

CHAPTER - 3

PART 4 · ≈ 60 MIN

Connected and Constrained Particle Systems

A connection restricts the motion the bodies are allowed to have. Equations of motion plus kinematic constraint relationships give a complete model — and neither half is enough alone.

4

Observe

›

Idealize

›

Model

›

Solve / Analyze

›

Validate

›

Report



The tractor pulls away and the rear cart follows. Nothing reaches back through the air — a steel drawbar simply will not let two points separate.

TOW HITCH COUPLING · CLOSE VIEW



What Part 3 already gave us

The force in the drawbar, once we knew the acceleration. But we obtained that acceleration from the whole train — we never had to ask how the two bodies were related.



What happens without that shortcut

Treat the two carts as separate bodies and the force equations run short by exactly one. The missing information is not about forces at all.

Observe

Idealize



Model



Solve / Analyze



Validate



Report



PREDICT

The tractor accelerates. Does the rear cart accelerate at the same rate — and what physically guarantees that it does?

Now replace the rigid drawbar with a slack chain. What changes?

In a two-to-one lifting rig, which end moves faster, and by what factor?



The first question is the one this Part exists to answer properly.

Observe



Idealize



Model



Solve / Analyze



Validate

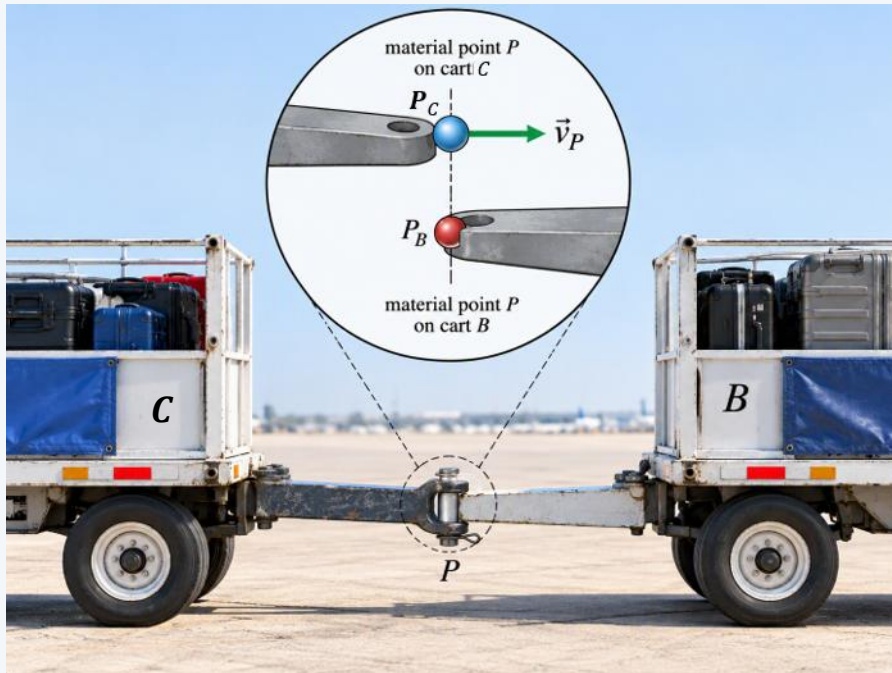


Report



Not that the bodies are the same. That one shared point must move as one point.

COMPATIBILITY AT THE SHARED POINT P



THE CONDITION AT THE CONNECTION

$$\vec{v}_P^{(C)} = \vec{v}_P^{(B)}$$

$$\vec{a}_P^{(C)} = \vec{a}_P^{(B)}$$

The pin does not let the two material points separate, so their velocities and accelerations must agree there.



Say it this way round, not the other

The connection constrains a point. It does not, by itself, say anything about the rest of either body — that comes next, and it depends on how each body is moving.

This distinction costs one slide now and saves a chapter later.

Observe



Idealize

**Model**

Solve / Analyze



Validate



Report



Not because they are connected. Because they are both translating.

1 The connection gives us P

The material points at the pin share a velocity and an acceleration. That is all the pin says.

2 Each cart is in translation

Every point on a translating body has the same velocity and the same acceleration as every other point on it.

3 So it propagates

The condition at P spreads to the whole of each body — and only then do the two cart accelerations become equal.

THE RESULT, WITH ITS CONDITION ATTACHED

$$\vec{a}_C = \vec{a}_B \quad \text{only because both bodies translate}$$



Stated as a consequence, never as a definition

A student who learns ‘connected means equal accelerations’ will apply it to a pin joining two rotating links in Chapter 6, and be wrong.

Observe



Idealize



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Solve / Analyze



Validate



Report

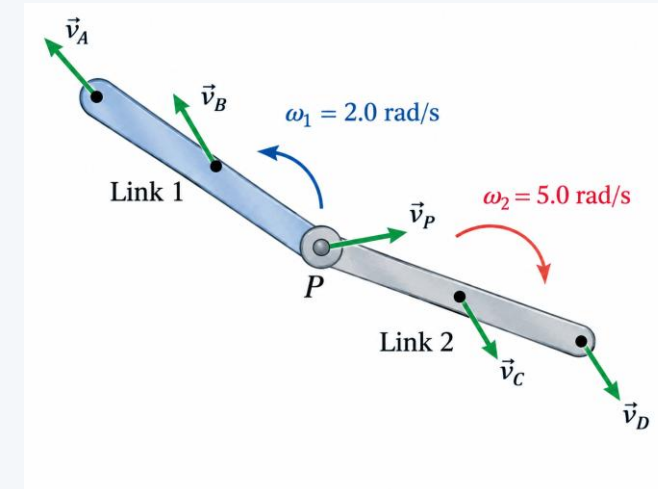


Plant this now; Chapter 6 collects it.

TWO TRANSLATING CARTS, JOINED AT P



TWO RIGID LINKS, PINNED AT P



The mathematics becomes richer later — but the physical principle does not change: connected bodies must have compatible motion at their common point.

Observe



Idealize

**Model**

Solve / Analyze



Validate

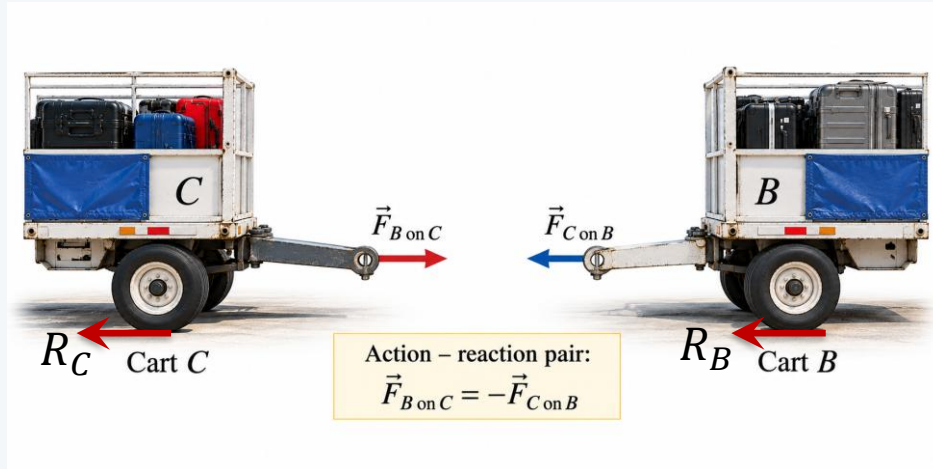


Report



Two bodies, two diagrams, one interaction pair — and the compatibility condition that closes it.

TWO FBDs AND TWO ma-DIAGRAMS



SOLVE

$$\text{Cart B: } \sum F_{x_B} = m_B a_B \Rightarrow P - F_{C \text{ on } B} - R_B = m_B a_B$$

$$\text{Cart C: } \sum F_{x_C} = m_C a_C \Rightarrow F_{C \text{ on } B} - R_C = m_C a_C$$

$$\text{Compatibility: } a_B = a_C$$

Three unknowns — the coupling force and the two accelerations. Two force equations, plus the compatibility condition. Three and three: closed.



The constraint is the third equation

It is not an extra trick bolted on. It is the information the force equations never contained.



Cross-check with Part 3

Cutting around the whole train gave the same acceleration in one equation. Two routes, one answer — an independent method check.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Count the unknowns, count the equations, name what is missing. Every system on its own.

System	Unknowns	Equations available
one cart alone	coupling force, acceleration — two	one component equation, plus the rolling-resistance model — two. Closed.
two carts, separate diagrams	coupling force, two accelerations — three	two component equations — two. Short by one.
two carts, plus the connection	coupling force, two accelerations — three	two component equations, plus compatibility — three. Closed.



No universal equation-counting formula is offered

A general rule is either wrong or so hedged it is useless, and a student who memorizes one stops looking at the system. Count each system as you meet it.

Equations of motion, plus kinematic constraint relationships, equals a complete model.

Observe



Idealize

**Model**

Solve / Analyze



Validate

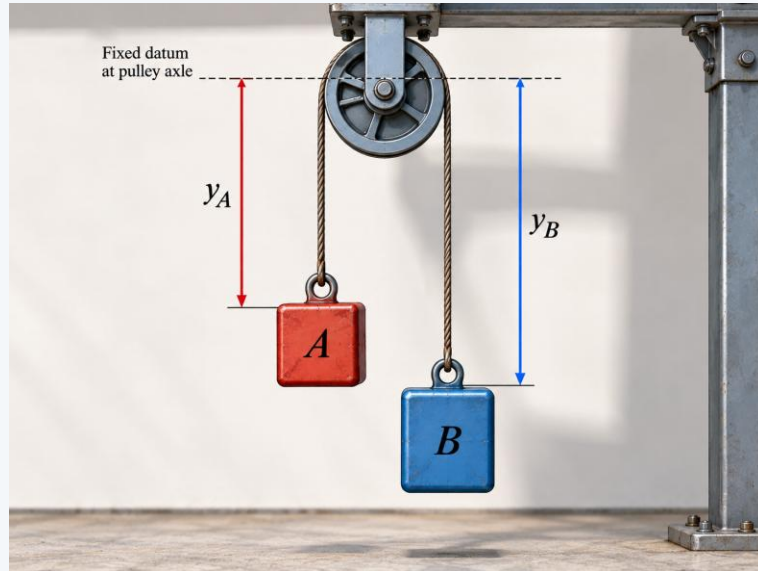


Report



A cable does not join two points. It fixes a total distance.

FIXED DATUM AND TWO COORDINATES



DIFFERENTIATE THE LENGTH

$$y_A + y_B = L$$

$$\dot{y}_A + \dot{y}_B = 0$$

$$\ddot{y}_A + \ddot{y}_B = 0$$

L is constant, so its derivatives vanish. The accelerations are equal and opposite.



The sign discipline from Chapter 2

Both coordinates measured from one fixed datum, in one consistent sense. Measure one up and one down and the signs will betray you.



The assumption you just made

Inextensible cable, massless cord, frictionless pulley. Write them down — Part 7 will come back for them.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



A cage and a counterweight on one cable over a drum. One motor, two masses.

ELEVATOR CAGE AND COUNTERWEIGHT



THE MODEL

Particles	the cage with its load, and the counterweight
Description	$x-y$, with one vertical coordinate for each, measured down from the sheave axle
Knowns	cage plus load 1200 kg; counterweight 900 kg; the drum drives the cable
Assumptions	massless inextensible cable; frictionless sheave; guide-rail friction neglected
Unknowns	the cable tension on each side and the acceleration
Constraint	the two coordinates sum to a constant cable length
Design question	what counterweight mass makes the motor's job easiest?

Modelling decision stated first: two bodies, one cable, so two diagrams and one constraint.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



The algebra is short. The design question is the point.

SOLVE

$$\begin{array}{l} \text{Cage: } \sum F_y = m_1 a \Rightarrow m_1 g - T = m_1 a \\ \text{Weight: } \sum F_y = m_2 a \Rightarrow T - m_2 g = m_2 a \end{array} \left. \vphantom{\begin{array}{l} \text{Cage: } \sum F_y = m_1 a \Rightarrow m_1 g - T = m_1 a \\ \text{Weight: } \sum F_y = m_2 a \Rightarrow T - m_2 g = m_2 a \end{array}} \right\} \begin{array}{l} 2 \text{ eqs. \& } \\ 2 \text{ unknowns} \end{array}$$

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2} = 1.40 \text{ m/s}^2$$

and $T = 10.3 \text{ kN}$ — which is neither weight.



The design question

Set the counterweight equal to the cage plus half the rated load. The motor then works hardest at the empty and full extremes, and hardly at all in between.



Read the sign

If the acceleration came out negative, the answer is telling you the assumed direction was wrong — not that you should restart the problem.



Validate with two limiting cases

Equal masses give zero acceleration. One mass going to zero gives an acceleration approaching g . Both are right.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



The prediction is clean. The measurement is a few per cent low, every time.

ATWOOD MACHINE



PREDICT, THEN MEASURE

Predict the acceleration from the two masses. Film the fall and track it. The measured value comes out consistently below the prediction — not scattered around it, but below it.



Why it is low

The pulley has mass, and it is moving too! Rotating.

Some of the driving effort goes into spinning it, which the massless-pulley model never accounted for.



A debt, named now

Chapter 7 gives the pulley a moment of inertia and pays it. Say so out loud — an unexplained discrepancy is worse than an explained one.



This is a rehearsal for Part 7

A systematic disagreement with a consistent sign is the signature of missing physics, not of measurement scatter. Part 7 makes that distinction its main subject.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Each one has a condition hiding in it.

1

“Connected means identical motion.”

Only the shared point is constrained. Equal body accelerations follow only when both bodies are translating.

2

“The tension equals the hanging weight.”

True only when the acceleration is zero. Part 1's elevator contradicted this before we ever drew a pulley.

3

“Both blocks always have the same acceleration.”

False as soon as the cable is reeved. Derive the constraint from the geometry; never assume it.

4

“The pulley changes the tension.”

Only a pulley with mass or bearing friction does. An ideal pulley changes the direction of the cable force and nothing else — which is exactly why the Atwood measurement disagrees.

5

“The answer came out negative, so I made a mistake.”

Not necessarily. A negative result for an assumed direction is the algebra telling you the true direction is the other one. Read it; do not restart.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



The one job in this Part worth handing over.

ACTIVITY C4

1. Write the constraint relation for your system from the geometry.
2. Differentiate it twice by hand.
3. Ask an AI to verify the differentiation and the sign convention — a narrow, checkable task.
4. Solve the coupled equations symbolically and confirm that the system is linear.



Why this task and not another

Differentiating a stated relation has a right answer that you can check independently. Choosing the boundary does not.



What the algebra is not

It is not where the engineering lives. The engineering was choosing the coordinates and writing the constraint.

Hand over what is verifiable. Keep what requires judgement.



Homework connection

One problem uses a constraint whose length changes geometrically — a tractor lifting a load through a boom.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Part 4 — connections restrict possible motion



You can now EXPLAIN

- what a pin or a hitch actually constrains
- why equal accelerations follow from translation, not connection
- why force equations alone leave a connected system open
- what the massless-cord and ideal-pulley assumptions buy
- why the Atwood measurement runs low, and what pays that debt



You can now DO

- write a compatibility condition at a shared connection point
- derive a cable constraint and differentiate it twice
- count unknowns against equations for a specific system
- solve a coupled force-and-constraint system with consistent signs
- read a negative result as a direction, not as an error

Next — Part 5: every system so far has moved in a straight line. Curve the path and the acceleration acquires a component nobody applied a force to produce — except that somebody must have.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



One more constraint geometry, for the students who want it.

**SNATCH BLOCK,
REEVED TWO TO ONE**



READ THE CONSTRAINT OFF THE DRAWING

$$2s_A + s_B = L$$

$$2\ddot{s}_A + \ddot{s}_B = 0$$

The hauler's end moves twice as fast as the load, so the load accelerates at half the rate. The rope force is halved and the distance pulled is doubled — which is what the rig actually buys, and what it costs.

Optional and deeper — not delivered live. Stop at one movable pulley; the source collection's three- and four-body trains are omitted entirely.

Observe



Idealize

**Model**

Solve / Analyze



Validate



Report