

CHAPTER - 3

PART 5 · ≈ 85 MIN

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# Forces Cause Curved-Path Motion — n–t Coordinates

The tangential net force changes speed. The normal net force changes direction. Everything in this Part is a consequence of those two sentences.

5

Observe

›

Idealize

›

**Model**

›

Solve / Analyze

›

Validate

›

Report



# Same corner, same motorcycle, same footage. A different question.

## RACETRACK CORNER · REUSE THE CHAPTER 2 ASSET

A motorcycle leaning through a corner has  $a_n = v^2/\rho$  toward the center.

We can compute it, we learned it in Chapter 2. Now, we can ask what force supplies it — tire friction, normal tangential forces, and what happens when it cannot.



Chapter 2 asked:

**How is the race motorcycle moving?**

Chapter 3 asks:

**What must the forces be doing to make it move that way?**



### You already have the kinematics

Chapter 2 gave  $a_t$  and  $a_n$  for exactly this corner. Nothing new is needed to describe the motion.



### What is missing is the cause

Those accelerations did not happen on their own. Something had to supply them.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



# Two acceleration components, each with a clear physical job.

## ALONG THE PATH

$$a_t = \dot{v}_{sp}$$

The rate of change of speed. Positive when speeding up, negative when slowing, zero at constant speed — whatever the path is doing.

## ACROSS THE PATH

$$a_n = \frac{v_{sp}^2}{\rho}$$

The rate of change of direction. Always directed toward the centre of curvature. Never negative, and never zero on a curve.

**Now ask the Chapter 3 question: what forces must produce those accelerations?**



### The axes ride with the particle

n and t are set on the body and turn as it moves. There is no origin — that contrast matters in Part 6.

Observe



Idealize

**Model**

Solve / Analyze



Validate



Report



## PREDICT

**If a car goes through a rounded corner at a constant 40 m/s.**

**Is it accelerating? In which direction?**

**And only then: which real interactions could possibly supply that?**

*Answer the first two before anyone draws a force. The order matters.*



Observe



Idealize



Model



Solve / Analyze



Validate



Report



# Constant speed, curved path: the acceleration points inward, so a net inward force must exist.

## STEP 1 — READ THE MOTION

$$a_t = 0, \quad a_n = \frac{v_{sp}^2}{\rho} \Rightarrow$$

*a net inward force of  $F_{net} = m \frac{v_{sp}^2}{\rho}$  is required*

*Nothing has been assumed about what supplies it. The requirement follows from the motion alone.*



## STEP 2 — now draw the diagram

Which real interactions have an inward component? Here: the friction from the road on the tyres, and — if the track is banked — part of the normal force.



## Why this order and not the other

Establishing the requirement before the forces makes the centripetal term a consequence of the motion rather than a new force to memorise — and it makes the centrifugal misconception much harder to form in the first place.

**Observed motion → required acceleration → required net force → which interactions supply it.**

Observe



Idealize

**Model**

Solve / Analyze



Validate



Report



# Two components, two jobs, and a third equation that says nothing leaves the plane.

## n-t EQUATIONS OF MOTION

$$\sum F_t = m a_t = m \dot{v}_{sp}$$

$$\sum F_n = m a_n = m \frac{v_{sp}^2}{\rho}$$

$$\sum \vec{F}_b = 0$$



### Tangential — changes SPEED

Zero whenever the speed is constant, however sharp the corner is.



### Normal — changes DIRECTION

Never zero on a curve. Always directed toward the centre of curvature, because  $a_n$  is.



### There is no third force here

The binormal equation says only that nothing accelerates out of the plane of the motion. Recognise it; you will not need to compute with it.

**The tangential net force changes speed. The normal net force changes direction.**

Observe



Idealize



Model



Solve / Analyze



Validate

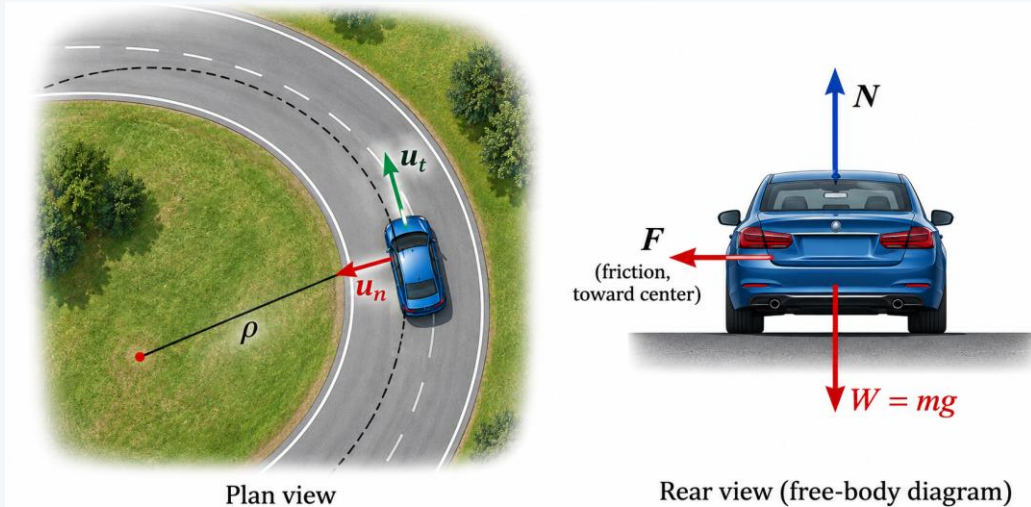


Report



# Constant speed on a level road: friction alone must supply the whole requirement.

## LEVEL HIGHWAY CURVE



## SOLVE

$$\sum F_n = ma_n \Rightarrow F_f = m \frac{v_{sp}^2}{\rho} \leq \mu_s mg$$

$$\rho \geq \frac{v_{sp}^2}{\mu_s g}$$

At 25 m/s with  $\mu_s = 0.70$  on dry asphalt, the radius must be at least 91 m. In the wet, with  $\mu_s = 0.35$ , it must be at least 182 m — twice as large for the same speed.



### The mass cancels

Both sides carry  $m$ . A loaded truck and an empty car have the same limiting radius — **which is why signs post speeds, not vehicle types.**



### Validate against practice

Compare your minimum radius with a published highway design table. Same order of magnitude means the model is behaving.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



# There is not one. Confront the word once, clearly, and move on.

## WHAT STUDENTS DRAW

An outward arrow on the car, named "**centrifugal force**", balancing the inward friction. If those two really balanced, the net force would be zero — and the car would go straight. It does not go straight. So, they do not balance, and one of them is not there.

**Centrifugal force does not exist** when a system is described relative to a non-rotating, stationary frame of reference.

**Centripetal force** is an inward pull (like a string or friction on tires) that forces an object into a circular path.

## WHAT IS ACTUALLY HAPPENING

In the road's frame there is one horizontal force on the car, and it points inward. The passenger feels the door pushing them inward, and interprets it backwards because their own body wants to continue straight.

That tendency is inertia, and inertia is not a force.



## Two minutes, not ten

The word is confronted rather than avoided, because students arrive with it and will use it. Then the Part moves on.

**Everything in Chapter 3 is written in an inertial frame. In an inertial frame there is no centrifugal force.**

Observe



Idealize



Model



Solve / Analyze

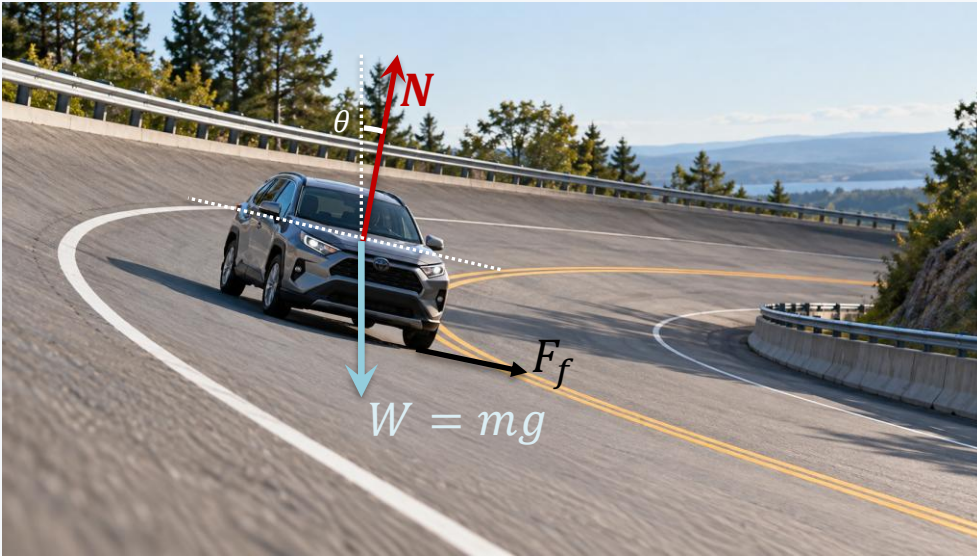
**Validate**

Report



# Tilt the road and the normal force acquires an inward component of its own.

## BANKED HIGHWAY CURVE



## AT THE DESIGN SPEED: NO FRICTION REQUIRED, BY DESIGN

$$\sum F_n = ma_n \Rightarrow N \sin(\theta) + F_f \cos(\theta) = ma_n$$

Note that the friction force  $0 \leq F_f \leq \mu_s N$ .

Is there a specific  $\theta$ , which will enable  $F_f = 0$ ?

$$N \sin \theta = m \frac{v_{sp}^2}{\rho}$$

$$N \cos \theta = mg$$

$$\tan \theta = v_{sp}^2 / (g \cdot \rho)$$

*The mass cancels again — the bank angle is a property of the road and the design speed.*



### Banking does not tilt gravity

It tilts the normal force, so that part of it points inward. The weight is exactly as vertical as it ever was.



### One bank angle, one speed

Above or below the design speed, friction has to make up the difference — which is why real roads are banked for a compromise.

Observe



Idealize



Model



Solve / Analyze



Validate

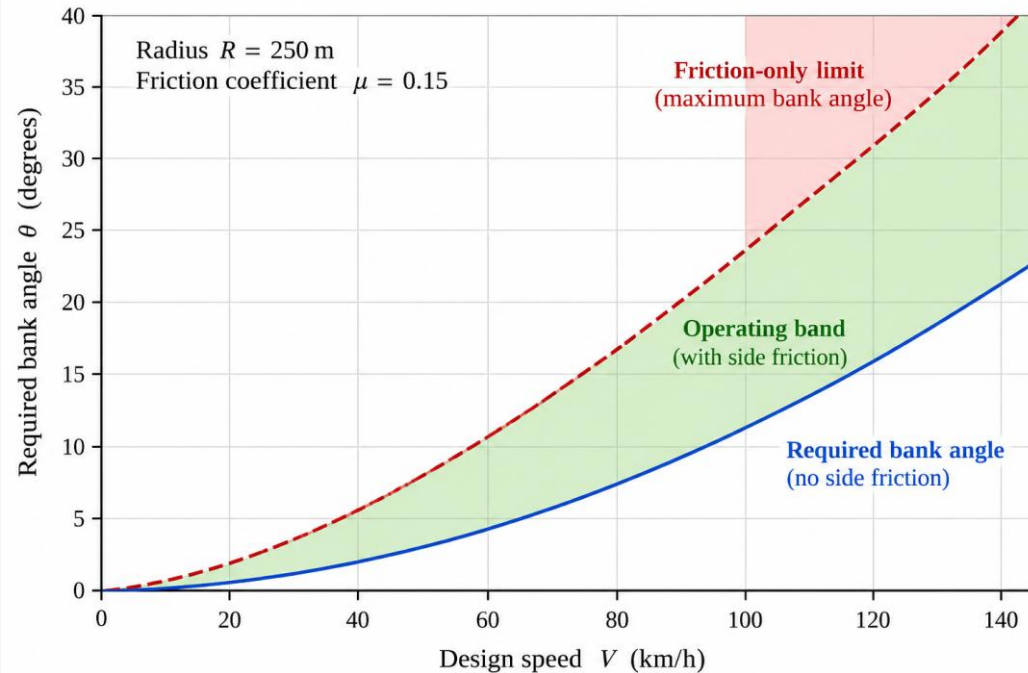


Report



# Sweep the design speed: Note the two limits close in on each other.

## BANK ANGLE VERSUS DESIGN SPEED



## ACTIVITY C5

1. Fix the radius. Sweep the design speed.
2. Plot the frictionless bank angle against speed.
3. Overlay the maximum speed friction alone could handle on a level road.
4. Shade the region where both are satisfied — that band is the design envelope.
5. Where do the two curves cross, and what does that crossing mean physically?



## AI as coding companion

Use it for the plotting. The engineering judgement is choosing the comfort limit and reading the envelope — and that stays with you.

Observe



Idealize



Model

**Solve / Analyze**

Validate

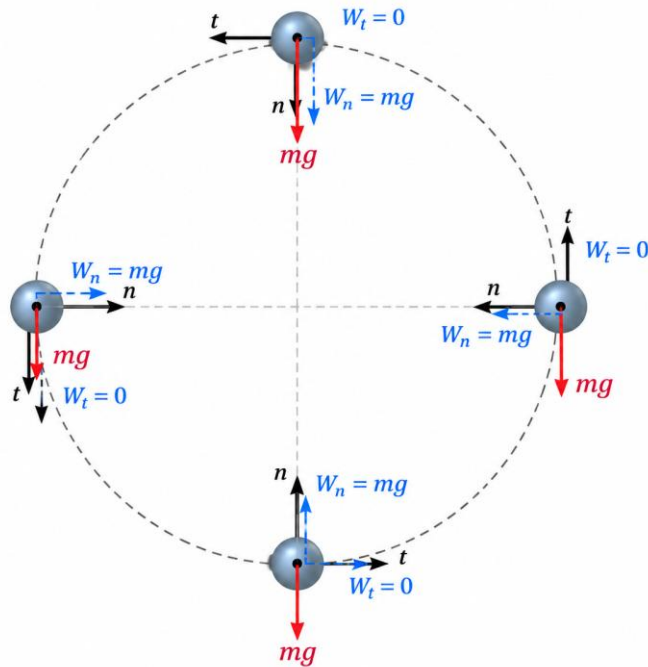


Report



# The weight now has a tangential component as well as a normal one — so the speed changes too.

**SAME CURVE,  
FOUR  
POSITIONS**



**AT THE TOP OF THE CIRCLE**

$$N + mg = m \frac{v_{sp}^2}{\rho}$$

**AT THE BOTTOM**

$$N - mg = m \frac{v_{sp}^2}{\rho}$$



**Apparent weight, from Part 1, doing real work**

At the bottom the seat must supply more than the weight; at the top it supplies less. Same equation, opposite sign on the same term.



**And the speed is changing**

The tangential component of the weight is non-zero except at the very top and bottom, so the body speeds up and slows down as it goes round.

Observe



Idealize



**Model**



Solve / Analyze



Validate



Report



# The cleanest $N = 0$ problem in the chapter.

## CAR AT A HUMP-BACK BRIDGE



**SOLVE:** (Note that curvature is looking down)

$$\sum F_n = ma_n \Rightarrow mg - N = m \frac{v_{sp}^2}{\rho}$$

$$N = m \left( g - \frac{v_{sp}^2}{\rho} \right)$$

*N falls as the speed rises, and reaches zero when the speed equals the square root of  $g\rho$ . For a 40 m crest radius that is about 20 m/s — roughly 72 km/h.*



### The scale reading again

N / W falls below one at a crest, and rises above one in a dip. The same ratio Part 1 introduced.



### Above that speed?

The equation returns a negative N. That is the subject of the next slide, and it is not an arithmetic problem.

Observe



Idealize



Model

**Solve / Analyze**

Validate



Report



# The first time in this course that an equation reports the failure of its own assumptions.

## THE CONTACT MODEL

$$N \geq 0$$

A surface in unilateral contact can push but never pull. That inequality is not a result — it is a condition on the validity of the model you wrote.

## AND THE SAME FOR A CORD

$$T \geq 0$$

A cord pulls but cannot push.  $T = 0$  is the instant it goes slack — and beyond that instant the body is a projectile, not a body on a circular path.



### How to read a negative $N$ , in one sentence

The assumed contact model is no longer physically valid: the body has left the surface, and a different model applies from that point on. It is not a sign error, and there is no such thing as a negative normal force.

**Contact loss, tension loss, slip onset — three names for one idea: the moment an assumption expires.**

Observe



Idealize

**Model**

Solve / Analyze



Validate

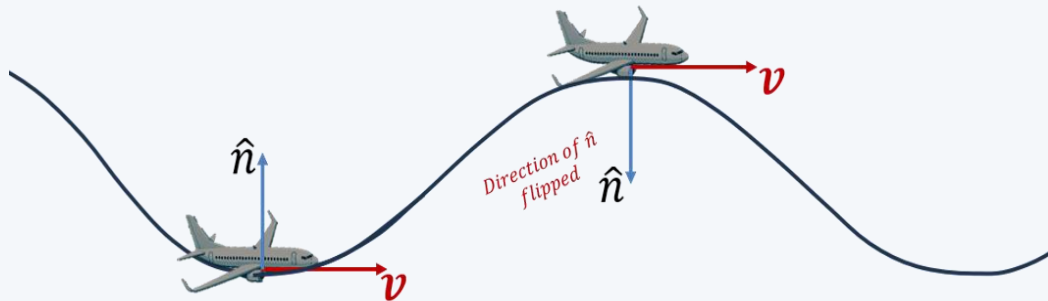


Report



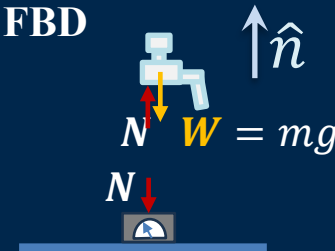
# The Zero-G aircraft: at the **BOTTOM** of the arc

## THE PARABOLIC ARC, AGAIN



## AT THE BOTTOM OF THE ARC

FBD



$$\begin{aligned}\Sigma F_n &= ma_n \\ N - W &= \left(m \frac{v^2}{\rho}\right) \\ N &= mg + \left(m \frac{v^2}{\rho}\right)\end{aligned}$$

The weight scale will show  $mg$  plus the term inside (..), i.e. a person feeling heavier



**The same picture as in Part-1, a genuinely new question**

Part 1 asked why the passengers float. Part 5 asks what shape the aircraft must fly for them to keep floating — and answers it in one line.

**One callback, once. If a slide here re-derives  $N = m(g + a_v)$ , it is a repeat and should be cut.**

Observe



Idealize



Model



Solve / Analyze



**Validate**

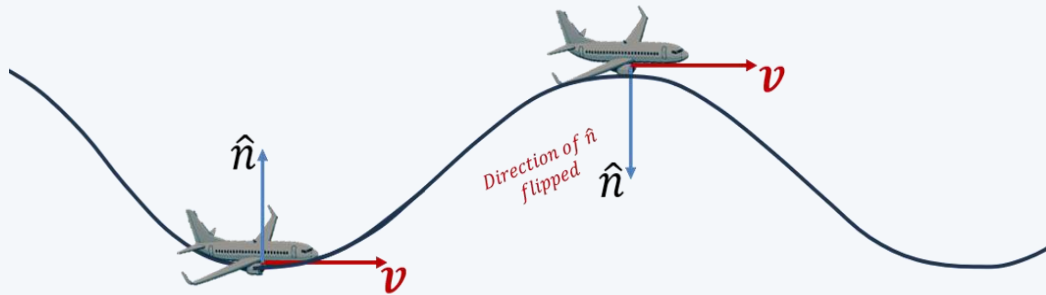


Report



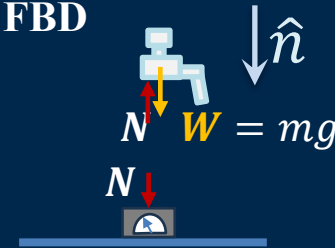
# The Zero-G aircraft: at the TOP of the arc.

## THE PARABOLIC ARC, AGAIN



## AT THE TOP OF THE ARC

FBD



$$\begin{aligned}\Sigma F_n &= ma_n \\ -N + W &= \left(m \frac{v^2}{\rho}\right) \\ N &= mg - \left(m \frac{v^2}{\rho}\right)\end{aligned}$$

- a) If  $mg > (\dots)$  he will be in his seat feeling less weight.
- b) If  $mg = (\dots)$  he will be floating weightless.
- c) If  $mg < (\dots)$  loose contact with the seat unless restrained!



### Curvature changed

From bottom to top the curvature becomes upside down ( $\hat{n}$  is flipped)

One callback, once. If a slide here re-derives  $N = m(g + a_v)$ , it is a repeat and should be cut.

Observe



Idealize



Model



Solve / Analyze



Validate

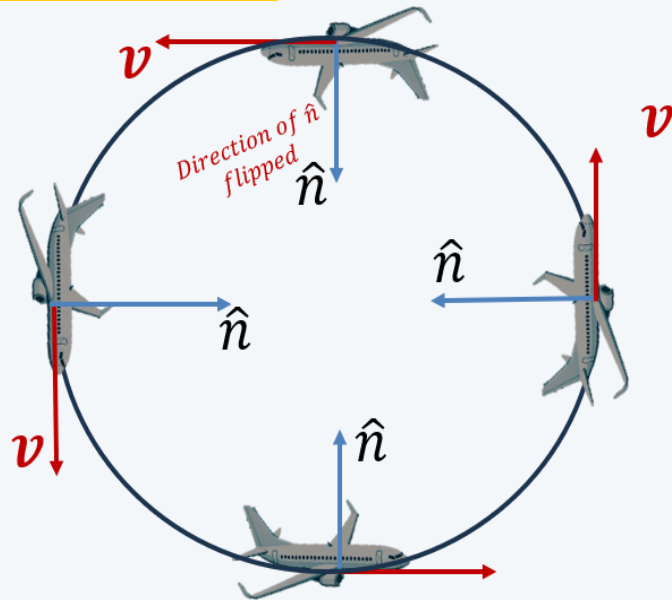


Report



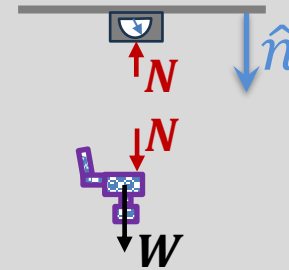
# What if an aircraft goes in a circular loop (FLIPPED).

## AIRPLANE IN A CIRCULAR LOOP



## AT THE TOP OF THE ARC

FBD



$$\begin{aligned}\sum F_n &= ma_n \\ N + W &= \left(m \frac{v^2}{\rho}\right) \\ N &= \left(m \frac{v^2}{\rho}\right) - W\end{aligned}$$

- a) If  $(..) > W$  he will be in his seat feeling less weight.
- b) If  $(..) = W$  floating weightless.
- c) If  $(..) < W$  loose contact with the seat unless restrained!



## The same picture, a genuinely new question

Part 1 asked why the passengers float. Part 5 asks what shape the aircraft must fly for them to keep floating — and answers it in one line.

One callback, once. If a slide here re-derives  $N = m(g + a_v)$ , it is a repeat and should be cut.

Observe



Idealize



Model



Solve / Analyze



Validate

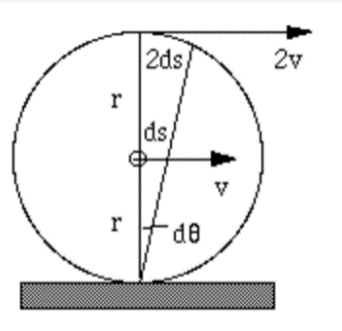
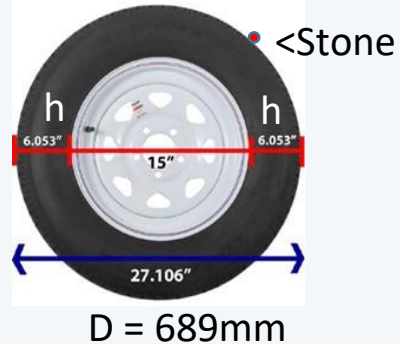


Report



# Instructor-created, and one of the best numbers in the course. Read the geometry off the sidewall.

## STONE LODGED IN THE TREAD



## 205 / 75 R 15 — WHAT THE SIDEWALL TELLS YOU



|                      |                                                                   |
|----------------------|-------------------------------------------------------------------|
|                      | <b>R – Radial Construction</b>                                    |
| <b>Tread width</b>   | 205 mm                                                            |
| <b>Aspect ratio</b>  | 75%, so the sidewall is<br>$205 \times 0.75 = 154\text{ mm}$      |
| <b>Rim</b>           | 15 in = 381 mm diameter                                           |
| <b>Overall</b>       | $381 + 2 \times 154 = 689\text{ mm}$ ,<br>so $r = 0.345\text{ m}$ |
| <b>Vehicle speed</b> | 65 mph = 29.06 m/s                                                |
| <b>Stone</b>         | 5 g, wedged in a groove                                           |



## Before the numbers, one observation

A tread point at the top of the tyre is moving at twice the vehicle speed relative to the ground. That is the example's best insight — and the next slide shows why it has to be handled with care.

## Tire size designation is made up of mixed units!

*mm, %, R, in*

Observe



Idealize



**Model**



Solve / Analyze



Validate

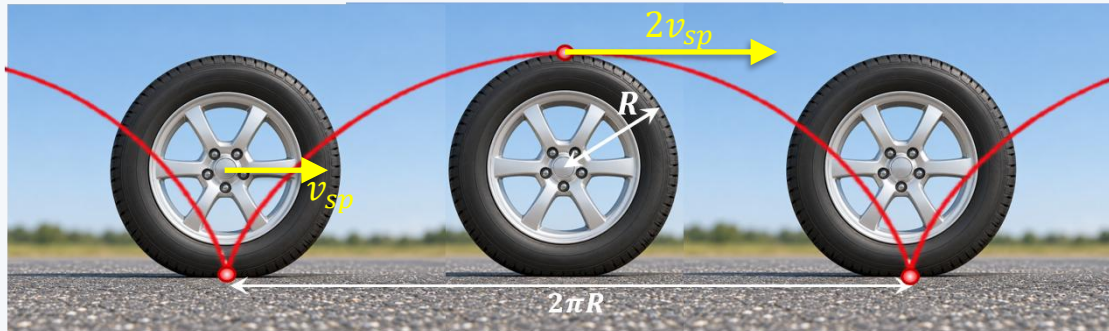


Report



# The tread point is moving at $2v$ . So what radius does it turn on?

## THE PATH OF A TREAD POINT

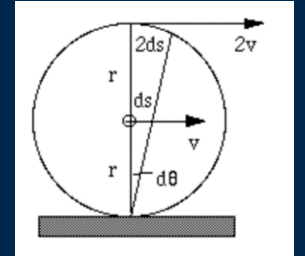


## TWO WAYS, ONE ANSWER

In the ground frame the path is a cycloid, whose radius of curvature at the top is  $4r$  — not  $r$ :

$$a_n = \frac{(2v_{sp})^2}{4r} = \frac{v_{sp}^2}{r}$$

In the wheel-centre frame — which translates at constant velocity, so it is also inertial — the stone simply travels a circle of radius  $r$  at speed  $v$ . Same answer, one line sooner.



## The trap, named

Pairing the ground-frame speed  $2v$  with the wheel radius  $r$  overstates the acceleration by a factor of four. Speed and radius of curvature must be read in the same frame.

**Forward link — Chapter 6 calls this point the instantaneous centre and makes the whole construction routine.**

Observe



Idealize



Model



Solve / Analyze



Validate



Report



# Five grams, 250 times its own weight, at 13 hertz.

What is the unbalance force of a 5g stone stuck on this tire's groove when the vehicle is going steady 65mph? (65mph=29.06 m/s)

**SOLVE**

$$a_n = \frac{v_{sp}^2}{r} = \frac{(29.06)^2}{0.345} = 2450 \text{ m/s}^2$$

$$F_n = m a_n = 0.005 \times 2450 = 12.3 \text{ N}$$

*A 5 g stone needs 12.3 N to stay in the groove — about 250 times its own weight.*



## Rotation rate

$\omega = v / r = 84.4 \text{ rad/s}$ , which is 13.4 revolutions per second (Hz).



## So the wheel bearing sees

A 12.3 N force rotating at 13.4 Hz — a harmonic forcing function.



## Forward link — Chapter 10

This is exactly the rotating-unbalance forcing that drives a suspension into resonance. The stone is a Chapter 3 kinetics problem and a Chapter 10 vibration source, and it is worth saying so here.

**Sweep the vehicle speed and the force goes as the square of it — which is why the thumping gets dramatically worse on the motorway.**

Observe



Idealize



Model



**Solve / Analyze**



Validate



Report



# As supplied it predicts a tension. Adding a measurement turns it into a validation.

## SLOW-MOTION VIDEO · INSTRUCTOR ASSET

Slow-motion video of a tennis ball swung on a 0.6 m string. Add an in-line spring scale or load cell at the hand end so the tension is measured independently of the prediction.



## PREDICT FROM THE VIDEO

Count 10 revolutions, get  $\dot{\theta} = 13.4 \text{ rad/s}$ , so the speed is

$$v_{sp} = r \dot{\theta} = 8.04 \text{ m/s}$$

With a 56.4 g ball the predicted tension (neglecting gravity) is

$$T = m r \dot{\theta}^2 = 6.1 \text{ N.}$$



## How about effect of gravity?

$$\sum F_n = m a_n$$

At the TOP:  $T + mg = m (r \dot{\theta}^2) \Rightarrow T = m(r \dot{\theta}^2 - g)$

At the Bottom:  $T - mg = m (r \dot{\theta}^2) \Rightarrow T = m(r \dot{\theta}^2 + g)$

At the 90° Sides:  $T = m (r \dot{\theta}^2) \Rightarrow T = m(r \dot{\theta}^2)$   
 $r \dot{\theta}^2 = 108 \text{ vs } g = 9.81$



## The modelling challenge in this experiment:

Because the ball moves in a vertical circle, gravity contributes differently to the normal-force balance as the ball moves around the circle. Therefore, **the string tension is not constant.**

Observe



Idealize



Model



Solve / Analyze



Validate



Report



# Two of these were wrong on the old slides. That is why they are here.

## 1 Constant speed round a corner

Is it accelerating? Yes — the direction is changing, and that is an acceleration.

## 2 Scale reading at a crest

Less than on level road, and zero at the contact-loss speed. Above that, the wheels leave the surface.

## 3 Forces at the top of a loop

Weight down, and the track or cord pushing or pulling inward. There is no upward force holding it there.

## 4 Does the bank angle depend on mass?

No. Both equations carry  $m$ , and it cancels. The road is banked for a speed, not for a vehicle.

## 5 Where is the outward force?

There is none. In the road's frame the only horizontal force on the car points inward.

## 6 $N$ came out negative

Not an algebra error. The contact assumption has expired and a different model applies.

Observe



Idealize



Model



Solve / Analyze

**Validate**

Report



## Part 5 — forces on a curved path



### You can now EXPLAIN

- which force component changes speed and which changes direction
- why the requirement is read from the motion before forces appear
- why no outward force appears on a cornering body
- what a negative normal force is actually telling you
- why the top of a rolling wheel does not turn on radius  $r$



### You can now DO

- apply  $\Sigma F_t = m a_t$  and  $\Sigma F_n = m a_n$  correctly oriented
- find the minimum radius or maximum speed for a friction-limited turn
- find a frictionless bank angle, and say why mass cancels
- find a contact-loss or tension-loss condition and interpret it
- predict a tension and then test the prediction against a measurement

**Next — Part 6:  $n$ - $t$  works when the path is known and the axes can ride along it. When a machine sets the geometry, the natural description is anchored to the machine's own centre.**

Observe



Idealize



Model



Solve / Analyze



Validate

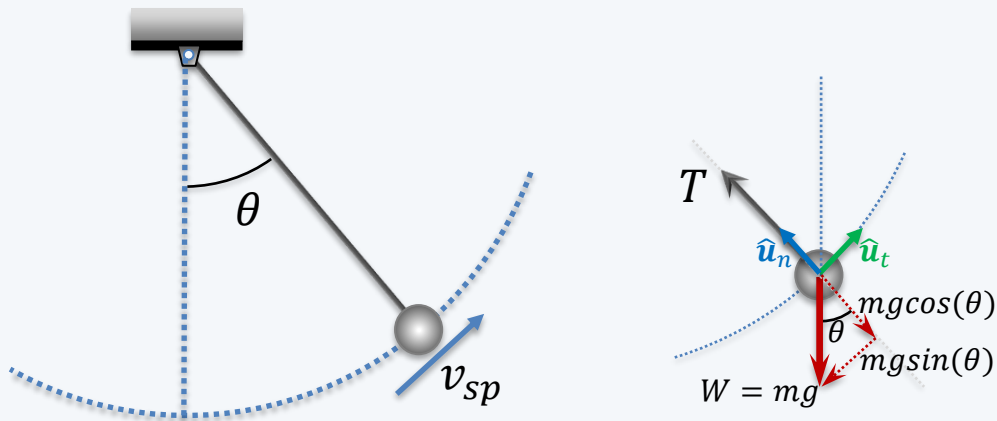


Report



# Both components active, all the way round.

## VERTICAL CIRCLE, ONE FULL SWING



## AT AN ARBITRARY ANGLE

$$\sum F_n = ma_n \rightarrow T - mg \cos \theta = m \frac{v_{sp}^2}{\rho}$$

$$\sum F_t = ma_t \rightarrow -mg \sin \theta = m \dot{v}_{sp}$$

Both components are active at every position except the very top and bottom.  $T = 0$  gives the angle at which the cord goes slack — beyond it the ball is a projectile.



### Where this Part deliberately stops

Asking for the speed at a second angle requires integrating a varying tangential acceleration. It is possible and it is punishing — and it is trivial with energy methods. That frustration is Chapter 4's motivation, so it is left intact.

Optional and deeper — not delivered live.

Observe



Idealize



Model



Solve / Analyze



Validate



Report