



CHAPTER 2 — REVIEW PROBLEMS

WORKED EXAMPLES & SOLUTIONS

Curvilinear Particle Kinematics

Taner Onsay, Ph.D. · © 2026 AI First Engineering

HOW TO USE THESE WORKED EXAMPLES

These worked examples summarize Chapter 2 problems. Use them to study the problem-solving process, not simply the final numerical answers.

For each problem, follow our course workflow:

SEE → THINK → MODEL → SOLVE → ANSWER → VALIDATE

Every problem is the one studied or worked earlier. Each begins with the concept it reviews and the problem statement with its original figure, then walks through the six workflow steps and closes with a *What to Watch For* note on the error most often made on that type of problem.

PROBLEM 1 — Constant Speed on the Wheel — Is the Bead Accelerating, and by How Much?

CONCEPT BEING REVIEWED

Constant speed on a curve is not zero acceleration: $a_n = v_{sp}^2/\rho = \omega^2 r$ changes direction; $a_t = dv_{sp}/dt = r\alpha$ changes speed.

PROBLEM



The bead is $r = 0.30$ m from the spindle. From the 120 fps video you count $N = 12$ revolutions in $\Delta t = 10.0$ s at steady speed.

- (A) Predict: if the wheel were spun at twice the rate, by what factor would v_{sp} change? And a_n ?
- (B) Compute f , ω (rad/s), the bead speed v_{sp} , and a_n ; express a_n in g 's. Is a_t zero here?
- (C) Plausibility: is “about 1.7 g on a bicycle bead” believable?
- (D) The brake is applied and ω falls from its steady value to half of it in 5.0 s, roughly linearly. Estimate α , a_t , and a_n at the end of the 5 s. Which component dominates $|\vec{a}|$?

STEP 1 — SEE

Given: $r = 0.30$ m; $N = 12$ rev in $\Delta t = 10.0$ s; later ω halves in 5.0 s. The path is a circle of fixed radius — direction changes every instant; speed does not (until the brake).

Sketch: two bead positions a short time apart with $\vec{v}_1$ and $\vec{v}_2$ (equal length, different direction); slide them to a common tail; $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ points inward.

STEP 2 — THINK

Before calculating, ask yourself: the bead's speed is constant. Is it accelerating — yes, no, or only if the wheel speeds up? Decide, then say in one sentence what “acceleration” means.

$\vec{a} = d\vec{v}/dt$ is the rate of change of the velocity vector.

A turning vector of constant length still has a derivative — perpendicular to itself, toward the concave side.

Prediction for part A:

$$v_{sp} = \omega r \propto \omega, \quad a_n = \omega^2 r \propto \omega^2$$

so twice the rate gives $v_{sp} \times 2$ and $a_n \times 4$.

STEP 3 — MODEL

n–t frame riding on the bead: $\hat{u}_t$ along the path in the direction of motion, $\hat{u}_n$ toward the spindle (centre of curvature, $\rho = r$).

Assume ω constant over the counting interval; bead treated as a particle at radius r .

$$\vec{v} = v_{sp}\hat{u}_t, \quad \vec{a} = a_t\hat{u}_t + a_n\hat{u}_n$$
$$a_t = dv_{sp}/dt = r\alpha, \quad a_n = v_{sp}^2/\rho = \omega^2 r$$

STEP 4 — SOLVE

$$f = N/\Delta t = 12/10.0 = 1.2 \text{ Hz}$$
$$\omega = 2\pi f = 2\pi(1.2) = 7.54 \text{ rad/s}$$
$$v_{sp} = \omega r = (7.54)(0.30) = 2.26 \text{ m/s}$$
$$a_n = \omega^2 r = (7.54)^2(0.30) = 17.1 \text{ m/s}^2 \approx 1.74 g$$

Steady speed $\Rightarrow a_t = dv_{sp}/dt = 0$. Braking (part D): ω falls from 7.54 to 3.77 rad/s in 5.0 s.

$$\alpha \approx \Delta\omega/\Delta t = (3.77 - 7.54)/5.0 = -0.75 \text{ rad/s}^2$$
$$a_t = r\alpha = (0.30)(-0.75) = -0.23 \text{ m/s}^2$$

→ **backward along the path**

$$a_n \text{ at the end} = \omega^2 r = (3.77)^2(0.30) = 4.26 \text{ m/s}^2$$

→ **down from 17.1 m/s²**

STEP 5 — ANSWER

ANSWER

Steady wheel: $v_{sp} = 2.26 \text{ m/s}$, $a_n = 17.1 \text{ m/s}^2 \approx 1.74 g$ toward the spindle, $a_t = 0$.
Braking: $\alpha \approx -0.75 \text{ rad/s}^2$, $a_t \approx -0.23 \text{ m/s}^2$, a_n falls to 4.26 m/s^2 — the normal part dominates throughout.

STEP 6 — VALIDATE

Units: $(\text{rad/s})^2 \cdot \text{m} = \text{m/s}^2$ (the radian is dimensionless).

Scaling: halving ω divides a_n by four — $17.1 \rightarrow 4.3 \checkmark$.

Direction: an unglued bead would leave along the *tangent*, not radially outward — nothing supplies the inward acceleration (Chapter 3).

Alternate description: $x = R \cos \omega t$, $y = R \sin \omega t$ gives $\vec{a} = -\omega^2 \vec{r}$, magnitude $\omega^2 R$, inward — the same number.

WHAT TO WATCH FOR

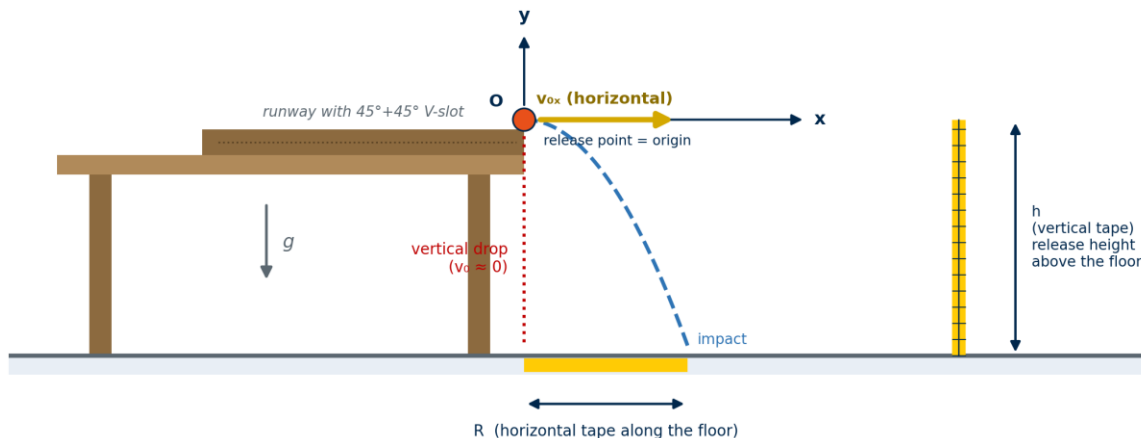
“Constant speed $\Rightarrow \vec{a} = 0$.” Speed is $|\vec{v}|$; a changing direction alone requires $a_n = v_{sp}^2/\rho$. And a small a_t does not mean the brake is “doing nothing” — a_t sets the rate of speed loss; a_n is set by how fast the wheel turns *right now*.

PROBLEM 2 — The V-Slot Launch — Flight Time, Release Speed, Impact Velocity

CONCEPT BEING REVIEWED

Projectile model: $a_x = 0$, $a_y = -g$ on fixed axes; the launch speed sets the range, never the flight time.

PROBLEM



Launch geometry: origin at the release point, x along the flight, y up.

A tennis ball rolls off the V-slot at the table edge, $h = 0.90$ m above the floor, and lands $R = 1.10$ m from the point directly below the edge. Use the ideal model, $g = 9.81$ m/s².

- Predict: if the ball had left twice as fast, would the flight time double, halve, or stay the same? Would R double?
- Compute the flight time t_f .
- Infer the release speed v_{0x} from R and t_f .
- Compute the velocity vector just before impact: components, magnitude, and angle below horizontal.
- Estimate the relative uncertainty in t_f for ± 2 frames at 30 fps and at 120 fps.

STEP 1 — SEE

Given: $h = 0.90$ m, $R = 1.10$ m, $g = 9.81$ m/s².

Origin at the release point (table edge), x along the flight, y up, so $y = -h$ at the floor.

Release velocity is horizontal: $v_{0y} = 0$.

STEP 2 — THINK

Before calculating, ask yourself: three balls leave the same edge at the same instant — one dropped, one slow, one fast. Which hits the floor first? Then: which equation contains v_{0x} , and which does not?

Only gravity acts after release $\Rightarrow$ the vertical motion is Chapter 1 free fall and “does not know” about v_{0x} . Prediction: t_f unchanged, R doubles.

STEP 3 — MODEL

Particle (mass centre); uniform g ; negligible drag and spin; exactly horizontal release at the edge; level floor.

$$a_x = 0 \Rightarrow x = v_{0x}t, \quad a_y = -g \Rightarrow v_y = -gt, \quad y = -\frac{1}{2}gt^2$$

Floor: $y = -h \Rightarrow t_f = \sqrt{2h/g}$; $R = v_{0x}t_f$

STEP 4 — SOLVE

$$t_f = \sqrt{2h/g} = \sqrt{2 \cdot 0.90/9.81} = 0.428 \text{ s}$$

$$v_{0x} = R/t_f = 1.10/0.428 = 2.57 \text{ m/s}$$

$$v_y = -g t_f = -9.81(0.428) = -4.20 \text{ m/s}$$

$$|\vec{v}| = \sqrt{v_{0x}^2 + v_y^2} = \sqrt{2.57^2 + 4.20^2} = 4.92 \text{ m/s}$$

$$\text{angle below horizontal} = \tan^{-1}(|v_y|/v_{0x}) = \tan^{-1}(4.20/2.57) = 58.6^\circ$$

Frame-count uncertainty (part E): $\pm 2 \text{ frames} = \pm 2/30 = \pm 0.067 \text{ s}$ at 30 fps $\Rightarrow \pm 16\%$ of t_f ;
 $\pm 2/120 = \pm 0.017 \text{ s}$ at 120 fps $\Rightarrow \pm 3.9\%$. $v_{0x} = R/t_f$ inherits the same relative uncertainty.

STEP 5 — ANSWER

ANSWER

$t_f = 0.428 \text{ s}$ (independent of launch speed); $v_{0x} = 2.57 \text{ m/s}$;

impact $\vec{v} = 2.57\hat{i} - 4.20\hat{j} \text{ m/s}$, $|\vec{v}| = 4.92 \text{ m/s}$ at 58.6° below horizontal.

STEP 6 — VALIDATE

Second route to the impact speed (no time): $|\vec{v}|^2 = v_{0x}^2 + 2gh = 6.60 + 17.66 = 24.26 \Rightarrow 4.93 \text{ m/s} \checkmark$.

Limit: $h \rightarrow 0 \Rightarrow t_f \rightarrow 0, R \rightarrow 0$.

Sign: $v_y < 0$ at impact with y up $\checkmark$.

Draw $\vec{v}$ at the landing point — steep and downward, matching the sketch.

WHAT TO WATCH FOR

“The faster ball has more velocity, so it stays in the air longer.” This inserts v_{0x} into the vertical equation; $t_f = \sqrt{2h/g}$ contains no v_{0x} .

And do not blame the model for a 5 % timing disagreement that sits inside a ± 2 -frame budget.

PROBLEM 3 — Two Highway Curves — Predict, Then Calculate

CONCEPT BEING REVIEWED

$a_n = v_{sp}^2/\rho$: doubling the speed quadruples the required radius; a_t and a_n are perpendicular, so $|\vec{a}|$ is a root-sum-square.

PROBLEM

Curve 1 is posted at 40 km/h with radius 50 m.

Curve 2 is posted at 80 km/h with radius 150 m.

(A) Commit first: which curve produces the larger sideways (normal) acceleration at the posted speed, and roughly by what factor?

(B) Compute a_n for each curve in m/s² and as a fraction of g .

(C) What radius would Curve 2 need for the same a_n as Curve 1? State the general rule.

(D) A car takes Curve 1 at the posted speed but is *braking* at 2.0 m/s². Find $|\vec{a}|$ and its direction relative to the tangent. Is $|\vec{a}|$ simply $a_t + a_n$?

STEP 1 — SEE

Two circular arcs; constant speed on each (part B); the path is known and the question is about acceleration across the path $\Rightarrow$ n–t is the natural language.

Speeds are in km/h — convert first.

STEP 2 — THINK

Before calculating, ask yourself: twice the speed on three times the radius — larger or smaller a_n ? Decide before any arithmetic.

$a_n \propto v_{sp}^2/\rho$. Speed doubles ($\times 4$), radius triples ($\div 3$) $\Rightarrow$ Curve 2 larger by 4/3.

STEP 3 — MODEL

$$\vec{a} = a_t \hat{u}_t + a_n \hat{u}_n, \quad a_n = v_{sp}^2/\rho, \quad |\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

$\hat{u}_n$ toward the centre of the curve; $a_t = 0$ while the speed is constant, $a_t = -2.0$ m/s² while braking (part D).

STEP 4 — SOLVE

$$v_{sp,1} = 40 \text{ km/h} \times (1000 \text{ m/km})/(3600 \text{ s/h}) = 11.11 \text{ m/s}$$

$$v_{sp,2} = 80/3.6 = 22.22 \text{ m/s}$$

$$a_{n1} = v_{sp,1}^2/\rho_1 = (11.11)^2/50 = 2.47 \text{ m/s}^2 = 0.25 g$$

$$a_{n2} = v_{sp,2}^2/\rho_2 = (22.22)^2/150 = 3.29 \text{ m/s}^2 = 0.34 g$$

$\rightarrow$ **Curve 2 larger, $\times 1.33$ ✓**

$$\text{Same } a_n \text{ as Curve 1: } \rho = v_{sp,2}^2/a_{n1} = (22.22)^2/2.47 = 200 \text{ m}$$

$\rightarrow$ **doubling the speed quadruples the radius**

$$\text{Braking on Curve 1: } |\vec{a}| = \sqrt{a_t^2 + a_n^2} = \sqrt{2.0^2 + 2.47^2} = 3.18 \text{ m/s}^2$$

$$\text{direction from the (backward) tangent: } \tan^{-1}(a_n/|a_t|) = \tan^{-1}(2.47/2.0) = 51^\circ$$

$\rightarrow$ **toward the inside of the curve**

STEP 5 — ANSWER

ANSWER

$a_{n1} = 2.47 \text{ m/s}^2$ (0.25 g); $a_{n2} = 3.29 \text{ m/s}^2$ (0.34 g); Curve 2 would need $\rho = 200 \text{ m}$ to match Curve 1;

braking on Curve 1: $|\vec{a}| = 3.18 \text{ m/s}^2$ at 51° from the backward tangent, inward — not $2.0 + 2.47$.

STEP 6 — VALIDATE

Units: $(\text{m/s})^2/\text{m} = \text{m/s}^2 \checkmark$. Plausibility: highway design tables sit near 0.25–0.30 $g \checkmark$.

Vector check: draw $a_t \hat{u}_t$ (backward) and $a_n \hat{u}_n$ (inward) tip-to-tail — the resultant is the hypotenuse, so the magnitude must be less than the sum.

WHAT TO WATCH FOR

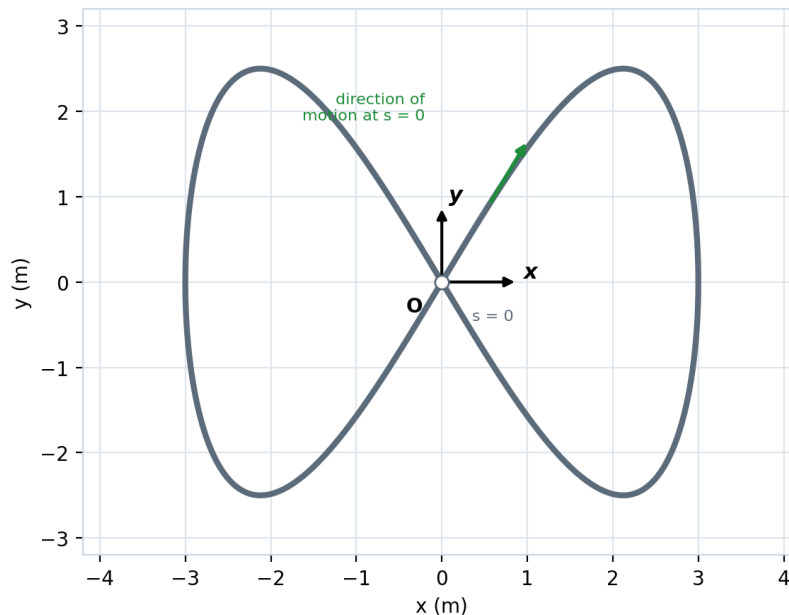
Adding perpendicular components as scalars ($|\vec{a}| = a_t + a_n$), and forgetting the $\text{km/h} \rightarrow \text{m/s}$ conversion before squaring.

PROBLEM 4 — The Figure-8 — Where Is ρ Smallest, and Is That Where a_n Is Largest?

CONCEPT BEING REVIEWED

ρ and $\hat{u}_n$ are local properties of the path; $a_n = v_{sp}^2/\rho$, so with changing speed the largest a_n need not occur at the smallest ρ .

PROBLEM



The path is $x = A \sin u$, $y = B \sin u \cos u$ with $A = 3$ m and $B = 5$ m; one lap is 24.8 m long. The particle starts at the crossing O ($s = 0$) with speed $v_{sp}(t) = v_0 + a_t t$ (SPEEDING UP), so in $T = 12$ s it completes several laps. On a curved path $\vec{a} = a_t \hat{u}_t + a_n \hat{u}_n$ with $a_t = dv_{sp}/dt$ and $a_n = v_{sp}^2/\rho$, where ρ is the local radius of curvature and $\hat{u}_n$ points toward the local centre of curvature O'.

C1: Where on the track is ρ smallest? In this speeding-up run, will the largest a_n of the whole run occur at the same place *and* the same time as the smallest ρ ? What are ρ and a_n at the crossing O?

C3: Does the Figure-8 have ONE fixed centre of curvature? On which side of the path is $\hat{u}_n$ in the right lobe, and in the left lobe?

C4: Using $a_n = v_{sp}^2/\rho$, which factor makes a_n larger at the last pass of a tightest bend than at the first pass?

STEP 1 — SEE

Look at the track, not the equations. The lobes are tightest at their tops and bottoms; the two diagonals through O are nearly straight; the lobe tips ($\pm 3, 0$) are the flattest curves. The speed grows lap after lap.

STEP 2 — THINK

Before calculating, ask yourself: point at the track — where would you *feel* the largest sideways push? Now, is that the same instant as the tightest bend if the car keeps speeding up?

ρ is a property of the *path* — the same at every pass of the same point. $a_n = v_{sp}^2/\rho$ also depends on the *speed at that instant*. So: smallest $\rho \Rightarrow$ largest a_n only at constant speed.

STEP 3 — MODEL

At each point the path is locally a circle of radius ρ centred at O' on the concave side; $\hat{u}_n$ points from the particle toward O' . At an inflection (the crossing) $\rho \rightarrow \infty$, $\hat{u}_n$ is undefined and $a_n \rightarrow 0$ — but $\vec{a} = a_t \hat{u}_t \neq 0$ while the speed changes.

STEP 4 — SOLVE (with the figure)

$\rho_{\min}$ occurs at the tops and bottoms of each lobe, near $(\pm 2.2, \pm 2.5)$ m — four points per lap.

→ for the module's preset run, $\rho_{\min} \approx 0.42$ m

At the crossing O : $\rho \rightarrow \infty$, $a_n \rightarrow 0$, $\vec{a} = a_t \hat{u}_t$

→ tangential only

One fixed centre of curvature? No — O' moves along with the particle. In the right lobe $\hat{u}_n$ points to the driver's right; in the left lobe to the driver's left.

→ the sense of turning reverses through O

Speeding up: $a_{n,max}$ of the whole run occurs at a $\rho_{\min}$ point on the *last* pass (lap 4 in the preset run), because it is the same ρ with a larger v_{sp}^2 .

Ratio check: $R_1 = a_{n,max}/a_n$ (first pass) must equal $R_2 = (v_{sp,last}/v_{sp,first})^2$ if ρ is the same.

→ for the 3.0 m/s, 0.6 m/s² preset run: $R_1 = 242.9/31.2 = 7.78$, $R_2 = (10.12/3.63)^2 = 7.77$ ✓

STEP 5 — ANSWER

ANSWER

$\rho_{\min}$ at the four lobe tops/bottoms (same ρ at every pass); $\rho \rightarrow \infty$ and $a_n = 0$ at O , where $|\vec{a}| = |a_t|$;

no single centre — O' sits inside the right lobe on the driver's right and inside the left lobe on the driver's left;

with speed increasing, $a_{n,max}$ is at the *last* pass of a $\rho_{\min}$ point — the v_{sp}^2 factor, not ρ , decides it. At constant speed the two would coincide.

STEP 6 — VALIDATE

Circle contrast: on a circle of radius R , O' is the fixed centre and $\rho = R$ everywhere.

Limiting case: constant speed $\Rightarrow a_n \propto 1/\rho$ only $\Rightarrow$ maximum a_n exactly at $\rho_{\min}$.

The ratio test $R_1 = R_2$ is the VEL validation you performed in Homework 3, item C4.

WHAT TO WATCH FOR

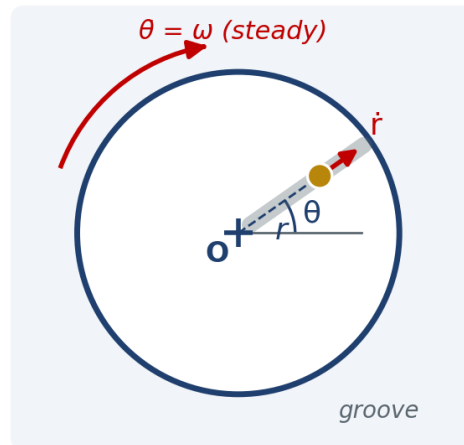
“Smallest ρ automatically means largest a_n ” — true only at constant speed. And “ $a_n = 0$ at the crossing, so $\vec{a} = 0$ ” — the tangential part survives.

PROBLEM 5 — Telescoping Aerial-Lift Boom — Polar Terms and What the Worker Feels

CONCEPT BEING REVIEWED

Polar description: $\vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$; $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$ — the $-r\dot{\theta}^2$ and $2\dot{r}\dot{\theta}$ terms exist even when $\dot{r}$ and $\dot{\theta}$ are constant.

PROBLEM



Opening: a turntable spins at a steady rate $\dot{\theta} = \omega$; a ball sits in a straight radial groove. Held at fixed r — is it accelerating? Now it slides outward at a steady $\dot{r}$ while $\dot{\theta}$ stays constant — is there any acceleration in the $\hat{u}_\theta$ direction?

Problem: a boom pivots about O in a horizontal plane. At one instant the platform at its tip is $r = 10$ m from O, the boom is extending at $\dot{r} = 0.40$ m/s (constant) and slewing at $\dot{\theta} = 0.20$ rad/s (constant).

- (A) Predict: $\ddot{r} = 0$ and $\ddot{\theta} = 0$ — is the platform's acceleration zero?
- (B) Fill in the six-number table $r, \dot{r}, \ddot{r}, \theta, \dot{\theta}, \ddot{\theta}$.
- (C) Assemble $\vec{v}$: components and speed.
- (D) Assemble $\vec{a}$: $a_r, a_\theta, |\vec{a}|$; interpret the sign of a_r and the origin of a_θ .
- (E) The operator also increases the slew rate at $\ddot{\theta} = 0.030$ rad/s²: recompute a_θ .

STEP 1 — SEE

The machine controls r and θ directly from a fixed pivot $\Rightarrow$ polar is the instrument's language. Six numbers: $r = 10$ m, $\dot{r} = 0.40$ m/s, $\ddot{r} = 0$, θ (any value — irrelevant to magnitudes), $\dot{\theta} = 0.20$ rad/s, $\ddot{\theta} = 0$.

STEP 2 — THINK

Before calculating, ask yourself: $\dot{r}$ is constant and $\dot{\theta}$ is constant. Is the acceleration zero — yes / no / only radial?

$\hat{u}_r$ and $\hat{u}_\theta$ rotate at $\dot{\theta}$, so their derivatives are nonzero: $d\hat{u}_r/dt = \dot{\theta}\hat{u}_\theta$, $d\hat{u}_\theta/dt = -\dot{\theta}\hat{u}_r$.

Two consequences: $-r\dot{\theta}^2$ (direction change seen from O — the bead on the wheel) and $2\dot{r}\dot{\theta}$ (moving outward *while* rotating: the transverse speed $r\dot{\theta}$ grows because r grows, and the radial velocity vector $\dot{r}\hat{u}_r$ turns). Prediction: $\vec{a} \neq 0$.

STEP 3 — MODEL

$$\vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$$

Radians throughout; $\hat{u}_r$ away from O, $\hat{u}_\theta$ 90° ahead in the direction of increasing θ .

STEP 4 — SOLVE

$$v_r = \dot{r} = 0.40 \text{ m/s}; \quad v_\theta = r\dot{\theta} = (10)(0.20) = 2.0 \text{ m/s}$$

$$v_{sp} = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0.40^2 + 2.0^2} = 2.04 \text{ m/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (10)(0.20)^2 = -0.40 \text{ m/s}^2$$

→ **negative ⇒ toward O: the turn**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.40)(0.20) = +0.16 \text{ m/s}^2$$

→ **forward, in the direction of rotation**

$$|\vec{a}| = \sqrt{a_r^2 + a_\theta^2} = \sqrt{0.40^2 + 0.16^2} = 0.43 \text{ m/s}^2$$

$$\text{With } \ddot{\theta} = 0.030 \text{ rad/s}^2: a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (10)(0.030) + 0.16 = 0.46 \text{ m/s}^2$$

→ **the $r\ddot{\theta}$ term (0.30) is about twice the $2\dot{r}\dot{\theta}$ term (0.16); neither is negligible**

STEP 5 — ANSWER

ANSWER

$$\vec{v} = 0.40\hat{u}_r + 2.0\hat{u}_\theta \text{ m/s}, \quad v_{sp} = 2.04 \text{ m/s};$$

$$\vec{a} = -0.40\hat{u}_r + 0.16\hat{u}_\theta \text{ m/s}^2, \quad |\vec{a}| = 0.43 \text{ m/s}^2.$$

The worker feels an inward pull ($-r\dot{\theta}^2$) and a sideways push in the direction of rotation ($2\dot{r}\dot{\theta}$) even with a perfectly steady slew.

STEP 6 — VALIDATE

Units: $\text{m} \cdot (\text{rad/s})^2 = \text{m/s}^2 \checkmark$.

Limits: stop the extension ($\dot{r} = 0$) $\Rightarrow a_\theta = 0$, $a_r = -r\omega^2$ — the bead on the wheel ($a_n = r\omega^2$ inward: same vector, opposite sign convention because $\hat{u}_r$ points outward); stop the slew ($\dot{\theta} = 0$, $\ddot{\theta} = 0$) $\Rightarrow \vec{a} = 0$ — Chapter 1 along a line.

Turntable answer: the held ball accelerates inward ($-r\dot{\theta}^2$); sliding outward adds a forward transverse acceleration $2\dot{r}\dot{\theta}$, backward if sliding inward.

WHAT TO WATCH FOR

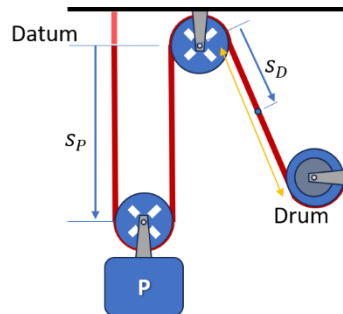
" $\dot{\theta}$ constant $\Rightarrow a_\theta = 0$ " (forgets $2\dot{r}\dot{\theta}$) and " $a_r = \ddot{r}$ " (forgets $-r\dot{\theta}^2$). Degrees inside $\dot{\theta}^2$ give nonsense: $0.20 \text{ rad/s} = 11.5^\circ/\text{s}$, and $(11.5)^\circ$ is not an acceleration.

PROBLEM 6 — Fixed-Geometry Hoist — Count the Segments That Change Length

CONCEPT BEING REVIEWED

Inextensible cable: write L as the sum of the segments that can change length, differentiate once for velocity and again for acceleration, read the signs from the datum.

PROBLEM



A construction hoist uses one fixed sheave and one movable sheave attached to the platform. The drum takes in cable at $v_D = 0.80$ m/s and is speeding up at $a_D = 0.20$ m/s².

- (A) Commit before any equation: the platform rises at $0.80 / 0.40 / 1.60$ m/s — which pieces of cable get shorter, and how many?
- (B) With the datum shown and s positive downward, write $L = \dots$ keeping only the segments that can change length.
- (C) Differentiate once and twice; with $v_D = +0.80$ m/s find v_P and a_P ; interpret the signs.
- (D) Measuring the platform upward from the floor instead — does the answer change?
- (E) Cable speed to lift the platform 12 m in 30 s at steady speed?

STEP 1 — SEE

Datum: the fixed top sheave axis; s_P and s_D measured DOWN from it. Two vertical segments run from the fixed level down to the movable sheave; the anchored segment and the arcs around the sheaves never change length.

STEP 2 — THINK

Before calculating, ask yourself: the drum takes in 0.80 m/s of cable. Does the platform rise at 0.80, 0.40 or 1.60 m/s? Say in words which pieces of cable get shorter.

One metre of platform rise shortens *two* segments by one metre each $\Rightarrow$ the drum must take in two metres: the platform moves at half the cable speed.

STEP 3 — MODEL

Cable taut and inextensible; sheaves ideal; vertical segments. Constraint: total length L constant.

$$L = 2s_P + s_D + (\text{constant arcs})$$

STEP 4 — SOLVE

differentiate once: $0 = 2v_P + v_D \Rightarrow v_P = -v_D/2 = -(0.80)/2 = -0.40$ m/s

$\rightarrow$ negative with s down $\Rightarrow$ UP

differentiate again: $0 = 2a_P + a_D \Rightarrow a_P = -(0.20)/2 = -0.10$ m/s²

$\rightarrow$ up, and speeding up

datum changed to the floor, $s' = h - s_p$: $0 = -2v' + v_D \Rightarrow v' = +0.40 \text{ m/s up}$

→ **same physics, different sign bookkeeping**

12 m in 30 s $\Rightarrow v_p = 0.40 \text{ m/s} \Rightarrow$ the drum takes in cable at 0.80 m/s

STEP 5 — ANSWER

ANSWER

$v_p = 0.40 \text{ m/s upward}$, $a_p = 0.10 \text{ m/s}^2 \text{ upward (speeding up)}$; for 12 m in 30 s the drum must run at 0.80 m/s.

STEP 6 — VALIDATE

Sign: $v_D > 0$ (cable point moving down toward the drum) gives $v_p < 0$ — up, as the picture demands.

Consistency: velocity and acceleration ratios are both 2 because the geometry is fixed (contrast the towed cart of Self-Assessment Question 27, where the ratio ℓ/x moves and an acceleration appears from a constant winch speed).

WHAT TO WATCH FOR

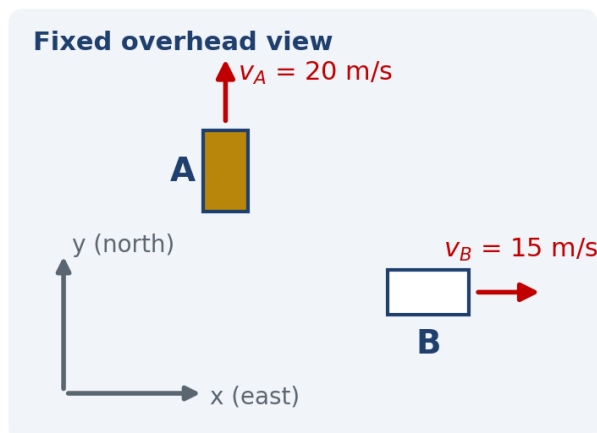
Counting the drum-side or the anchored segment as if it shortened, or taking the coefficient from the number of sheaves. The coefficient is the number of segments that change length.

PROBLEM 7 — Two Cars, One Radar — What “Relative To” Means

CONCEPT BEING REVIEWED

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$ and $\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$ for translating, nonrotating observer axes; a radar measures a difference and cannot say who is accelerating.

PROBLEM



Car A travels north at 20 m/s; car B travels east at 15 m/s. Both are on straight lanes at this instant. Use x east, y north, and translating (non-rotating) observer axes.

(A) Find $\vec{v}_{B/A}$: components, magnitude, direction.

(B) Find $\vec{v}_{A/B}$.

(C) Later, both cars are in the same lane and A's radar reports "closing at 5 m/s": is the car ahead braking? Is A accelerating? Can you tell from the radar alone?

(D) Still later, B rounds a curve of radius 100 m at a constant 15 m/s, heading east and turning toward the north; A continues north at a constant 20 m/s. What $\vec{a}_{B/A}$ does A's radar report?

(E) A's radar reports $\vec{a}_{B/A} = 0$ for a while — does that mean neither car is accelerating?

(F) State the caveat under which these relations are valid.

STEP 1 — SEE

Fixed overhead view: $\vec{v}_A = 20\hat{j}$ m/s, $\vec{v}_B = 15\hat{i}$ m/s. The observer is car A; its axes translate with A and do not rotate (straight lane).

STEP 2 — THINK

Before calculating, ask yourself: A's radar says "closing at 5 m/s." Who is braking — B, A, or can't tell?

"B relative to A" is what A would measure with its own instruments: the vector *difference*. Draw $\vec{v}_A$ and $\vec{v}_B$ tail to tail; $\vec{v}_{B/A}$ runs from the tip of $\vec{v}_A$ to the tip of $\vec{v}_B$. The subscript after the slash names the observer.

STEP 3 — MODEL

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A; \quad \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

Valid for observer axes that translate without rotating. A turning car's own frame is a rotating frame — Chapter 6.

STEP 4 — SOLVE

$$\vec{v}_{B/A} = (15, 0) - (0, 20) = (15, -20) \text{ m/s}$$

$$|\vec{v}_{B/A}| = \sqrt{15^2 + 20^2} = 25 \text{ m/s, direction } \tan^{-1}(20/15) = 53.1^\circ \text{ south of east}$$

→ **B appears to move east and fall back**

$$\vec{v}_{A/B} = -\vec{v}_{B/A} = (-15, 20) \text{ m/s}$$

closing at 5 m/s means only $v_B - v_A = -5$ m/s along the lane — who is accelerating: cannot tell

→ **A's own speedometer/accelerometer (the absolute half of the relation) settles it**

B on the curve: $a_n = v_{sp}^2/\rho = 15^2/100 = 2.25 \text{ m/s}^2$ toward the centre (north), $a_t = 0 \Rightarrow \vec{a}_B = (0, 2.25) \text{ m/s}^2$;

$$\vec{a}_A = 0 \Rightarrow \vec{a}_{B/A} = (0, 2.25) \text{ m/s}^2$$

→ **the radar reports B accelerating north although B's speed is constant**

STEP 5 — ANSWER

ANSWER

$$\vec{v}_{B/A} = 15\hat{i} - 20\hat{j} \text{ m/s (25 m/s, } 53.1^\circ \text{ south of east); } \vec{v}_{A/B} = -\vec{v}_{B/A};$$

a closing rate alone cannot say who is accelerating; on the curve $\vec{a}_{B/A} = 2.25\hat{j} \text{ m/s}^2$;

$\vec{a}_{B/A} = 0$ means only $\vec{a}_B = \vec{a}_A$ — both could be braking hard together.

STEP 6 — VALIDATE

Limit: same heading and speed $\Rightarrow \vec{v}_{B/A} = 0$, B looks parked to A ✓.

Reverse the subscripts $\Rightarrow$ the vector flips: $\vec{v}_{A/B} = -\vec{v}_{B/A}$ ✓.

Direction check on the sketch: B is east of A and pulling away east while A outruns it north — south-east relative motion ✓.

WHAT TO WATCH FOR

Writing $\vec{v}_{B/A} = \vec{v}_A - \vec{v}_B$ (reversed order), and forgetting that constant speed on a curve still gives B an acceleration that the radar reports.

PROBLEM 8 — Which Language Would You Start In — and Why?

CONCEPT BEING REVIEWED

Same physical motion, different mathematical descriptions: choose from what is known, what is asked, and what the instrument measured — no description is universally better.

PROBLEM

For each situation, choose the description you would begin with — Cartesian x–y, normal–tangential n–t, or polar r– θ — and give a one-sentence justification built on the three questions: *What do I know? What am I asked? What did the instrument measure?* Then name one piece of information that would flip your choice.

SOLUTION — SITUATION BY SITUATION

Before reading the “Start in” column, decide for yourself — and ask what single change would flip your choice.

Situation	Start in	Reason	Flips to ... if
Roller-coaster car on a known track; comfort limit stated as an acceleration	n–t	path known; question is a_n against a limit	Cartesian, if the data were GPS points
Ship tracked by harbor radar (range and bearing), asked for its acceleration	polar	that is what the radar measures	Cartesian, for a rendezvous with a second ship
Tracker video of a bouncing ball, velocity between bounces	Cartesian	Tracker gives $x(t)$, $y(t)$; between bounces $a_x = 0$, $a_y = -g$	n–t, if the curvature at the apex were asked
Bead sliding outward along a spoke of the spinning wheel	polar	r and θ both change; $2\dot{r}\dot{\theta}$ is real	n–t would need the spiral's ρ — stay polar
Cloverleaf car with a GPS logger $x(t)$, $y(t)$; did the ride exceed $0.3\ g$ sideways?	Cartesian, then n–t	compute in the data's language, project $\vec{a}$ onto $\hat{u}_n$ to answer	— (two descriptions in one problem)
Pendulum bob on a fixed cord — then the cord is reeled in	n–t or polar (they coincide)	on a circle about the pivot $\hat{u}_n = -\hat{u}_r$, $\hat{u}_t = \hat{u}_\theta$	polar only, once $\dot{r} \neq 0$

STEP 5 — ANSWER

ANSWER

No description is universally better — the wrong one gives the right answer slowly. Compute in the instrument's language, answer in the question's language, validate in the third.

STEP 6 — VALIDATE

Same motion, three languages: the steady bead gives $|\vec{v}| = 2.26\ \text{m/s}$ and $|\vec{a}| = 17.1\ \text{m/s}^2$ in x–y, n–t and polar alike (Problem 1). If two descriptions disagree, the error is in the algebra, never in the physics.

WHAT TO WATCH FOR

Choosing coordinates by habit (“always x–y”) instead of from the data and the question; and treating $a_r = -r\dot{\theta}^2$ and $a_n = v_{sp}^2/\rho$ as different physics — same vector, $\hat{u}_n = -\hat{u}_r$.