

CHAPTER - 4

PART 2 · ≈ 55 MIN

How Does Work Change Motion?

We calculated the net work done on a particle. What does that tell us about its motion — without ever finding the acceleration or the time?

2

Observe



Idealize



Model



Solve / Analyze



Validate

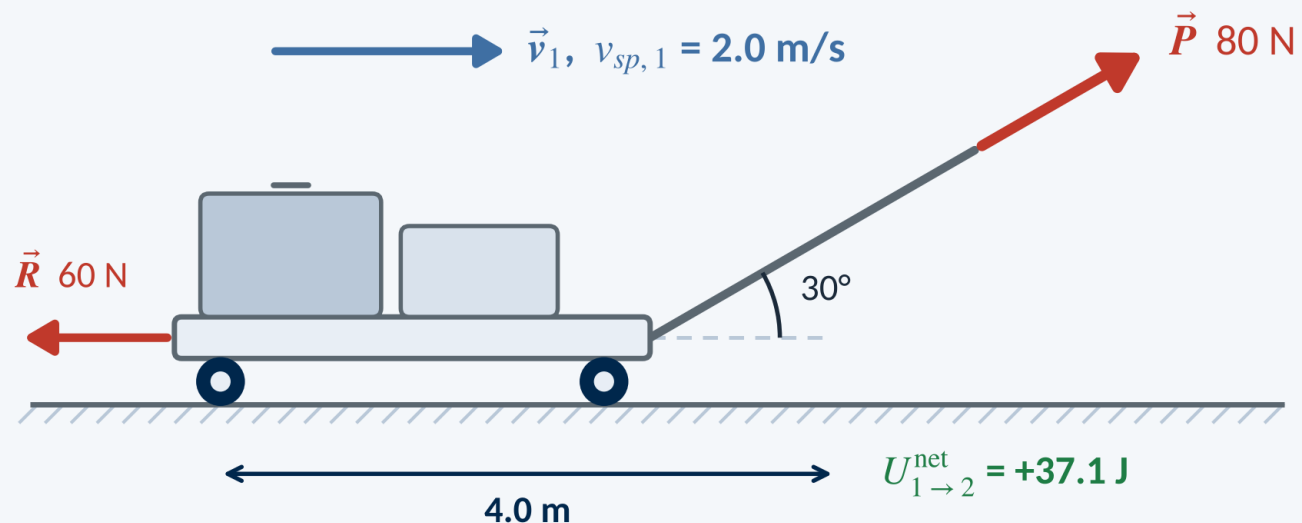


Report



What Did Those 37.1 Joules Do?

SAME CART · NOW ALREADY ROLLING



GIVEN — FROM PART 1

$$m = 25 \text{ kg}$$

$$v_{sp,1} = 2.0 \text{ m/s} \quad 4.0 \text{ m of straight travel}$$

$$\text{Net work on the cart: } U_{1 \rightarrow 2}^{\text{net}} = +37.1 \text{ J}$$

PREDICT

1. Does the cart's speed increase, decrease, or stay the same?
2. Can you find the final speed without the acceleration or the time?



The question this Part answers

We calculated the net work done on a particle. What does that tell us about its motion?

Observe

Idealize

Model

Solve / Analyze

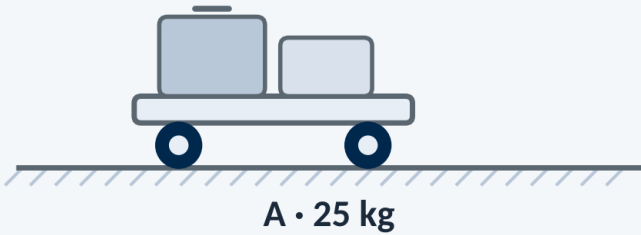
Validate

Report



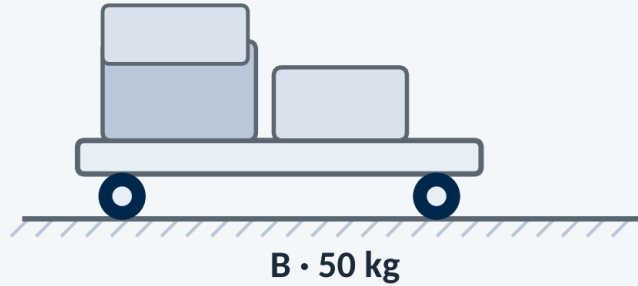
Same Work, Different Carts

A 25 kg, starts at rest



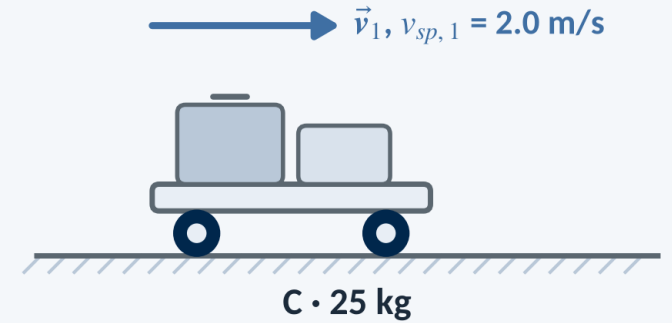
Net work received: **+37.1 J**

B 50 kg, starts at rest



Net work received: **+37.1 J**

C 25 kg, already moving at 2.0 m/s



Net work received: **+37.1 J**

PREDICT

Predict each cart's final speed. Then decide:

Does B end at half of A's speed?

Does C gain as much speed as A?

Observe

Idealize

Model

Solve / Analyze

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From Force to Motion—Without Time

1

Chapter 3, tangential direction

$$\sum F_t = m a_t$$

2

Chain rule: trade time for position

$$a_t = \frac{dv_{sp}}{dt} = \frac{dv_{sp}}{ds} \frac{ds}{dt} = v_{sp} \frac{dv_{sp}}{ds}$$

3

Multiply both sides by ds and Integrate from state 1 to state 2

$$\sum F_t = m \left(v_{sp} \frac{dv_{sp}}{ds} \right) \rightarrow \int_1^2 \sum F_t ds = \int_1^2 m v_{sp} dv_{sp}$$

4

Result:

$$U_{1 \rightarrow 2}^{\text{net}} = \frac{1}{2} m v_{sp,2}^2 - \frac{1}{2} m v_{sp,1}^2$$

The left side is the net work — Part 1's ledger, summed. The right side is new.

CONDITIONS

- Inertial reference frame
- Constant-mass particle, or a body in pure translation
- Net work counts **every** force on the particle
- The result gives **speed**, not the direction of $\vec{v}$



Where did F_n go?

It changes direction, not speed (Part 1, slide 4), so it never enters.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



What Is Kinetic Energy?

KINETIC ENERGY

$$T = \frac{1}{2} m v_{sp}^2$$

THE WORK-ENERGY PRINCIPLE

$$T_1 + U_{1 \rightarrow 2}^{\text{net}} = T_2$$

- a scalar, never negative
- grows with mass and with the **square** of speed
- independent of the direction of $\vec{v}$
- measured in joules: $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ N} \cdot \text{m}$

BACK TO SLIDE 2 — EACH CART RECEIVES +37.1 J

Cart	m	$v_{sp,1}$	T_1	T_2	$v_{sp,2}$	Δv_{sp}
A	25 kg	0	0	37.1 J	1.72 m/s	+1.72 m/s
B	50 kg	0	0	37.1 J	1.22 m/s	+1.22 m/s
C	25 kg	2.0 m/s	50.0 J	87.1 J	2.64 m/s	+0.64 m/s

B is not half of A: for equal net work from rest, $v_{sp} \propto 1/\sqrt{m}$. C gains least: equal work gives equal ΔT , not equal Δv_{sp} .



Why equal work $\neq$ equal speed change

Because T grows with v_{sp}^2 , each extra joule buys less speed when the particle is already moving fast.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



What Does the Sign of Net Work Tell Us?

+ positive

$$v_{sp,2} > v_{sp,1}$$

the particle ends faster

- negative

$$v_{sp,2} < v_{sp,1}$$

the particle ends slower

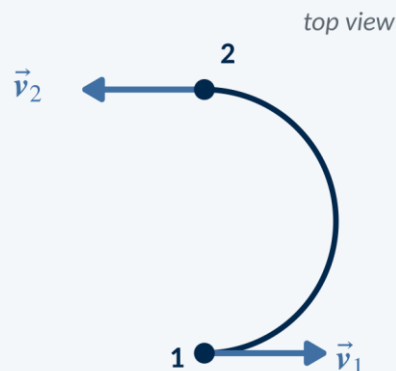
0 zero

$$v_{sp,2} = v_{sp,1}$$

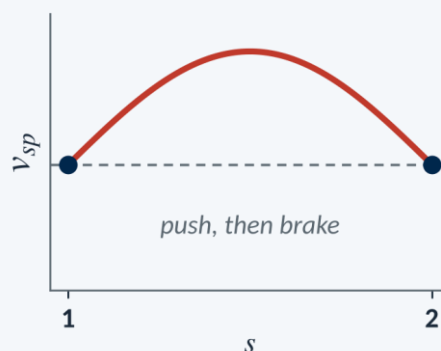
the endpoint speeds are equal

Equal endpoint speeds do **not** mean constant speed throughout, constant velocity, or zero acceleration.

ZERO NET WORK — TWO CASES



same speed, reversed $\vec{v}$; $a_n \neq 0$



same endpoint speed; not constant between

Limiting case

$$U_{1 \rightarrow 2}^{\text{net}} = -T_1 \Rightarrow T_2 = 0$$

The particle just comes to rest at 2.

If you compute $T_2 < 0$

No real speed exists. The endpoint and the force/path model are incompatible as posed — re-examine the model.

Observe

Idealize

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Choosing the System and Accounting for Work

1 Choose the system (one particle) and states 1, 2

2 Draw the free-body diagram

3 List every force on the particle

4 Find each force's work $U_{1 \rightarrow 2}^{(i)}$

5 Add the works algebraically

6 Apply $T_1 + U_{1 \rightarrow 2}^{\text{net}} = T_2$

7 Validate the result

THE PART 1 LEDGER, READY FOR STEP 6

Force on the cart	Work of each force
Pull $P^{\rightarrow}$	+277.1 J
Resistance $R^{\rightarrow}$	-240.0 J
Weight $W^{\rightarrow}$	0 J
Normal $N^{\rightarrow}$	0 J
Net work $U_{1 \rightarrow 2}^{\text{net}}$	+37.1 J



Count every force — exactly once

Zero-work forces still get a row: it proves you considered them. Never add magnitudes, and never count one force twice.

Observe

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The Baggage Cart—Now Solve It

ROUTE 1 — WORK-ENERGY (THIS PART)

$$T_1 = \frac{1}{2}(25 \text{ kg})(2.0 \text{ m/s})^2 = 50.0 \text{ J}$$

$$T_2 = T_1 + U_{1 \rightarrow 2}^{\text{net}} = 50.0 + 37.1 = 87.1 \text{ J}$$

$$v_{sp,2} = \sqrt{\frac{2T_2}{m}} = 2.64 \text{ m/s}$$

No acceleration, no time — only the states and the net work.

ROUTE 2 — FORCE AND ACCELERATION (CHAPTER 3)

$$\Sigma F_t = (80 \text{ N}) \cos 30^\circ - 60 \text{ N} = 9.28 \text{ N}$$

$$a_t = \frac{9.28 \text{ N}}{25 \text{ kg}} = 0.371 \text{ m/s}^2$$

$$v_{sp,2}^2 = v_{sp,1}^2 + 2a_t \Delta s = 6.97 \text{ m}^2/\text{s}^2$$

$$v_{sp,2} = 2.64 \text{ m/s}$$

Constant forces → constant acceleration, so Chapter 1 kinematics applies.

Both routes: 2.64 m/s. Two methods, one answer.

The mass was not needed for the work of $\vec{P}$ (Part 1). It **is** needed for the speed.

Observe



Idealize



Model



Solve / Analyze



Validate



Report

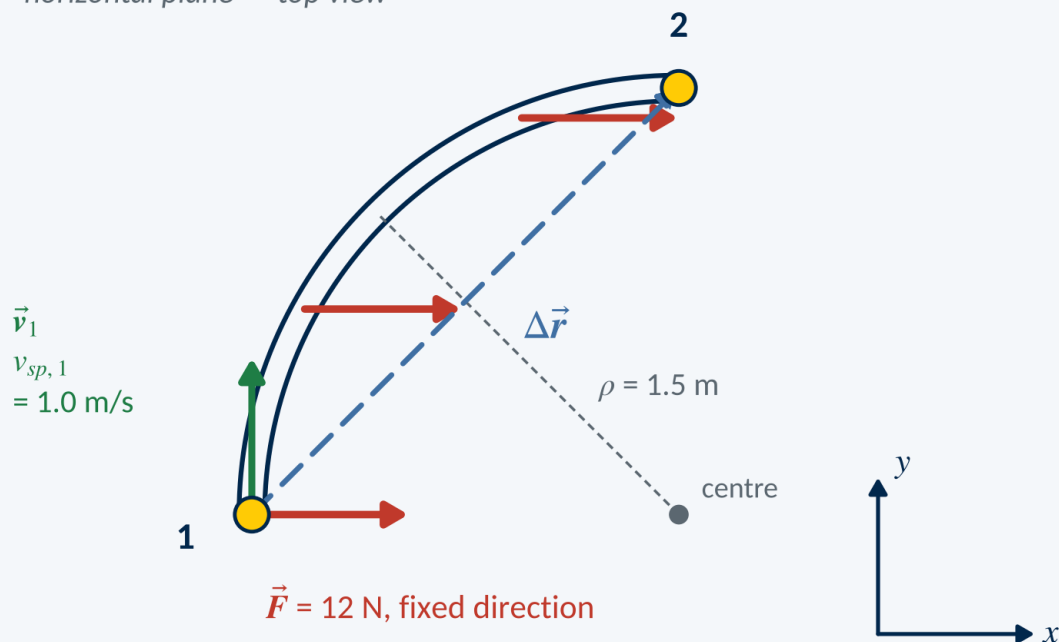


Work–Energy on a Curved Path

SLIDER IN A CURVED GUIDE

the guide as a two-sided, frictionless slot that can constrain the slider in either normal direction. The existing two-rail illustration can support this interpretation.

horizontal plane — top view



Smooth, stationary guide · quarter circle, $\rho = 1.5 \text{ m} \cdot 1 \rightarrow 2 \cdot m = 2.0 \text{ kg}$

SOLVE

$$U_{1 \rightarrow 2}^{(F)} = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = (12 \text{ N})(1.5 \text{ m}) = 18.0 \text{ J}$$

$$U_{1 \rightarrow 2}^{\text{net}} = U_{1 \rightarrow 2}^{(F)} = 18.0 \text{ J}$$

$$T_1 = \frac{1}{2}(2.0)(1.0)^2 = 1.0 \text{ J}, \quad T_2 = 19.0 \text{ J}$$

$$v_{sp,2} = \sqrt{2T_2/m} = 4.36 \text{ m/s}$$

Guide normal, weight, vertical support: zero work, so only $\vec{F}$ counts.

Can energy determine the force the guide exerts?

Not by itself — the guide force does no work. Energy gives v_{sp} ; n–t (Chapter 3): $\sum F_n = mv_{sp}^2/\rho$ — the guide supplies whatever $\vec{F}$ does not.

Observe

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Validate

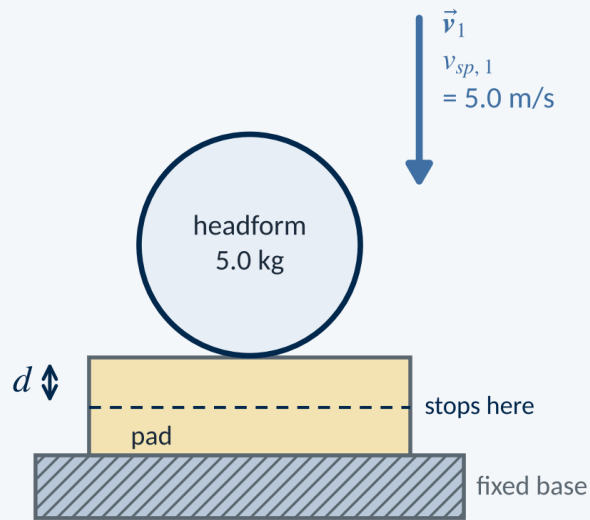
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Why Does Stopping Distance Matter?

TEST HEADFORM ON A PAD

controlled vertical drop — not to scale



Idealized translating headform — not a complete head-injury model.

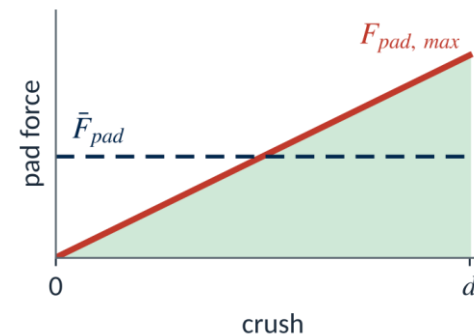
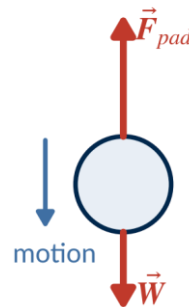
$$T_1 = (1/2)(5.0)(5.0)^2 = 62.5 \text{ J}$$

$$\bar{F}_{\text{net}} = T_1/d \quad (\text{stops: } T_2 = 0)$$

Pad + weight together. Pad alone (weight does $+mgd$):

$$\bar{F}_{\text{pad}} = \frac{T_1}{d} + mg$$

Distance-averaged — not the peak, not the time-average.



Stopping distance d

0.020 m

0.040 m

$$\bar{F}_{\text{net}} = T_1/d$$

3125 N

1563 N

$$\bar{F}_{\text{pad}} = T_1/d + mg$$

3174 N

1612 N

$$\text{Linear pad: } F_{\text{pad}, \text{max}} = 2\bar{F}_{\text{pad}}$$

6348 N

3223 N

Double d : F_{net} halves.

Double $v_{sp,1}$: $T_1 \times 4$.

Research context

Pellman et al., Neurosurgery 53(4), 2003 — reconstructed concussive NFL impacts: mean peak translational head acceleration $\approx 98 \text{ g}$. An observation, not a universal threshold; later work adds rotation. A peak — not comparable with our average.

Observe

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What Can Energy Tell Us—and What Can't It?

ENERGY ALONE CAN DETERMINE

- ✓ Changes in kinetic energy
- ✓ Final speed, when the net work is known
- ✓ Total work needed to stop a particle
- ✓ Distance-averaged net stopping force (under the stated assumptions)

Energy alone cannot determine	Also needed
Detailed force history	A force model or measurement
Peak impact force	A force–displacement model (pad test)
Impact duration	Impulse–momentum (Chapter 5)
Acceleration history	Force history and $\sum \vec{F} = m\vec{a}$
Rotational head response	Rigid-body kinetics (Chapter 7)
Guide normal forces on a curve	n–t kinetics (Chapter 3)

A correct calculation is not necessarily a complete model of the physical event.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



Can We Trust the Energy Result?

1

Dimensions

Every term in joules: $\text{kg}\cdot\text{m}^2/\text{s}^2 = \text{N}\cdot\text{m}$.

2

Sign of net work

Does the speed change in the predicted direction?

3

Kinetic energy ≥ 0

$T_2 < 0$ means the model cannot reach state 2.

4

Limiting cases

$U_{1 \rightarrow 2}^{\text{net}} = 0 \rightarrow$ same speed; $= -T_1 \rightarrow$ just stops.

5

Complete ledger

Every force on the FBD — once, with its sign.

6

Assumptions

Particle? Inertial frame? Constant forces where assumed?

7

Independent check

Chapter 3 gave the cart 2.64 m/s too (slide 7).

A PROPOSED SHORTCUT

$$T_2 - T_1 = \frac{1}{2}m(v_{sp,2} - v_{sp,1})^2$$

Your diagnosis first (2 min): which check exposes it? Then test it on the cart.

THE CORRECTION

$$T_2 - T_1 = \frac{1}{2}m(v_{sp,2}^2 - v_{sp,1}^2) = 37.1 \text{ J } \checkmark$$

$$\text{Shortcut: } \frac{1}{2}(25)(0.64)^2 = 5.1 \text{ J } \times$$

$a^2 - b^2 \neq (a - b)^2$. It can never be negative, so checks 2 and 4 fail; the ledger (5) and Route 2 (7) disagree. Then ask an AI to critique it — and judge its critique.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



What Have We Learned?

- 1 Why net work changes kinetic energy — speed, not direction
- 2 How T depends on mass and on the square of speed
- 3 How the theorem follows from $\sum F_t = ma_t$
- 4 Why net work is an algebraic sum of signed works
- 5 How to find a final speed without finding the time
- 6 Why energy cannot answer every dynamics question
- 7 How to validate an energy result

CARRY FORWARD

$$T = \frac{1}{2} m v_{sp}^2$$

$$T_1 + U_{1 \rightarrow 2}^{\text{net}} = T_2$$

speed, not velocity · all forces · inertial frame

Homework and self-assessment

Practise: final speed from a ledger, a two-route check, and one ‘what can’t energy tell us?’ question.

We now know what net work does to a particle’s motion. How do we calculate the work of gravity, springs, friction and other common forces — for any path or displacement?

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report