



Formula Summary for CHAPTERS 1–3

Rectilinear Motion → Curvilinear Motion → Relative Motion → Projectile Motion

Taner Onsay, Ph.D. · © 2026 AI First Engineering

Average Velocity and Speed

$$(v_{sp})_{avg} = \frac{\Delta s}{\Delta t}, \quad \vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}, \quad (v_{sp})_{avg} = \frac{s_T}{\Delta t}$$

Particle Rectilinear Motion

Variable acceleration

$$a = \frac{dv_{sp}}{dt}, \quad v_{sp} = \frac{ds}{dt}, \quad a ds = v_{sp} dv_{sp}$$

Constant acceleration: $a = a_c$

$$v_{sp} = v_{sp,0} + a_c t$$

$$s = s_0 + v_{sp,0} t + \frac{1}{2} a_c t^2$$

$$v_{sp}^2 = v_{sp,0}^2 + 2a_c(s - s_0)$$

Particle Curvilinear Motion:

Cartesian x, y

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} \quad v_x = \dot{x}, \quad v_y = \dot{y}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} \quad a_x = \ddot{x}, \quad a_y = \ddot{y}$$

$$v_{sp} = |\vec{v}| = \sqrt{v_x^2 + v_y^2}, \quad |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

Normal-Tangential n, t

$$\vec{v} = v_{sp}\hat{u}_t = \dot{s}\hat{u}_t$$

$$\vec{a} = a_n\hat{u}_n + a_t\hat{u}_t$$

$$a_t = \dot{v}_{sp} = v_{sp} \frac{dv_{sp}}{ds}, \quad a_n = \frac{v_{sp}^2}{\rho}$$

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

Polar r, θ

$$\vec{r} = r\hat{u}_r + z\hat{k}$$

$$\vec{v} = v_r\hat{u}_r + v_\theta\hat{u}_\theta \quad v_r = \dot{r}, \quad v_\theta = r\dot{\theta}$$

$$\vec{a} = a_r\hat{u}_r + a_\theta\hat{u}_\theta \quad a_r = \ddot{r} - r\dot{\theta}^2, \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

 angular velocity $\omega = \dot{\theta}$ (rad/s),

 angular acceleration $\alpha = \ddot{\theta}$ (rad/s²)

N – revolution per minute (rpm) ,

 f – cycles per second (Hertz – Hz), $f = N/3600$
 $\omega = N \cdot (2\pi)/3600$ (rad/s), $\omega = 2\pi \cdot f$

Projectile Motion

 For +y upward and constant vertical acceleration $a_y = a_c$:

$$v_y = v_{y0} + a_c t$$

$$y = y_0 + v_{y0} t + (1/2)a_c t^2$$

$$v_y^2 = v_{y0}^2 + 2a_c(y - y_0)$$

 For horizontal motion with $a_x = 0$:

$$x = x_0 + v_{x0} t$$

For ideal projectile motion near Earth's surface:

$$a_x = 0, \quad a_y = -g$$

For a horizontal launch from height h:

$$t_f = \sqrt{2h/g}, \quad R = v_{x0} \cdot t_f, \quad v_y = -g \cdot t_f$$

$$v_{sp} = \sqrt{v_x^2 + v_y^2}, \quad \theta = \tan^{-1}\left(\frac{|v_y|}{v_x}\right)$$

Relative Motion:

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Particle Kinetics

$$\sum \vec{F} = m \vec{a} \quad \text{Cartesian: } \sum F_x = ma_x, \quad \sum F_y = ma_y$$

$$n-t: \quad \sum F_n = ma_n, \quad \sum F_t = ma_t$$

$$\text{Weight: } W = mg$$

$$r-\theta: \quad \sum F_r = ma_r, \quad \sum F_\theta = ma_\theta$$

Common Force Models

 Static friction (no slip): $|f_s| \leq \mu_s N$, Kinetic friction (sliding): $f_k = \mu_k N$

 Quadratic drag MODEL: $F_D = \frac{1}{2} \rho_{\text{air}} C_D A v_{sp}^2$, opposing $\vec{v}$
 ρ_{air} = air (fluid) density, C_D = drag coefficient, A = frontal area. An approximate model, not an exact law.

Differentiation: for any two functions $u(t)$ and $v(t)$
Product rule:

$$\frac{d}{dt}(uv) = u \frac{dv}{dt} + v \frac{du}{dt}$$

Chain rule

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$$

Useful dynamics form:

$$\frac{dv_{sp}}{dt} = \frac{dv_{sp}}{ds} \frac{ds}{dt} = v_{sp} \frac{dv_{sp}}{ds}$$

Notation Used on This Sheet

 $\vec{r}$

position vector

 $\vec{v}$

velocity vector

 v_{sp}

speed, scalar magnitude of the velocity vector

 $\vec{a}$

acceleration vector

 $\hat{i}, \hat{j}, \hat{k}$

Cartesian unit vectors

 $\hat{u}_t, \hat{u}_n$

tangential and normal unit vectors

 $\hat{u}_r, \hat{u}_\theta$

polar unit vectors

 ρ

radius of curvature