

CHAPTERS 1–3 COMPREHENSIVE ASSESSMENT

Particle Kinematics and Kinetics



AI First Engineering

Taner Onsay, Ph.D. · © 2026 AI First Engineering LLC · 34 responses · 100 points · about 105 minutes

Purpose. This comprehensive assessment is a learning checkpoint. It lets you integrate and evaluate your understanding of the concepts, modelling approaches and problem-solving methods developed in Chapters 1–3: describing motion on straight and curved paths, choosing a coordinate description, and connecting forces to the motion they cause. The questions ask you to do what an engineer does — understand the situation, choose the model, compute, interpret, and decide whether the answer is believable.

How this checkpoint works

- Three parts: **A · Concept and Model Checkpoints** (10 responses, 20 points, ≈ 15 min) · **B · Setup and Calculate** (6 responses, 24 points, ≈ 25 min) · **C · Linked Engineering Scenarios** (18 responses, 56 points, ≈ 64 min).
- Type every answer in its labelled field. Where a field asks for **Equation used**, write the governing equation, with known values inserted if you like; it earns credit even if your arithmetic slips.
- In linked scenarios, a later answer is graded using *your own* earlier answer, so one early mistake is not penalised twice.
- Use $g = 9.81 \text{ m/s}^2$ and report three significant figures with units. Each question states its origin, axes and positive directions; use them.
- For the most useful self-assessment, complete the checkpoint independently using only the provided formula sheet and a calculator. Afterward, use AI as a tutor to investigate missed questions and strengthen your understanding.

Typing equations: use the mathematical symbols and notation shown in the provided formula sheet. Use ordinary keyboard equivalents when needed; put units after numerical answers.

Part A — Concept and Model Checkpoints

10 responses · 20 points · ≈ 15 min

Select one answer for each question. Each is worth 2 points.

A01 · Momentarily at rest (2 pts)

A positioning stage moves along a straight rail. Its position is $s(t) = 2t^3 - 9t^2 + 12t + 1$, with s in metres, t in seconds, and $+s$ to the right. Which statement is correct?

- A. It is momentarily at rest at $t = 1$ s and $t = 2$ s, and its acceleration is zero at both instants.
- B. It is momentarily at rest at $t = 1$ s and $t = 2$ s, with $a = -6$ m/s² at $t = 1$ s and $a = +6$ m/s² at $t = 2$ s.
- C. It is momentarily at rest only at $t = 1.5$ s, where $a = 0$.
- D. It is momentarily at rest at $t = 1$ s and $t = 2$ s, with $a = +6$ m/s² at $t = 1$ s and $a = -6$ m/s² at $t = 2$ s.
- E. It is never at rest, because $s(t)$ is never zero.

A01

Your choice (A–E)

A02 · Reading a v–t graph (2 pts)

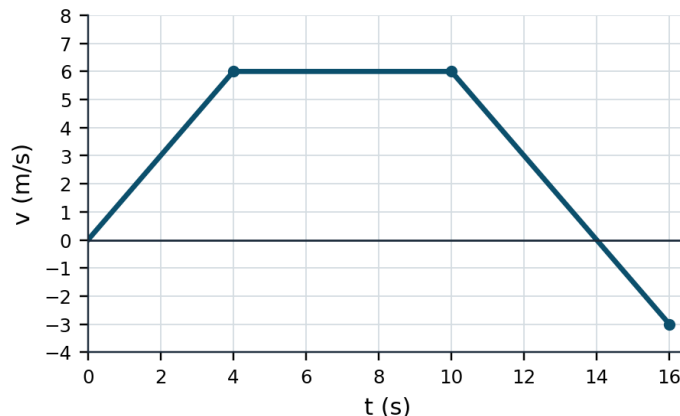


Figure A02 · Velocity of a cart on a straight track; $+s$ to the right.

The graph shows the velocity of a cart on a straight track ($+s$ to the right). Which statement is correct?

- A. The cart reverses direction at $t = 10$ s, where its velocity starts to decrease.
- B. The cart reverses direction at $t = 14$ s.
- C. The acceleration is zero at the instant the velocity is zero.
- D. Between $t = 14$ s and $t = 16$ s the cart is slowing down, because its acceleration is negative.
- E. The cart is back at its starting position at $t = 16$ s.

A02

Your choice (A–E)

A03 · A reversal (2 pts)

A cart on a straight track is at $s_0 = -2.0$ m at $t = 0$ moving to the right at 6.0 m/s. Its acceleration is constant, 1.5 m/s² to the left. Take +s to the right. Which statement describes the motion for $0 \leq t \leq 6.0$ s?

- A. The cart moves to the right for the whole 6.0 s and is at $s = 7.0$ m at $t = 6.0$ s.
- B. The cart stops at $t = 4.0$ s at $s = 10$ m, reverses, and at $t = 6.0$ s is at $s = 7.0$ m moving left at 3.0 m/s.
- C. The cart stops at $t = 4.0$ s, reverses, and at $t = 6.0$ s is at $s = 9.0$ m moving left at 3.0 m/s.
- D. The cart's displacement over the 6.0 s is negative because its acceleration is negative.
- E. The cart stops at $t = 4.0$ s and remains at rest, because nothing pushes it back.

A03

Your choice (A–E)

A04 · Constant speed on a circle (2 pts)

A survey drone flies a horizontal circle of radius 40 m at a constant speed of 12 m/s. Its acceleration is:

- A. zero, because its speed is constant.
- B. 3.6 m/s², tangent to the path.
- C. 3.6 m/s², directed toward the centre of the circle.
- D. 3.6 m/s², directed away from the centre of the circle.
- E. 0.30 m/s², directed toward the centre of the circle.

A04

Your choice (A–E)

A05 · Landing lower than launch (2 pts)

Figure A05 · A stone thrown from the cliff top at A lands on the beach at B; +x right, +y up.

Neglect air resistance. Which statement is correct?

- A. $|\vec{v}_B| > |\vec{v}_A|$ and $v_{Bx} = v_{Ax}$
- B. $|\vec{v}_B| = |\vec{v}_A|$ and $v_{Bx} = v_{Ax}$
- C. $|\vec{v}_B| < |\vec{v}_A|$ and $v_{Bx} < v_{Ax}$
- D. $|\vec{v}_B| > |\vec{v}_A|$ and $v_{Bx} > v_{Ax}$
- E. $|\vec{v}_B| = |\vec{v}_A|$ and $v_{By} = -v_{Ay}$

A05

Your choice (A–E)

A06 · Average speed on a two-loop track (2 pts)

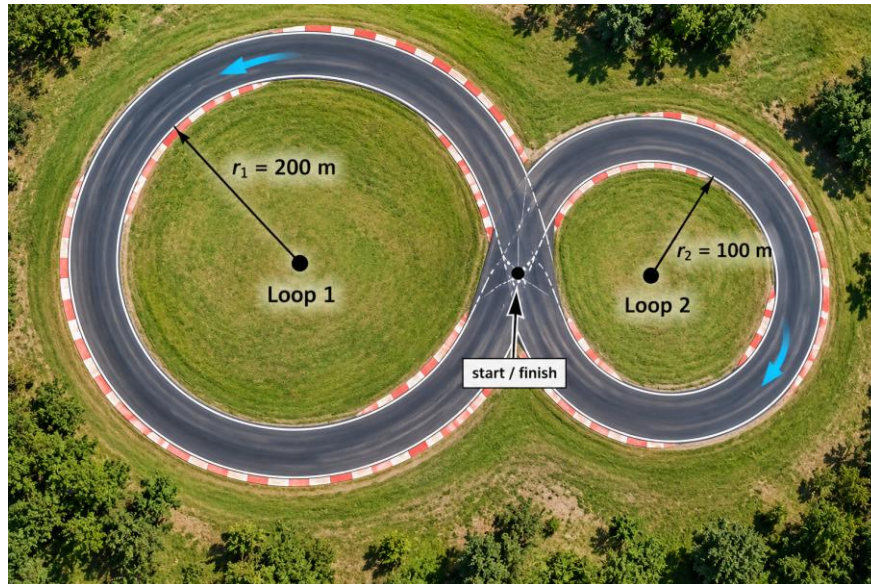


Figure A06 · Figure-8 test track of two tangent circular loops.

A test car drives Loop 1 once in 25 s, then Loop 2 once in 10 s. Let v_{sp1} , v_{sp2} and $v_{sp,track}$ be its average speeds on Loop 1, on Loop 2, and over the whole track. Which is correct?

- A. $v_{sp1} = v_{sp2} = v_{sp,track}$
- B. $v_{sp2} > v_{sp,track} > v_{sp1}$
- C. $v_{sp,track} = (v_{sp1} + v_{sp2})/2$
- D. $v_{sp1} > v_{sp2}$, because Loop 1 is longer
- E. All three average speeds are zero, because the car returns to its starting point.

A06

Your choice (A–E)

A07 · Choosing the description (2 pts)

A coastal radar station at O reports a ship's range r and bearing ϑ once per second. Engineers need the ship's acceleration. Which starting description is best justified?

- A. Cartesian x – y , because Cartesian components always give the most accurate acceleration.
- B. Normal–tangential, because every curved path must be analysed with $\hat{u}_t$ and $\hat{u}_n$.
- C. Polar r – ϑ with origin at O, because the instrument measures r and ϑ directly, so $\dot{r}$, $\ddot{r}$, $\dot{\vartheta}$, $\ddot{\vartheta}$ follow from the data.
- D. Normal–tangential, because the other descriptions give a different magnitude of acceleration.
- E. Polar r – ϑ , because $a_r = \ddot{r}$ can then be read straight from the range data.

A07

Your choice (A–E)

A08 · The free-body diagram (2 pts)

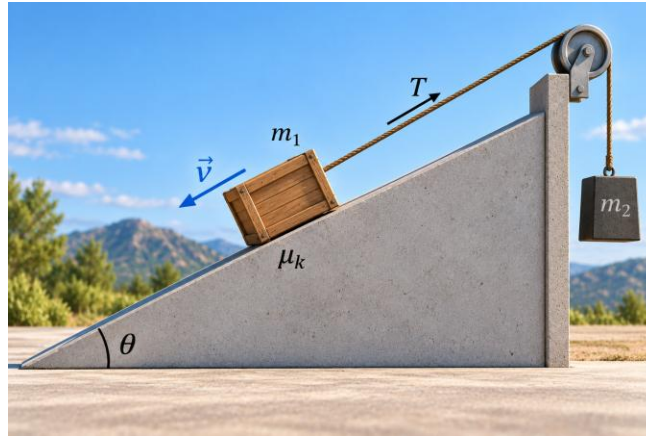


Figure A08 · A crate slides down a rough incline while a rope parallel to the slope pulls up-slope.

A crate slides down a rough incline while connected by a light inextensible cable over a frictionless pulley to a second mass. At the instant shown, the crate is moving down the incline with velocity $\vec{v}$. Which list gives the forces acting on the crate?

- A. W vertically down; N perpendicular to the slope; T up the slope; friction $\mu_k N$ down the slope.
- B. W vertically down; N perpendicular to the slope; T up the slope; friction $\mu_k N$ up the slope.
- C. W vertically down; N perpendicular to the slope; T up the slope; friction $\mu_k W$ up the slope.
- D. W vertically down; N perpendicular to the slope; T up the slope; friction $\mu_k N$ up the slope; and a force $m\vec{a}$ down the slope.
- E. W vertically down; N vertically up; T up the slope; friction $\mu_k N$ up the slope.

A08

Your choice (A–E)

A09 · The crate on the truck bed (2 pts)

A 50 kg crate rides on the flat bed of a truck that speeds up at 2.0 m/s^2 on a level road. The crate does **not** slide. Between crate and bed, $\mu_s = 0.40$ and $\mu_k = 0.30$. The friction force on the crate is:

- A. zero, because the crate and truck move together.
- B. 196 N, opposite to the truck's motion.
- C. 100 N, in the direction of the truck's acceleration.
- D. 196 N, in the direction of the truck's acceleration.
- E. 147 N, in the direction of the truck's acceleration.

A09

Your choice (A–E)

A10 · Where the disagreement comes from (2 pts)

An electric shuttle's speed was predicted with $\Sigma \vec{F} = m\vec{a}$ using a constant drive force and constant rolling resistance. Measured speeds were compared with the prediction over a 12 s run, five times:

t (s)	2	4	6	8	10	12
(measured – predicted)/predicted	–0.2 %	–0.8 %	–1.9 %	–3.2 %	–4.8 %	–6.6 %

The pattern is the same in all five runs. The speed sensor resolves ± 0.05 m/s; mass and drive force are known to ± 0.7 % and ± 1.5 %. Which is the best-supported first diagnosis?

- A.** Measurement: sensor noise, because real data never match a model exactly.
- B.** Model: a resistance that grows with speed, such as aerodynamic drag, is missing.
- C.** $\Sigma \vec{F} = m\vec{a}$ does not hold at the higher speeds.
- D.** Parameters: the mass is wrong.
- E.** Implementation: a units error in the calculation.

A10

Your choice (A–E)

Part B — Setup and Calculate

6 responses · 24 points · ≈ 25 min

Each response is worth 4 points. Write the governing equation you used, then the result with units. The equation earns credit on its own.

B01 · The shuttle leaves the stop line (4 pts)

An autonomous shuttle crosses a stop line at $t = 0$ and moves along a straight road. For $0 \leq t \leq 4.0$ s its velocity is $v(t) = 0.6t^2 + 1.2t + 2.0$ (v in m/s, t in s, positive in the direction of travel). At $t = 4.0$ s the controller switches to holding the speed constant.

B01.eq Equation used for the acceleration: $a(t) =$	
B01.res.a_before a just before $t = 4.0$ s (m/s ²)	
B01.res.a_after a just after $t = 4.0$ s (m/s ²)	

B02 · Average velocity on an arc (4 pts)

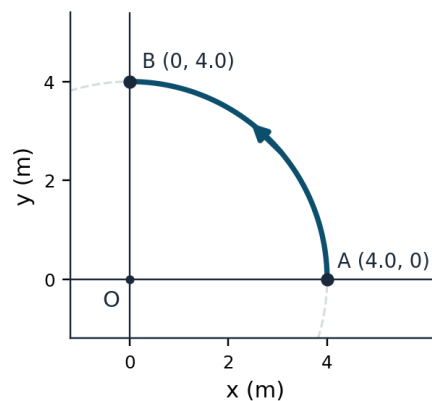


Figure B02 · Quarter-circle path of radius 4.0 m from A to B, counter-clockwise.

A robot moves along the quarter-circle from A to B in 2.0 s.

B02.eq Equation used: $\vec{v}_{avg} =$	
B02.res Result: $\vec{v}_{avg} = (\text{ ______ } , \text{ ______ })$ m/s	

B03 · What the car's sensor reports (4 pts)

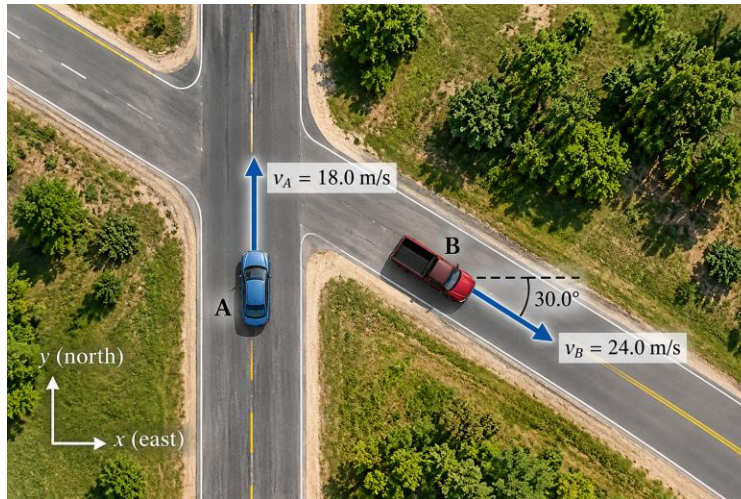


Figure B03 · Plan view: car A northbound, truck B heading 30.0° south of east; x east, y north.

Car A travels north at 18.0 m/s. Truck B travels at 24.0 m/s in the direction 30.0° south of east. Treat A as a translating (non-rotating) observer; use x east and y north.

B03.eq Equation used: $\vec{v}_{B/A} =$	
B03.res Result: $\vec{v}_{B/A} = (\text{ ______ } , \text{ ______ }) \text{ m/s}$	

B04 · Is the end-effector speeding up? (4 pts)

A robot end-effector moves in a plane with fixed x–y axes: $x(t) = 0.5t^3 - 3t$ and $y(t) = 4 - t^2$ (x, y in metres, t in seconds). Consider $t = 2.0$ s.

B04.eq Equation used for the tangential acceleration: $a_t =$	
B04.res.v_sp v_{sp} at $t = 2.0$ s (m/s)	
B04.res.a_t a_t at $t = 2.0$ s (m/s^2)	

B05 · Package on a chute (4 pts)

Figure B05 · Package on a straight chute inclined at 25.0° ; $\mu_k = 0.20$.

A package slides down the chute. It passes the top mark at 1.50 m/s. Find its speed 3.00 m further down the chute.

B05.eq Equation used (along the chute):	
B05.res Result: v_{sp} (m/s)	

B06 · Lifting a pallet (4 pts)

A 250 kg pallet hangs from a single vertical crane cable. At the instant considered the pallet accelerates upward at 1.20 m/s^2 .

B06.eq Equation used:	
B06.res Result: T (N)	

Part C — Linked Engineering Scenarios

18 responses · 56 points · ≈ 64 min

Each scenario is a chain. Answer in order. Later answers are graded using your own earlier answers.

Scenario 1 — A test ball launched onto a raised platform

S1-1 to S1-5 · 14 points · ≈ 17 min

A pneumatic launcher on an outdoor test range fires a smooth, solid rubber test ball from point A with speed 55.0 m/s at 20.0° above the horizontal. In the ideal projectile model (a particle with no air resistance), the ball lands exactly on the front edge B of a raised target platform, 180 m from A measured horizontally. An engineer uses the model to find how high B is above A, then checks whether the model can be trusted.

Take the origin at A, +x horizontal toward the platform, +y vertically up. Ball data (used only in S1-5): smooth sphere, mass 72.0 g, diameter 50.0 mm, drag coefficient $C_D = 0.47$; air density 1.20 kg/m³.

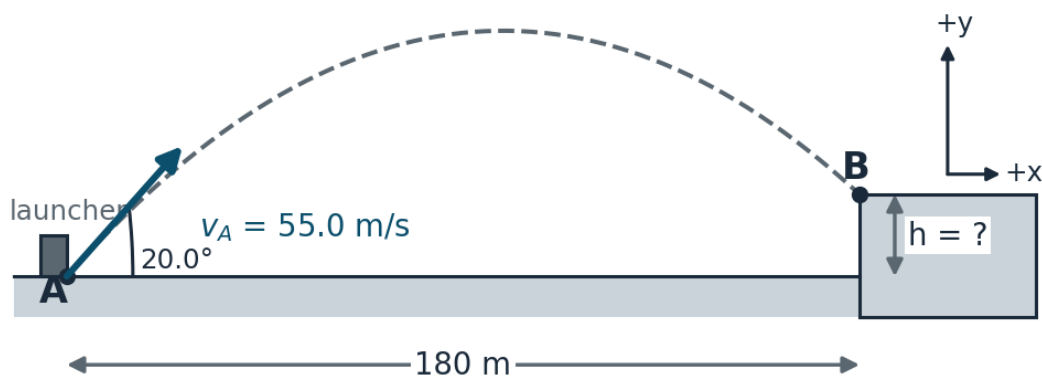


Figure S1 · Side elevation: launcher at A (origin), ideal trajectory to the front edge B of a raised platform; +x toward the platform, +y up.

S1-1 · Select the model (2 pts)

Which pair of equations describes the ideal flight in the stated axes?

- A. $x = (55.0 \cos 20.0^\circ)t$ and $y = (55.0 \sin 20.0^\circ)t - \frac{1}{2}gt^2$
- B. $x = 55.0t$ and $y = 55.0t - \frac{1}{2}gt^2$
- C. $x = (55.0 \cos 20.0^\circ)t - \frac{1}{2}gt^2$ and $y = (55.0 \sin 20.0^\circ)t - \frac{1}{2}gt^2$
- D. $x = (55.0 \cos 20.0^\circ)t$ and $y = (55.0 \sin 20.0^\circ)t + \frac{1}{2}gt^2$
- E. $x = (55.0^2 \sin 40.0^\circ)/g$ and $y = 0$ at landing

S1-1 Your choice (A–E)	
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S1-2 · Time to reach B (3 pts)

S1-2.eq Equation used:	
S1-2.res Result: t (s)	

S1-3 · Height of B above A (3 pts)

S1-3.eq Equation used:	
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S1-3.res Result: h (m)	
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S1-4 · Speed at B, checked by a second route (3 pts)

Find the speed at B **without using the time**, and state whether the ball is rising or descending there.

S1-4.eq Equation used:	
S1-4.res.v_sp Result: v_{sp} at B (m/s)	
S1-4.res.direction Rising or descending at B?	

S1-5 · Is neglecting drag reasonable? (3 pts)

Estimate the aerodynamic drag force on the ball just after launch using the Chapter 3 quadratic drag model, compare it with the ball's weight, and state in one sentence whether neglecting drag is reasonable for predicting h .

S1-5.F_D F_D just after launch (N)	
S1-5.ratio F_D / W	
S1-5.verdict Verdict (one sentence)	

Scenario 2 — Braking through an exit ramp, then a crest

S2-1 to S2-4 · 14 points · ≈ 15 min

A 1500 kg car leaves a highway onto a level, unbanked exit ramp that is a circular arc of radius $\rho = 120$ m. It enters the ramp at point A at 24.0 m/s and brakes so that its speed decreases at a constant rate, reaching 14.0 m/s at point C, 100 m of arc later. Treat the car as a particle; the road must supply the whole horizontal force through tyre friction.

Beyond C the road runs straight but passes over a crest D whose profile, in vertical section, is a circular arc of radius 40.0 m; the car crosses D at 14.0 m/s.

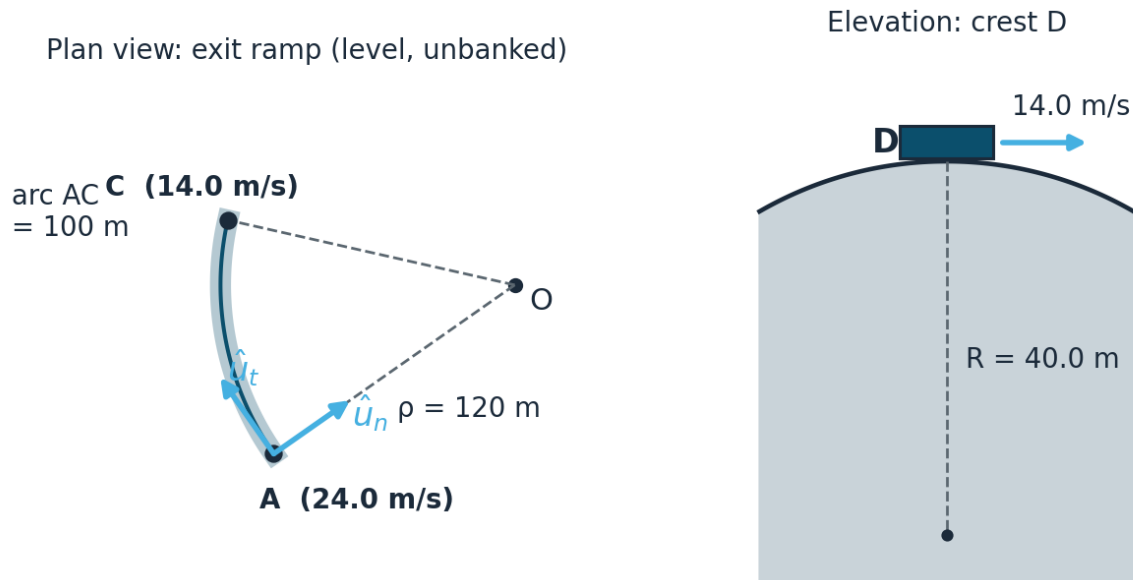


Figure S2 · Left: plan view of the circular exit ramp A→C. Right: elevation of the crest D.

S2-1 · Tangential acceleration (3 pts)

S2-1.eq Equation used:	
S2-1.res Result: a_t (m/s ²)	

S2-2 · Total acceleration at entry (4 pts)

S2-2.eq Equation used:	
S2-2.res.a_n a_n at A (m/s ²)	
S2-2.res.a_mag $ \vec{a} $ at A (m/s ²)	

S2-3 · Can the road supply it? (4 pts)

The wet ramp surface has $\mu_s = 0.60$ between tyres and road. Find the friction coefficient the manoeuvre requires at A, and give a verdict with the reserve, defined as $(\mu_s - \mu_{required})/\mu_s \times 100$ %.

S2-3.eq Equation used:	
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S2-3.res.mu_req μ required	
S2-3.res.reserve Reserve (%)	
S2-3.verdict Verdict: within capacity / exceeds capacity	

S2-4 · Over the crest (3 pts)

At what speed would the normal force at the crest D become zero? Then, in one sentence, say what a calculation that returns a **negative** normal force at a higher speed means physically.

S2-4.eq Equation used:	
S2-4.res v_{sp} at which $N = 0$ (m/s)	
S2-4.meaning Meaning of a negative N (one sentence)	

Scenario 3 — A crate and a load on a movable pulley

S3-1 to S3-5 · 14 points · ≈ 17 min

On a level loading dock, crate A ($m_A = 20.0$ kg) rests on the floor; the kinetic friction coefficient between crate and floor is $\mu_k = 0.25$. A cable is attached to A, runs horizontally to a small fixed pulley P at the dock edge, passes over P and runs vertically down to a movable pulley M, passes under M, and runs vertically up to a fixed anchor Q on the dock edge beside P. Load B hangs from M; the combined mass of B and M is $m_B = 30.0$ kg. The cable is inextensible and massless; the pulleys are small, massless and frictionless; both vertical falls stay vertical. The system is released from rest and moves with B descending.

Coordinates: the datum is the horizontal line through the axle of P (Q is on this line). s_A is the horizontal distance from P to A, positive away from P. s_B is the vertical distance from the datum down to the axle of M, positive downward.

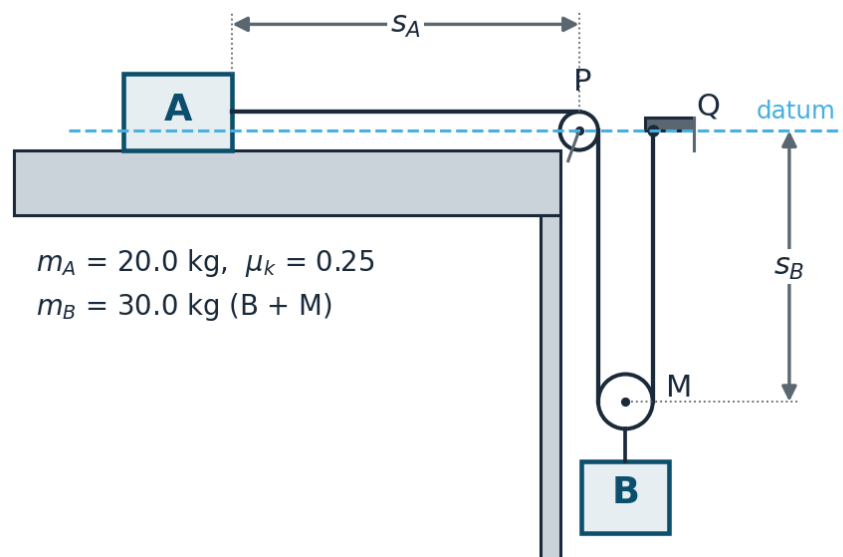


Figure S3 · Side view of the loading dock: crate A, fixed pulley P, movable pulley M carrying load B, fixed anchor Q; datum through P and Q.

S3-1 · The constraint (2 pts)

Write the constant-length relation for the cable in terms of s_A and s_B , and the acceleration relation it implies.

S3-1.length Length relation: _____ = constant	
S3-1.accel Acceleration relation:	

S3-2 · Equations of motion (3 pts)

From here on, let a_A be the crate's acceleration measured **toward P** and a_B the load's acceleration measured **downward**; T is the cable tension. Which pair of equations is correct?

- A. Crate: $T - \mu_k m_A g = m_A a_A$ · Load: $m_B g - 2T = m_B a_B$
- B. Crate: $T - \mu_k m_A g = m_A a_A$ · Load: $m_B g - T = m_B a_B$
- C. Crate: $T + \mu_k m_A g = m_A a_A$ · Load: $m_B g - 2T = m_B a_B$
- D. Crate: $T - \mu_k m_A g = m_A a_A$ · Load: $2T = m_B g$
- E. Crate: $T - \mu_k m_B g = m_A a_A$ · Load: $m_B g - 2T = m_B a_B$

S3-2

Your choice (A–E)

S3-3 · The accelerations (3 pts)**S3-3.eq**

Equation used (after combining):

S3-3.res.a_B a_B (m/s², downward)**S3-3.res.a_A** a_A (m/s², toward P)**S3-4 · The tension** (3 pts)**S3-4.eq**

Equation used:

S3-4.resResult: T (N)**S3-5 · Faster than free fall?** (3 pts)

A colleague claims that nothing in this system can ever accelerate faster than g . Replace the crate with a lighter one, $m_A = 5.00$ kg (everything else unchanged), compute a_A , and respond to the claim in one sentence.

S3-5.res a_A with $m_A = 5.00$ kg (m/s²)**S3-5.sentence**

Response to the claim (one sentence)

Scenario 4 — A slider on a rotating arm

S4-1 to S4-4 · 14 points · ≈ 15 min

A horizontal arm of an inspection fixture rotates about a fixed vertical axis through O. Its angle, measured counter-clockwise from a fixed reference line, is $\vartheta = 0.20t^2$ (ϑ in radians, t in seconds). A 2.50 kg slider moves along a smooth radial slot in the arm, driven by an actuator along the slot so that $r = 0.40 + 0.20t$ (r in metres from O). The motion is in a horizontal plane; weight and the vertical support force balance and do not enter the planar equations.

Consider the instant $t = 2.00$ s. Use polar coordinates with origin O: $\hat{u}_r$ from O toward the slider, $\hat{u}_\theta$ perpendicular to it in the direction of increasing ϑ .

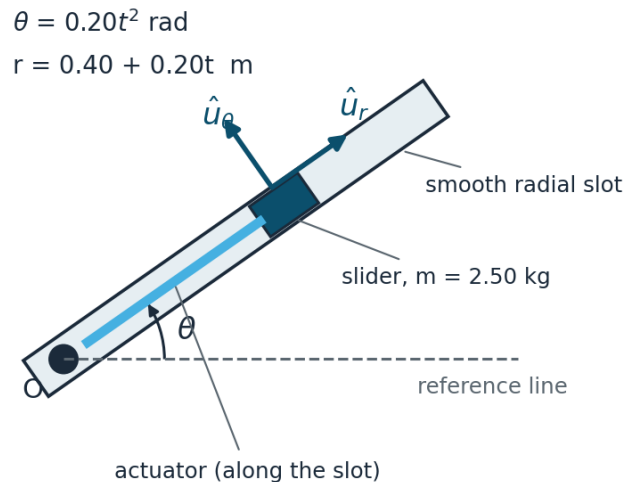


Figure S4 · Plan view of the rotating arm, radial slot and slider; ϑ measured counter-clockwise from the fixed reference line.

S4-1 · Which terms are present? (2 pts)

At $t = 2.00$ s, which of the four polar acceleration terms $\ddot{r}$, $-r\dot{\theta}^2$, $r\ddot{\theta}$ and $2\dot{r}\dot{\theta}$ are **non-zero**?

- A. Only $-r\dot{\theta}^2$
- B. $-r\dot{\theta}^2$, $r\ddot{\theta}$ and $2\dot{r}\dot{\theta}$
- C. All four
- D. Only $r\ddot{\theta}$ and $2\dot{r}\dot{\theta}$; the radial acceleration is zero because $\ddot{r} = 0$
- E. Only $-r\dot{\theta}^2$ and $r\ddot{\theta}$

S4-1

Your choice (A–E)

S4-2 · Acceleration components (4 pts)

S4-2.eq

Equations used: $a_r =$ _____, $a_\theta =$ _____

S4-2.res.a_r

a_r (m/s²)

S4-2.res.a_theta

a_θ (m/s²)

S4-3 · Forces on the slider (4 pts)

Find the radial and transverse components of the horizontal force on the slider, and name which part of the fixture supplies each.

S4-3.eq Equations used:	
S4-3.res.F_r F_r (N)	
S4-3.res.F_theta F_θ (N)	
S4-3.sources Supplied by (F_r / F_θ)	

S4-4 · Clamping the slider (4 pts)

At $t = 2.00$ s the actuator stops and clamps the slider at $r = 0.80$ m, while the arm continues with the same $\dot{\theta}$ and $\ddot{\theta}$ at that instant. Find the new transverse force on the slider and, in one sentence, explain which acceleration term's load has disappeared and why.

S4-4.res F_θ with the slider clamped (N)	
S4-4.sentence Explanation (one sentence)	

Reading your results

Mark each response you missed, then use the table to find the chapter Part to revisit. Rework missed items first without help; then use an AI tutor that gives hints rather than answers.

Response IDs	Revisit
A01–A03, B01	Chapter 1 — rectilinear kinematics, graphs, constant acceleration
A06, B02	Chapter 1–2 — displacement, distance, average velocity and speed
A04, S2-1, S2-2	Chapter 2 Part 4 — normal–tangential kinematics
A05, S1-1 to S1-4	Chapter 2 Part 3 — projectile motion
A07, B04	Chapter 2 Parts 2 and 6 — Cartesian description and choosing a description
B03, S3-1	Chapter 2 Part 7 — relative and constrained motion
S4-1, S4-2	Chapter 2 Part 5 — polar kinematics
A08, A09, B05, B06	Chapter 3 Parts 1–3 — free-body diagrams, friction, Cartesian kinetics
S3-2 to S3-5	Chapter 3 Part 4 — connected particles
S2-3, S2-4	Chapter 3 Part 5 — normal–tangential kinetics and contact
S4-3, S4-4	Chapter 3 Part 6 — polar kinetics
A10, S1-5	Chapter 3 Part 7 — model adequacy

The goal is not 100 points. The goal is to be able to say: “Given a moving machine, I can choose the description, draw the right diagram, compute what matters, and decide whether I believe the answer.”

AI assists. The engineer observes, models, validates, decides, and approves.