

CHAPTER - 3

PART 3 · ≈ 80 MIN

Cartesian Kinetics — Cause and Effect

Net force is the cause; acceleration is the effect. The same free-body diagram runs forward to predict motion and backward to infer the force that motion required.

Observe

›

Idealize

›

Model

›

Solve / Analyze

›

Validate

›

Report



Before any calculation: in each case, what is the net force doing?

AIRPORT TOW SYSTEM · THREE CLIPS OR THREE STILLS



1

Steady speed across the apron

Moving, and not changing. What is the net force?

2

Pulling away from rest

Which way does the net force point?
Which way does the acceleration point?

3

Slowing to a stop

Still moving forward. Same two questions.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



PREDICT

The train crosses the apron at a steady 4 m/s.

What is the net force on it?

Most people do not answer zero. Say why you did or did not.



Ten minutes of this before a single equation. It is the whole Part in miniature.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Net force is the cause. Acceleration is the effect. Say it precisely or it teaches the wrong thing.

**Net force does NOT cause velocity.
Net force causes a CHANGE in velocity.**

WHEN THE FORCES BALANCE

$$\sum \vec{F} = 0 \implies \vec{a} = 0$$

The velocity does not change. It is not zero — it simply stops changing. Case 1 on the previous slide: 4 m/s, and no net force at all.

WHEN THEY DO NOT

$$\sum \vec{F} \neq 0 \implies \vec{a} \neq 0$$

The velocity changes — in magnitude, in direction, or in both. Cases 2 and 3, and every remaining slide of Chapter 3.

This is Newton's first law arriving where you can feel it, rather than as a definition.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



The same diagram and the same equations. Only the direction of the reasoning changes.

FORWARD · PREDICTION

known
forces



acceleration



predicted
motion

Given the drive force and the resistances, how fast is the train moving after 40 m? This is the direction textbooks emphasize.

INVERSE · INFERENCE

observed
motion



acceleration



required
net force

It stopped in 18 m from 6 m/s. What friction force was required, and can the tyres supply it? This is the direction engineers spend most of their time in.



Say it the right way round

The inverse direction is inference from an effect back to its required cause. The observed acceleration tells us what net force must have been acting. Acceleration never causes force.

Recognising that these are one machine, not two topics, cuts the apparent size of this chapter in half.

Observe



Idealize



Model



Solve / Analyze



Validate

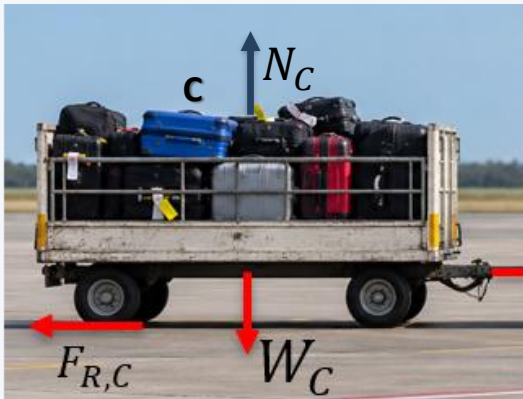


Report



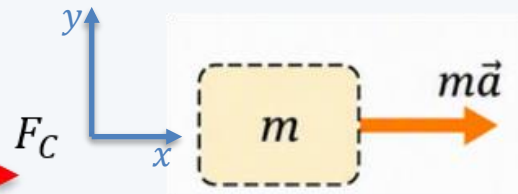
The mathematics is the bookkeeping. The diagram is the physics.

FBD ↔ ma-DIAGRAM ↔ EQUATIONS



Free-body diagram

External forces only. Front cart B exerts forward coupling force



$$m = m_C$$

ma diagram (cart C only)
Only mass of cart C

CARTESIAN EQUATIONS OF MOTION

$$\sum F_x = m a_x$$

$$\sum F_y = m a_y$$



A third equation, if you need it

In three dimensions the z-component joins them. Everything in this Part is planar.



The equations do not know what the force depends on

That is the next decision, and it decides whether you need algebra, calculus, or a numerical solution.

Keep the positive directions you chose in Part 2. The kinematics that follows must use the same ones.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Three levels. The first is the one to master; the third is the one to experience.

Level	The force is	What changes	Tool you need	Status
1 · MUST MASTER	constant, or known	acceleration is constant	Chapter 1 kinematics	core
2 · MUST UNDERSTAND	a function of time $F(t)$	acceleration is a function; the constant-a formulas become illegal	integration	core
3 · SHOULD EXPERIENCE	a function of speed or position	the equation of motion becomes a differential equation	one numerical solution	core, one example



This Part is not a course in ODEs

Level 3 gets exactly one example, chosen so the numerical solution obviously earns its place.



Nor a parade of crate problems

The source collection has six near-identical ones. One survives, as homework.

Observe



Idealize



Model

**Solve / Analyze**

Validate



Report

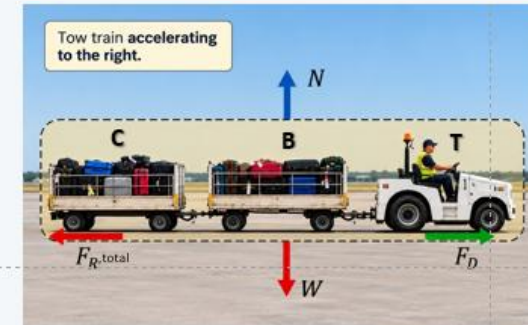


Level 1. We are solving the diagram Part 2 finished.

THE MODEL

Particle	the train as one system, then the rear cart alone
Description	x–y; the motion is rectilinear and horizontal
Knowns	tractor 900 kg, cart B 550 kg, cart C 325 kg; drive force 2.4 kN; rolling resistance 1.5% of each normal force
Assumptions	level apron; rigid couplings; wheels roll freely; no aerodynamic drag at 4 m/s
Unknowns	the acceleration, and the force in the rear coupling
Two cuts	whole train for the acceleration, then cart C alone for the coupling

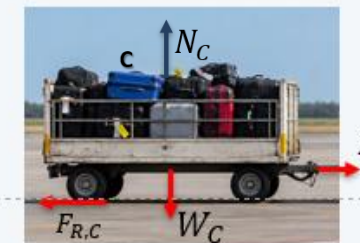
TWO CUTS, ALREADY DRAWN



Free-body diagram (entire tow train)
External forces only. Tractor + both carts treated as one system.

m
 $\rightarrow m\vec{a}$
 $m = m_C + m_B + m_T$

ma-diagram (entire tow train)
One mass, one acceleration vector.



Free-body diagram
External forces only. Front cart B exerts forward coupling force

m
 $\rightarrow m\vec{a}$
 $m = m_C$

ma diagram (cart C only)
Only mass of cart C

Modelling decision stated first: the path is straight and horizontal, so x–y is the natural description.

Observe



Idealize

**Model**

Solve / Analyze



Validate



Report



One equation gives the motion; a second cut gives the design number.

CUT 1 — THE WHOLE TRAIN

$$\sum F_y = 0 \Rightarrow N - W = 0 \Rightarrow N = m_{total} g$$

$$\sum F_x = m a_x \Rightarrow F_D - F_R = (900 + 550 + 325) a_x$$

$$2400 - (1.5\%) [1775(9.81)] = 1775 a_x$$

$$a_x = 1.21 \text{ m/s}^2$$

CUT 3 — THE REAR CART ALONE

$$\sum F_y = 0 \Rightarrow N_C - W_C = 0 \Rightarrow N_C = m_C g$$

$$\sum F_x = m a_x \Rightarrow F_C - (1.5\%) [325(9.81)] = 325(1.21)$$

$$F_C = 440$$

Illustrative numbers — replace with your own values.



Interpret — what the number is for

440 N is the load the rear drawbar carries while accelerating. At steady speed it drops to 47 N. The coupling is sized by the first number, not the second.



Validate

Units: N / kg = m/s². Sign: positive, so the cart is pulled forward. Limit: with no rolling resistance the acceleration rises to 1.35 m/s², as it should.



Notice what did the work

One free-body diagram per cut and one component equation each. Nothing in this example is new except the numbers.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



The motion is measured. The force is what we are trying to find out about.

OBSERVED MOTION → ACCELERATION → NET FORCE

A 1500 kg vehicle stops in 18 m from 6 m/s, braking steadily on level asphalt.

$$v_{sp}^2 = v_{sp,0}^2 + 2a_x s \Rightarrow a_x = -1.00 \text{ m/s}^2$$

$$F_{req} = m \cdot |a_x| = 1500 \text{ N}$$

Required friction coefficient: $1500 / (1500 \times 9.81) = 0.10$. Dry asphalt supplies roughly 0.7, so the stop was gentle.



This is the question engineers actually ask

Not “what will happen?” but “the vehicle did this — was the road able to supply what it needed?”



Friction arrives without an incline

A braking vehicle introduces friction through a question students care about, instead of through a block on a slope.

Same diagram, same equations, opposite direction of reasoning — and a verdict rather than a number.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



The most common error in this chapter has its own slide.

WHAT NOT TO DO

$$v_{sp} = v_{sp,0} + a_x t$$

Legal only when the acceleration is constant. If the force is a function of time, the acceleration is a function of time and this formula is simply false — it will still produce a number, which is what makes it dangerous.

WHAT TO DO INSTEAD

$$m \frac{dv_{sp}}{dt} = F(t)$$

Separate and integrate. The acceleration is known as a function, so the velocity follows from an integral rather than from a formula.



The teaching point is the illegality, not the integration

Most students can integrate. Far fewer stop to ask whether the constant-acceleration formulas apply — so ask it out loud, every time.

Before using any Chapter 1 formula, say one sentence: “the acceleration here is constant, because the net force is constant.”

Observe



Idealize



Model

**Solve / Analyze**

Validate

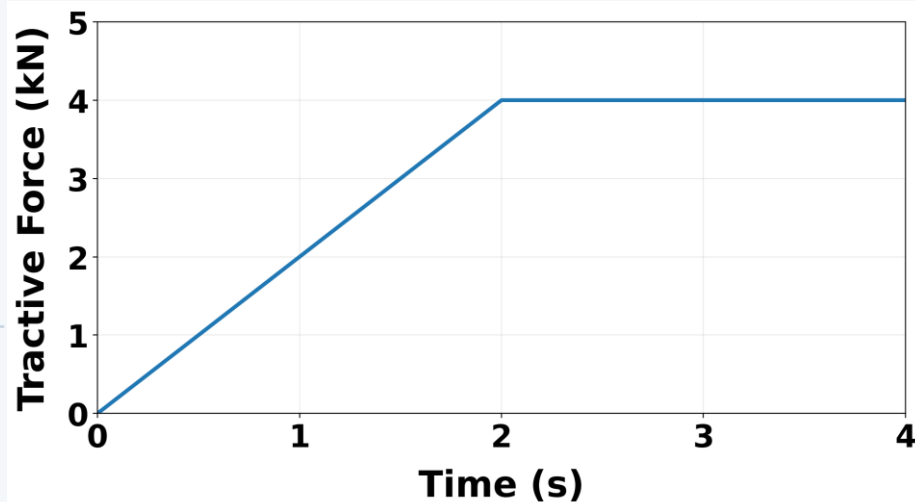


Report



Level 2. Integration is unavoidable, and the force curve is a real one.

DRIVE FORCE VERSUS TIME



SOLVE

1600 kg vehicle, from rest, resistances neglected over the first two seconds.

$$F(t) = 2000t \Rightarrow a_x(t) = 1.25t$$

$$v_{sp} = 0.625 t^2 \Rightarrow v_{sp}(2\text{ s}) = 2.50\text{ m/s}$$

A constant-acceleration formula using the final acceleration would have predicted 5.0 m/s — exactly twice the truth.



Why the factor of two

The vehicle spent the first second at less than half its final tractive force. Averages matter when the force ramps.



Validate

Units: N/kg integrated over s gives m/s. Limit: a flat force curve returns the constant-acceleration answer.

Observe



Idealize



Model

**Solve / Analyze**

Validate



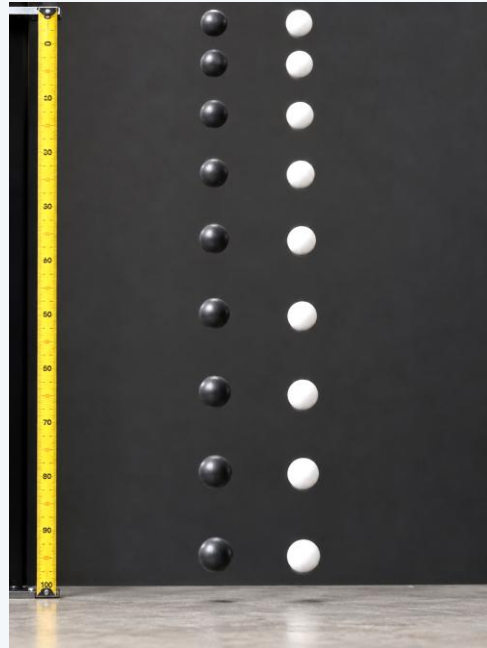
Report



Two balls of the same diameter and very different mass, dropped together.

SIMULTANEOUS DROP · 240 FPS

The two balls have the same diameter but very different masses, so they experience similar aerodynamic drag at the same speed but have very different weights. As their speed increases, air resistance affects the lighter ball much more strongly, causing the two trajectories to separate.



What Chapter 2 predicted

The projectile model has no mass in it. Both balls should fall identically and land together.



What we see

They separate visibly within two meters. The model the class was taught to trust has just failed.



This is not a trick — it is the motivation

A model is not wrong for failing outside its assumptions. It is wrong to keep using it there without noticing.

Something acts on these two bodies differently. It depends on speed, and it is the last force model of this Part.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



The equation of motion stops being an equation to rearrange and becomes one to solve.

FALLING WITH QUADRATIC DRAG

$$m \frac{dv_{sp}}{dt} = mg - cv_{sp}^2$$

The unknown appears on both sides. There is no rearrangement that produces $v(t)$ — this is a differential equation, and for the two-ball drop we will solve it numerically.

TERMINAL SPEED

$$v_{sp} \rightarrow \sqrt{\frac{mg}{c}}$$

Not a separate formula: it is the state where the acceleration reaches zero, read off the same equation.



Why the two balls separate

Same c , same size, very different m . The drag-to-weight ratio is what differs, and it is much larger for the foam ball.



Where the square law is valid

In a speed regime. At very low speed a linear law fits better. Stating the regime is part of the model.

At terminal speed the forces balance and the velocity stops changing — it does not stop. Newton's first law, again.

Observe



Idealize

**Model**

Solve / Analyze



Validate



Report



The chapter's one full computational activity.

ACTIVITY C3

1. Integrate $m \, dv/dt = mg - c \, v^2$ numerically for both balls.
2. Plot speed and height against time; mark the terminal speed.
3. Sweep c over a plausible range and see the family of curves.
4. Overlay your own tracked drop data and pick the c that fits.
5. Check that $c \rightarrow 0$ reproduces the Chapter 2 result exactly.



AI as coding companion

Use it for the integrator and the plotting. You choose the drag model, you choose the parameter, and you deliver the verdict.



One script is deliberately wrong

In the supplied AI-generated code the drag term does not reverse sign on the way down. Catch it from the shape of the plot, not by reading the code.



MEASURE · your own two-ball drop

Same diameter, different mass, filmed at 120 or 240 fps with a tape in frame. Tracker gives height against time for both — a direct extension of the Chapter 1 four-ball drop.

Observe



Idealize



Model

**Solve / Analyze**

Validate



Report



Name the move you are using each time.

1 Units

c has units of kg/m, so $c v^2$ comes out in newtons. If it does not, the model is mis-specified before any arithmetic.

2 Limiting case

Set $c = 0$ and the solution must reproduce the Chapter 2 projectile result exactly. If it does not, the integrator is wrong.

3 Order of magnitude

A skydiver's terminal speed is about 55 m/s. Does your fitted c give anything like that for a human-sized body?

4 Comparison with measurement

Overlay the tracked data. Quantify the disagreement rather than eyeballing it — and notice that fitting c until the curves match is not the same as validating the model.



That last distinction is Part 7

A fitted parameter must be checked against an independent estimate or a second experiment before it counts as evidence. Hold the thought — Part 7 makes it the main event.

Observe



Idealize



Model



Solve / Analyze

**Validate**

Report



Decide first, discuss second.

1 Constant velocity

The train crosses the apron at a steady 4 m/s. What is the net force? Zero — and it is still moving.

2 Is this formula legal?

You are given $F(t)$ and asked for the speed at 3 s. Can you use $v = v_0 + at$? No — the acceleration is not constant.

3 At terminal speed

What is the net force on the falling ball? Zero. And the speed is 30 m/s. Both statements are true at once.

4 Read the force from the motion

Given a measured speed-time curve with a sharp kink, sketch the force history that produced it. The kink is where the net force changed abruptly.

5 The driving wheel — held back from Part 2 on purpose

The tractor accelerates forward, so the net force on it points forward, so something pushes it forward. What? Friction from the road on the tyres — acting in the direction of motion.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Part 3 — cause and effect in x and y



You can now EXPLAIN

- why a body can move steadily with no net force on it
- why net force causes a change in velocity, not velocity
- why prediction and inference are one machine, not two topics
- when a constant-acceleration formula stops being legal
- what terminal speed is, in terms of the equation of motion



You can now DO

- solve $\Sigma F_x = m a_x$ and $\Sigma F_y = m a_y$ from a correct diagram
- infer a required force from a measured motion, and judge it
- integrate the equation of motion for a time-varying force
- recognize a speed-dependent force as a differential equation
- solve one numerically and validate it four ways

Next — Part 4: put two carts on one hitch and ask for the coupling force, and the force equations alone no longer determine the answer. The two bodies are not free to move independently.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Separation of variables, for the case where it works.

STRAIGHT-LINE FALL FROM REST

$$v_{sp}(t) = \sqrt{\frac{mg}{c}} \tanh \left(\sqrt{\frac{gc}{m}} t \right)$$

Valid only while the motion is downward and one-dimensional. As soon as the body is thrown rather than dropped, the drag direction reverses at the apex and the closed form no longer applies — which is precisely why the live core uses the numerical solution.



Use it once, as a check

Compare it against your numerical solution at three times. If they agree, trust the integrator on the cases with no closed



Also optional

The position-dependent case, $F(x)$, solved with the Chapter 1 relation $a \, ds = v \, dv$. One homework problem uses it.

Optional and deeper — not delivered in the live lecture. Available as reading and as homework scaffolding.

Observe



Idealize



Model

**Solve / Analyze**

Validate



Report