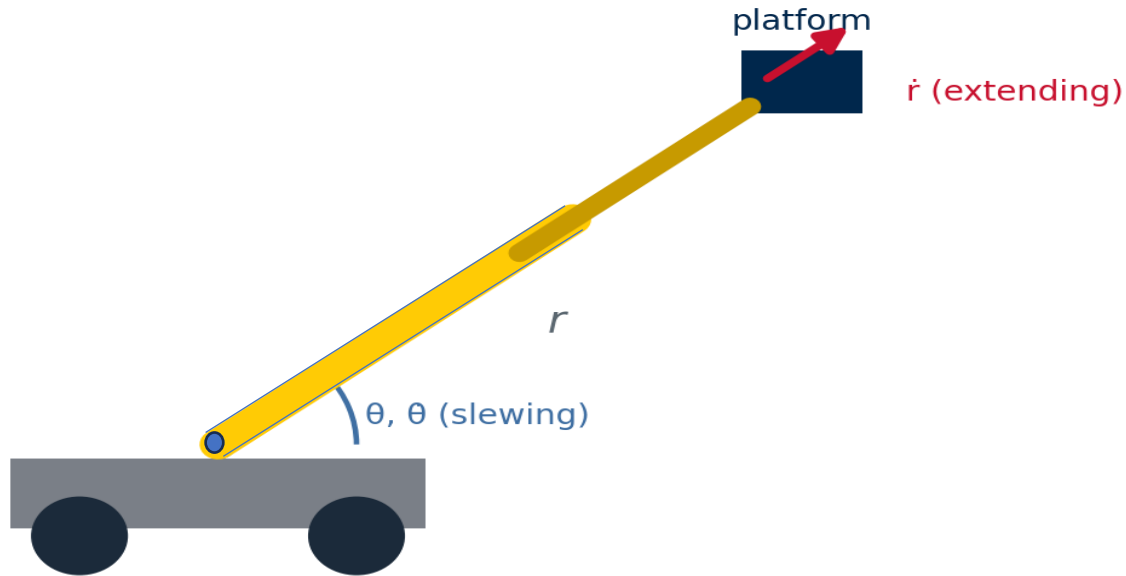
AI as Validator

"Here is my polar acceleration for straight-line flight past a station. Convert it to Cartesian and confirm it is zero. If it is not, find my error."

The instrument chose the description. The physics chose the answer. Part 6 makes this trade explicit.



The boom extends at 0.5 m/s while slewing at 0.15 rad/s. What does the worker on the platform feel?



Interpret: with a perfectly steady slew, the worker still feels a sideways push — from extending while turning. Add a slew acceleration of 0.02 rad/s^2 and a_θ becomes $0.24 + 0.15 = 0.39 \text{ m/s}^2$: the $2\dot{r}\dot{\theta}$ term is as large as the $r\ddot{\theta}$ term.

THE MODEL

Particle	the platform (worker + basket) at the boom tip
Description	polar about the pivot — the machine controls r and θ directly
Knowns	$r = 12 \text{ m}$, $\dot{r} = 0.5 \text{ m/s}$, $\ddot{r} = 0$ $\dot{\theta} = 0.15 \text{ rad/s}$, $\ddot{\theta} = 0$
Unknowns	v_{sp} , a_r , a_θ ; which term dominates the sideways acceleration

SOLVE

$$v_r = 0.5, \quad v_\theta = 1.8 \rightarrow |v| = \sqrt{v_r^2 + v_\theta^2} = 1.87 \text{ m/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 12(0.15)^2 = -0.27 \text{ m/s}^2 \text{ (inward)}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.5)(0.15) = +0.15 \text{ m/s}^2 \text{ (sideways)}$$



Cylindrical coordinates (3D) – (animation):



Observe ›

Idealize ›

Model ›

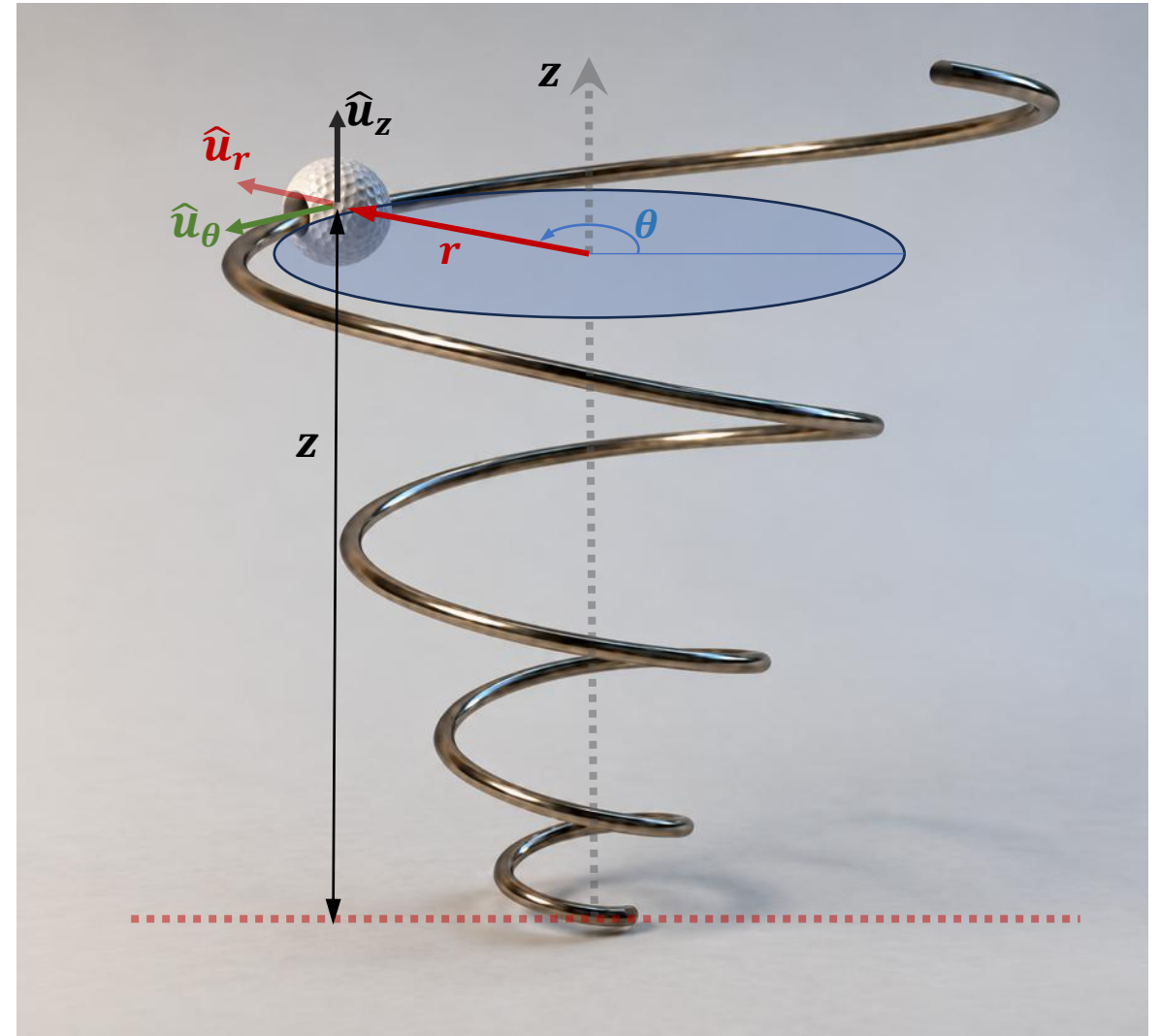
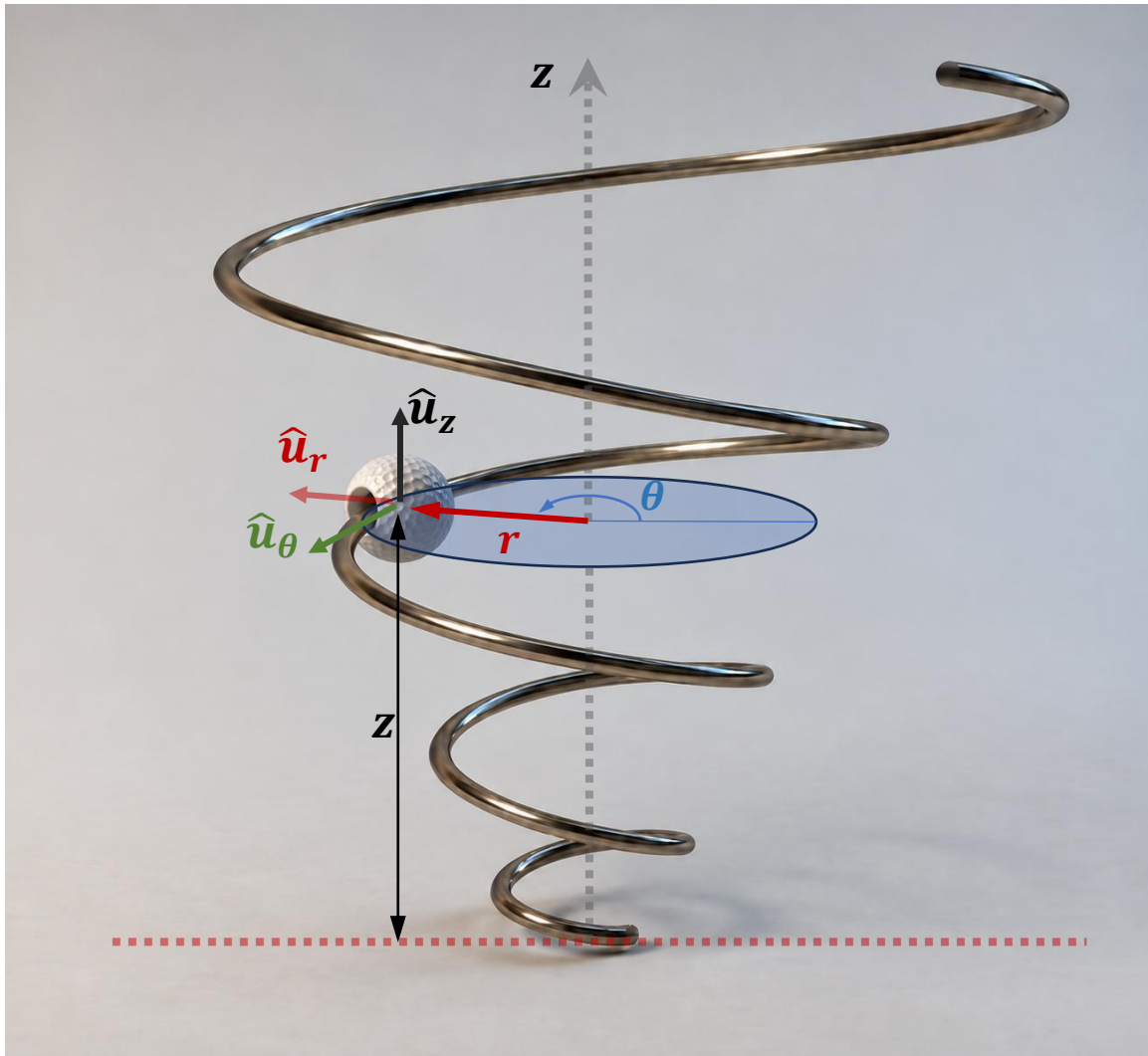
Solve / Analyze ›

Validate ›

Report



Cylindrical coordinates: (r, θ, z) : Unit vectors $\hat{u}_r$ & $\hat{u}_\theta$ change, $\hat{u}_z$ is constant in time.





Cylindrical coordinates: add z $\hat{u}_z$. Nothing else changes.

CYLINDRICAL (r, θ, z) — u_z IS CONSTANT

$$\vec{r}_p = r \hat{u}_r + z \hat{u}_z$$

$$\vec{v} = \dot{r} \hat{u}_r + r \dot{\theta} \hat{u}_\theta + \dot{z} \hat{u}_z$$

$$\vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{u}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \hat{u}_\theta + \ddot{z} \hat{u}_z$$

$\hat{u}_z$ never rotates, so its derivative is zero — the z -motion is Chapter 1 along a vertical line.



Spiral slide

r constant, θ increasing, z decreasing:

$$\vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{u}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \hat{u}_\theta + \ddot{z} \hat{u}_z$$



Robot arm

rotates (θ), extends (r), lifts (z). All three at once → all terms, including $2\dot{r}\dot{\theta}$.



Naming

Planar: **POLAR**. With z : **CYLINDRICAL**. Do not let the old textbook name override the planar priority of this Part.

Main Chapter 2 stays planar. 3-D worked cases live in Appendix E and one optional homework challenge.



Before the synthesis

$$\vec{v} = \dot{r} \hat{u}_r + r\dot{\theta} \hat{u}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{u}_\theta$$

1

Circle about O, constant rate

The bead's radial acceleration is...

- A) zero — r is constant
- B) $-r\dot{\theta}^2$, inward
- C) $+r\dot{\theta}^2$, outward
- D) $\ddot{r}$

2

Groove on a steady turntable

Slider moves outward at constant $\dot{r}$, table at constant $\dot{\theta}$. Its transverse acceleration is...

- A) zero — $\dot{\theta}$ is constant
- B) $r\dot{\theta}$
- C) $2\dot{r}\dot{\theta}$
- D) $\dot{r}\dot{\theta}$

3

Polar vs Cartesian components

They differ because...

- A) $\hat{u}_r, \hat{u}_\theta$ rotate; $\hat{i}, \hat{j}$ do not
- B) polar uses degrees
- C) the origin moves
- D) same numbers, new names

Answers in the notes. For each, name the physical situation on the wheel or the turntable that settles it.

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Part 5 — what you can now explain and do



You can now EXPLAIN

- Why $\hat{u}_r$ and $\hat{u}_\theta$ rotate, and why that gives $\dot{\hat{u}}_r = \dot{\theta} \hat{u}_\theta$ and $\dot{\hat{u}}_\theta = -\dot{\theta} \hat{u}_r$ — the same idea as Part 4
- What each of the four acceleration terms means physically, especially $-r\dot{\theta}^2$ and $2\dot{r}\dot{\theta}$ (two pictures)
- Why the wheel's $a_r = -r\omega^2$ and Part 4's $a_n = +r\omega^2$ are the same vector with different labels
- When polar is the natural description: motion seen from a point, or prescribed along a rotating line



You can now DO

- Build the six-number derivative table and assemble v and a at an instant, radians throughout
- Handle r given as a function of θ with the chain rule
- Validate a polar result by a limiting case or by converting to another description (zero acceleration for straight flight)
- Extend to cylindrical coordinates by adding $\dot{z}$, $\ddot{z}$ — and recognize when that is needed

Next — Part 6: three descriptions of one motion. Which would YOU choose, and why? The bead gives the same numbers in all three; the engineer decides which language is easiest.



Every step, once

START FROM $\vec{v} = \dot{r} \hat{u}_r + r\dot{\theta} \hat{u}_\theta$ **AND USE** $\dot{\hat{u}}_r = \dot{\theta} \hat{u}_\theta$, $\dot{\hat{u}}_\theta = -\dot{\theta} \hat{u}_r$

$$\frac{d}{dt} (\dot{r} \hat{u}_r) = \ddot{r} \hat{u}_r + \dot{r} \dot{\hat{u}}_r = \ddot{r} \hat{u}_r + \dot{r} \dot{\theta} \hat{u}_\theta$$

$$\frac{d}{dt} (r\dot{\theta} \hat{u}_\theta) = \dot{r} \dot{\theta} \hat{u}_\theta + r\ddot{\theta} \hat{u}_\theta + r\dot{\theta} \dot{\hat{u}}_\theta = \dot{r} \dot{\theta} \hat{u}_\theta + r\ddot{\theta} \hat{u}_\theta - r\dot{\theta}^2 \hat{u}_r$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \hat{u}_r + (r\ddot{\theta} + \dot{r}\dot{\theta} + \dot{r}\dot{\theta}) \hat{u}_\theta = (\ddot{r} - r\dot{\theta}^2) \hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{u}_\theta$$

The two $\dot{r}\dot{\theta}$ contributions come from different places — one from differentiating $(\dot{r} \hat{u}_r)$ (its direction rotates), one from differentiating $(r\dot{\theta} \hat{u}_\theta)$ (r grows). That is the "two pictures" of the $2\dot{r}\dot{\theta}$ term.



How did we get $\dot{\theta} = -\frac{d \cdot v_{sp}}{r^2}$

We have $\tan(\theta) = \frac{d}{x}$, also recall that $d = \text{constant}$, $x = v_{sp}t$.

Take time derivative of both sides:

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{d}{x}\right)$$

which gives,

$$\sec^2 \theta \cdot \dot{\theta} = -\frac{d \cdot \dot{x}}{x^2} = -\frac{d \cdot v_{sp}}{x^2}$$

But, from geometry $\sec^2 \theta = 1 + \tan^2 \theta$, since $\tan(\theta) = \frac{d}{x}$

$$\sec^2 \theta = 1 + \frac{d^2}{x^2} =$$

$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + \frac{d^2}{x^2} = \frac{x^2 + d^2}{x^2}.$$

But

$$x^2 + d^2 = r^2,$$

so

$$\sec^2 \theta = \frac{r^2}{x^2}.$$

Substitute:

$$\frac{r^2}{x^2} \dot{\theta} = -\frac{dv_{sp}}{x^2}.$$

The x^2 's cancel:

$$\boxed{\dot{\theta} = -\frac{dv_{sp}}{r^2}}$$



AI as can explain the details

No need to remember all the derivation steps



A pick-and-place arm that rotates, extends and lifts at once

THE MODEL

Given $\theta = 0.8t \text{ rad}, \quad r = 0.6 + 0.2t \text{ m},$
 $z = 0.1t^2 \text{ m}$

Instant $t = 2 \text{ s}$

Table $r = 1.0, \quad \dot{r} = 0.2, \quad \ddot{r} = 0;$
 $\dot{\theta} = 0.8, \quad \ddot{\theta} = 0;$
 $\dot{z} = 0.4, \quad \ddot{z} = 0.2$

Unknowns v, a of the gripper

ASSEMBLE

$$v = 0.2 \hat{u}_r + 0.8 \hat{u}_\theta + 0.4 \hat{u}_z \rightarrow 0.92 \text{ m/s}$$

$$\vec{a} = -0.64 \hat{u}_r + 0.32 \hat{u}_\theta + 0.2 \hat{u}_z \rightarrow 0.74 \text{ m/s}^2$$

a_r is inward although $\ddot{r} = 0$;
 a_θ exists although $\ddot{\theta} = 0$, $a_\theta = 2\dot{r}\dot{\theta}$, a_z is as in Chapter 1.



AI as Coding Companion

"Write a Python script that takes $r(t), \theta(t), z(t)$ as expressions and animates the path with $\hat{u}_r, \hat{u}_\theta$ drawn, printing the six-number table at a chosen instant."* Verify its numbers against the table above before trusting the animation.