

CHAPTER - 4

PART 3 · ≈ 57 MIN

Work of Common Forces

Gravity, springs, friction and cables each do work in their own way. What does the work of each one depend on?

Observe

Idealize

Model

Solve / Analyze

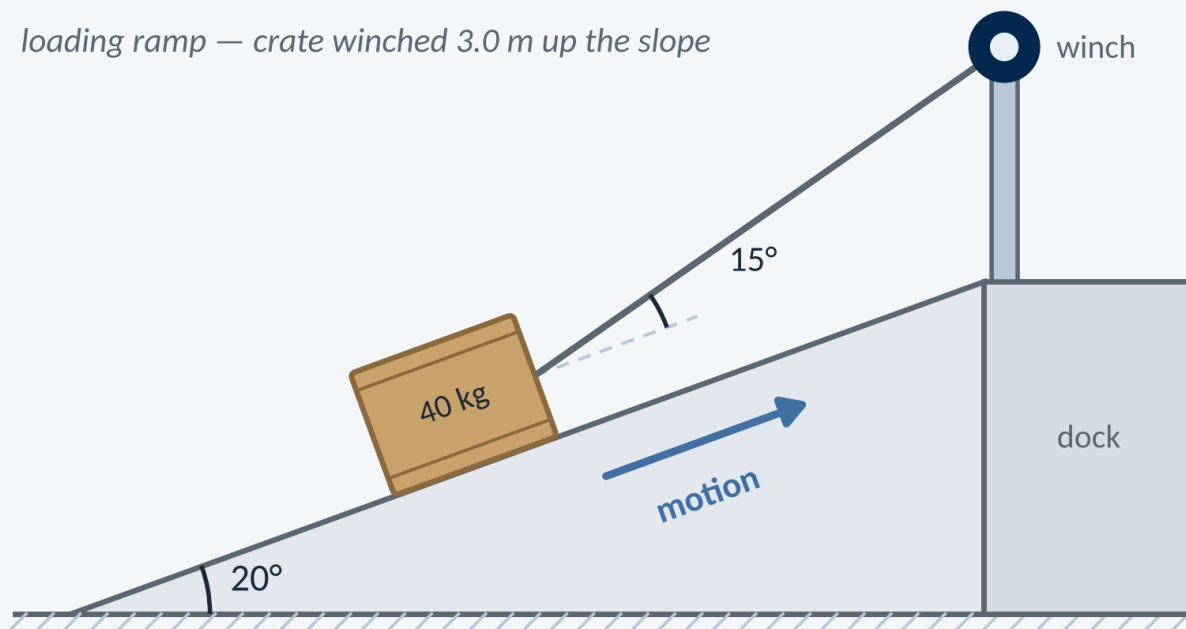
Validate

Report



Same Motion, Different Forces

loading ramp — crate winched 3.0 m up the slope



THE SITUATION

A 40 kg crate is winched up a rough 20° ramp. The cable pulls at 15° above the ramp surface. The crate slides 3.0 m up the slope.

NAME THEM

Name every force acting on the crate while it slides.

Which way does each one point?



The question this Part answers

Each force here does work in its own way. How do we calculate the work of each kind of force?

Observe

Idealize

Model

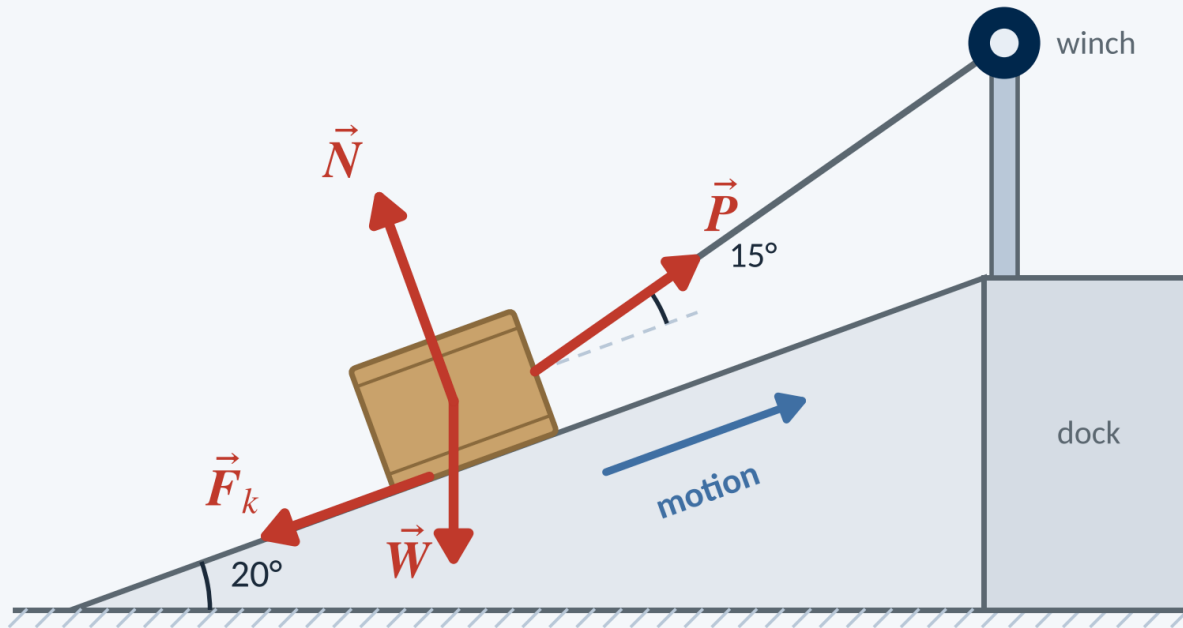
Solve / Analyze

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Which Forces Actually Do Work?



Each click adds one force to the crate.

YOUR VOTE: SIGN OF EACH FORCE'S WORK

Force	+	-	0
Cable pull P			
Weight W			
Normal N			
Friction F_k			

PREDICT

Vote +, -, or 0 for each force as the crate slides up.

Then: which votes change if the crate were sliding **down** the ramp?

Observe

Idealize

Model

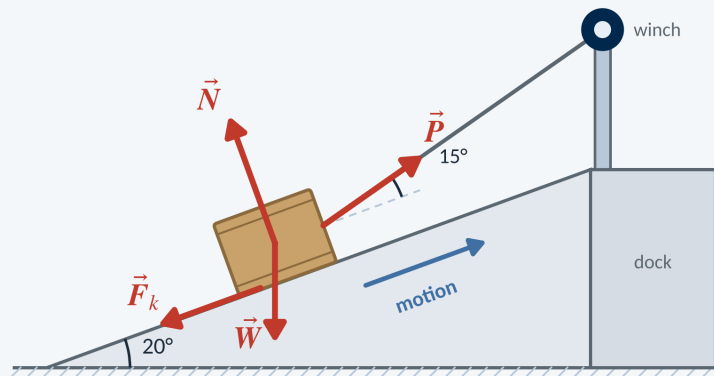
Solve / Analyze

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Report



One Work Ledger, Several Force Models



All four forces on the crate, sliding up.

THE LEDGER FOR THE CRATE — SLIDING UP

Force	Sign	Its work depends on ...	Model
Cable pull P	+	magnitude, its angle to the motion, distance	slide 10
Weight W	–	change in height only	slides 4–5
Normal N	0	zero: always perpendicular to the motion	Part 1
Friction F_k	–	N and the distance slid	slides 8–9
Net work		$U_{1 \rightarrow 2}^{\text{net}} = \text{sum of all four rows}$	Part 2

THE LEDGER (PART 2)

$$U_{1 \rightarrow 2}^{\text{net}} = \sum_i U_{1 \rightarrow 2}^{(i)}$$

$$T_1 + U_{1 \rightarrow 2}^{\text{net}} = T_2$$



Trap: is $N = mg \cos 20^\circ$ here?

Not necessarily. The cable pulls partly away from the ramp. Find N from the free-body diagram — slide 11 shows how much it matters.

Observe

Idealize

Model

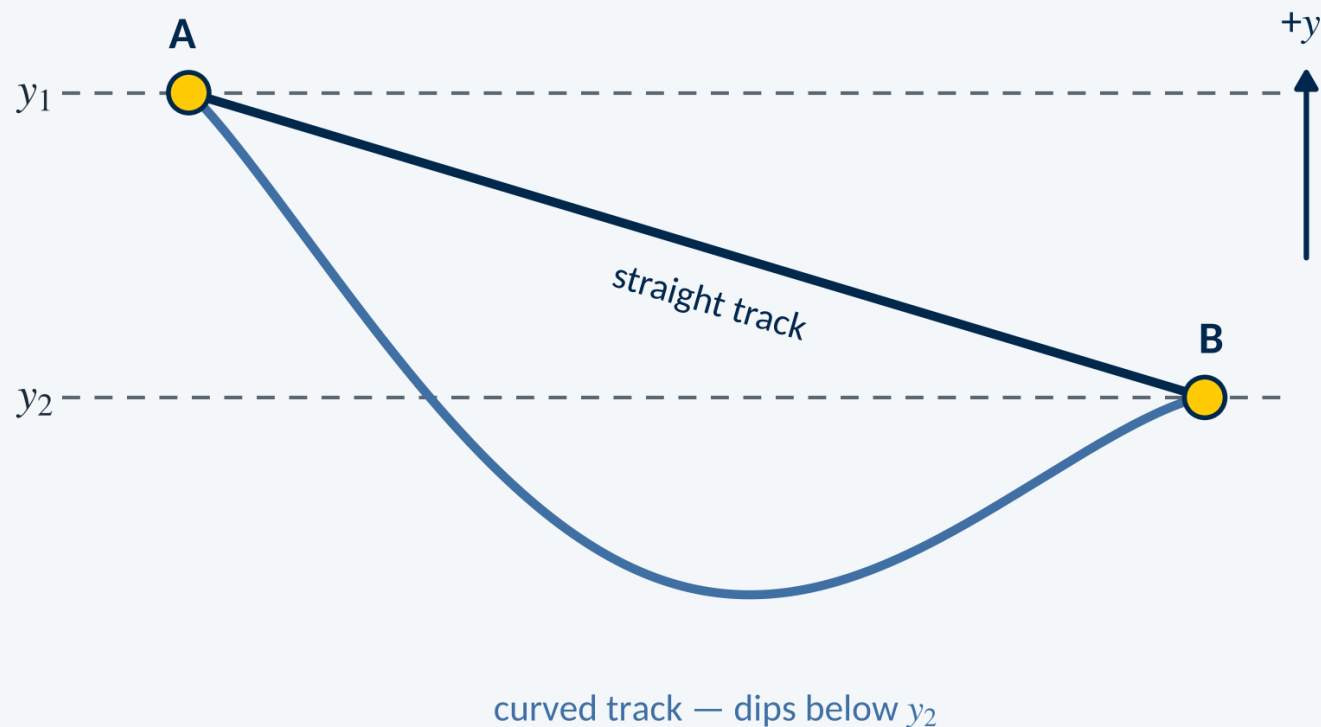
Solve / Analyze

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Report



Gravity: Does the Path Matter?



Same crate, same start A, same end B. Two different tracks.

PREDICT

As the crate moves from A to B, the work done by gravity is ...

- A** larger on the curved track
- B** the same on both tracks
- C** larger on the straight track



Give a reason

What about the curved track might change gravity's work — its length, or its dip?

Observe

Idealize

Model

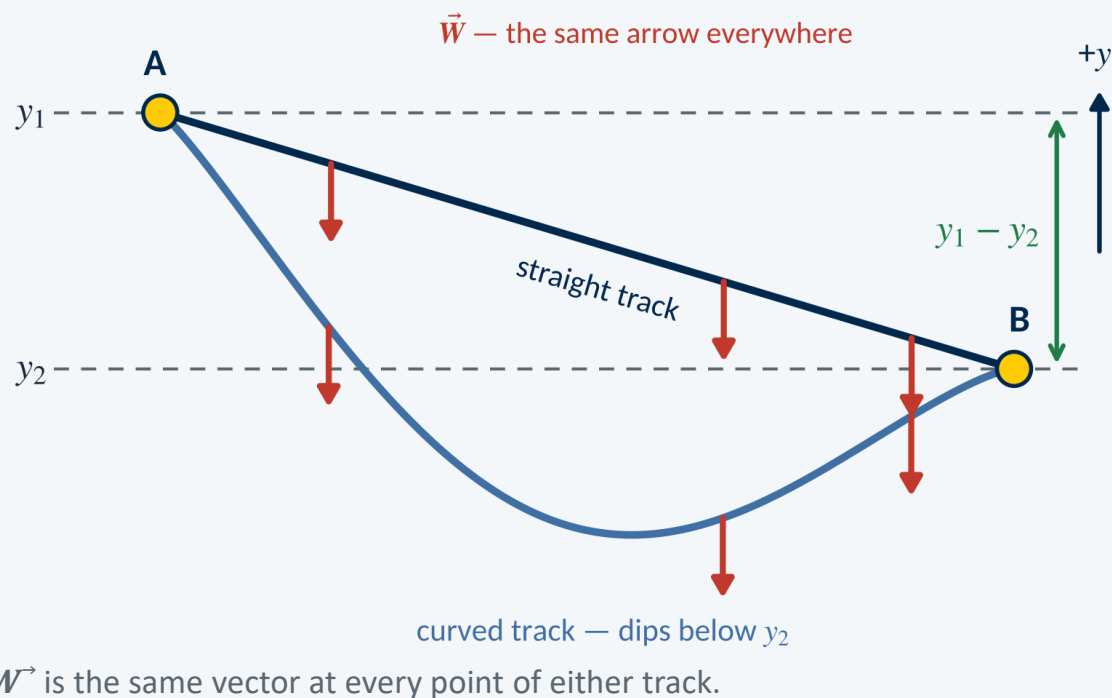
Solve / Analyze

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Work Done by Gravity



DERIVATION (+y UPWARD)

$$\vec{W} = -mg \hat{j}$$

$$dU_{1 \rightarrow 2}^{(g)} = \vec{W} \cdot d\vec{r} = -mg dy$$

$$U_{1 \rightarrow 2}^{(g)} = -mg(y_2 - y_1)$$

$$U_{1 \rightarrow 2}^{(g)} = mg(y_1 - y_2)$$

Only the height change appears. Path shape and length never enter.

Crate (slide 1): rises $3.0 \sin 20^\circ = 1.026$ m, so $U_{1 \rightarrow 2}^{(g)} = -(40)(9.81)(1.026) = -402.6$ J



Round trip, A $\rightarrow$ dip $\rightarrow$ A: $U_{1 \rightarrow 2}^{(g)} = 0$

Observe ›

Idealize ›

Model ›

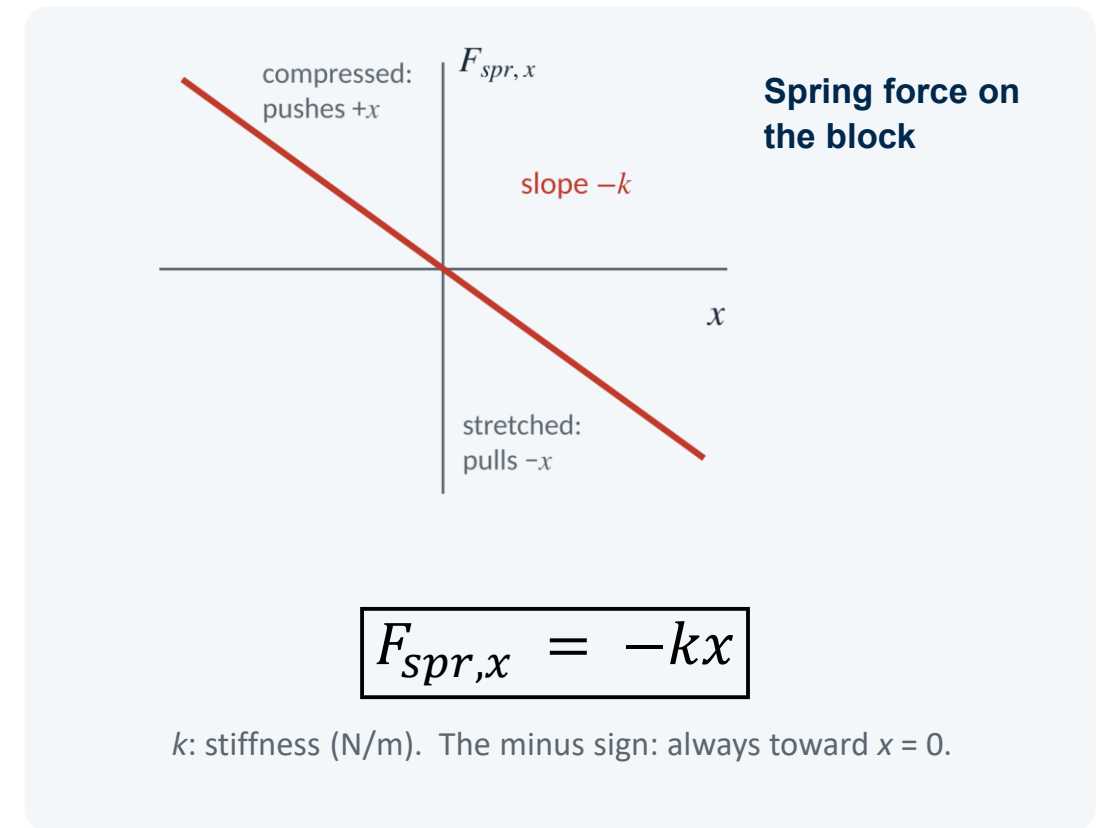
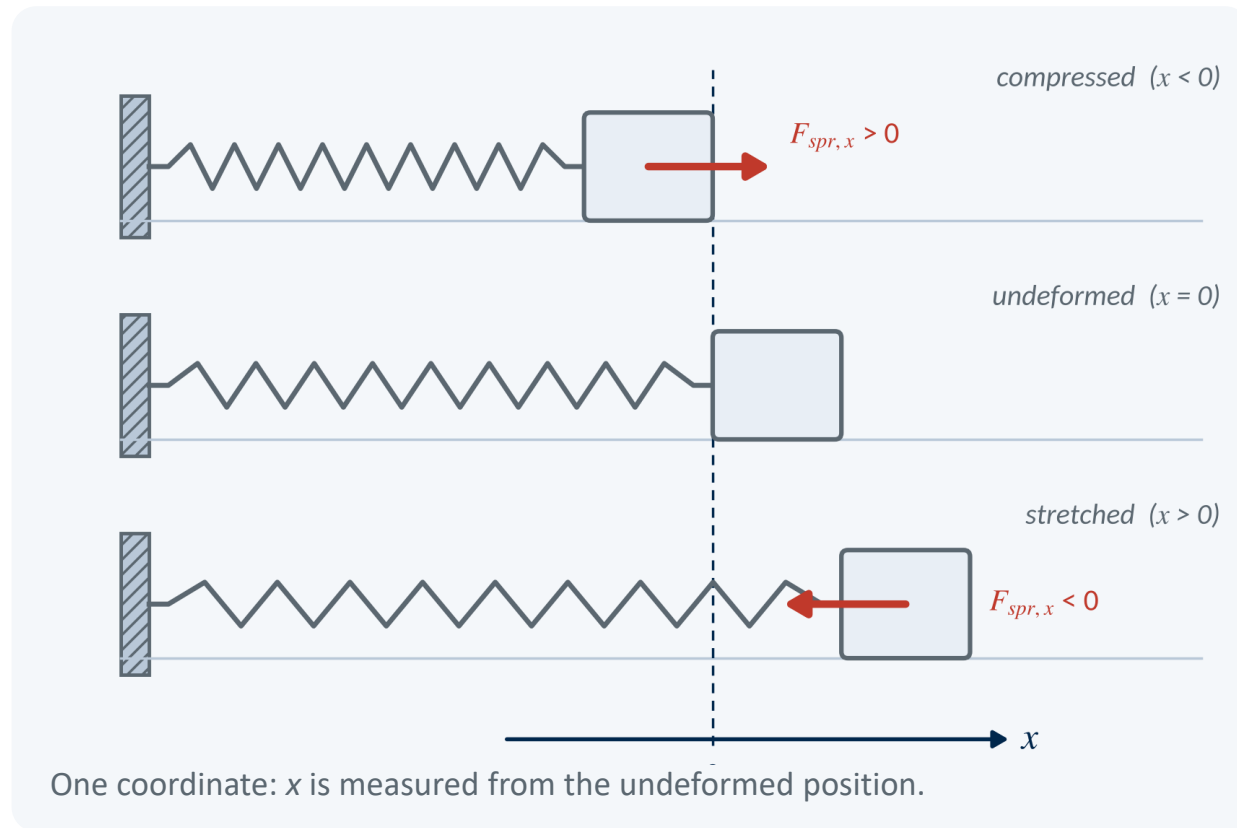
Solve / Analyze ›

Validate ›

Report



Springs: Force (applied on the block) Changes with Position



Hold the block at $x = -0.10$ m ($k = 800$ N/m): spring on block $F_{spr,x} = +80$ N · your hand on block -80 N · opposite signs

Observe

Idealize

Model

Solve / Analyze

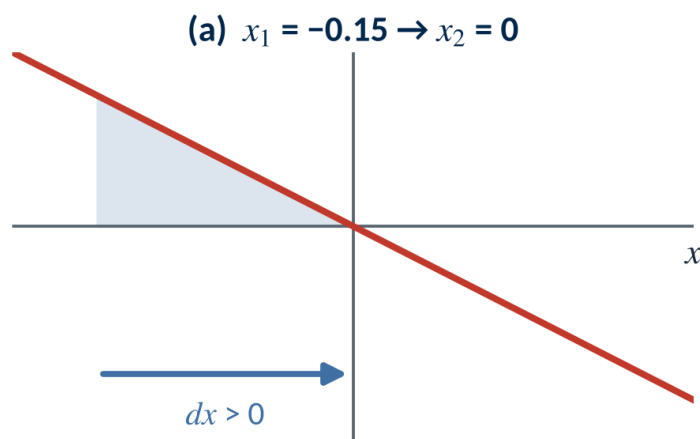
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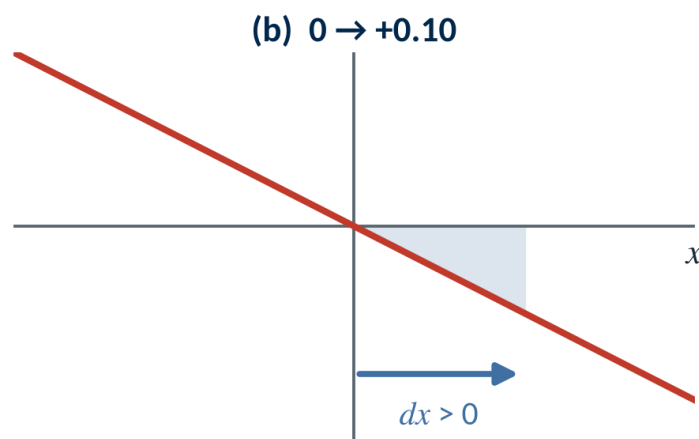


Work Done by a Spring

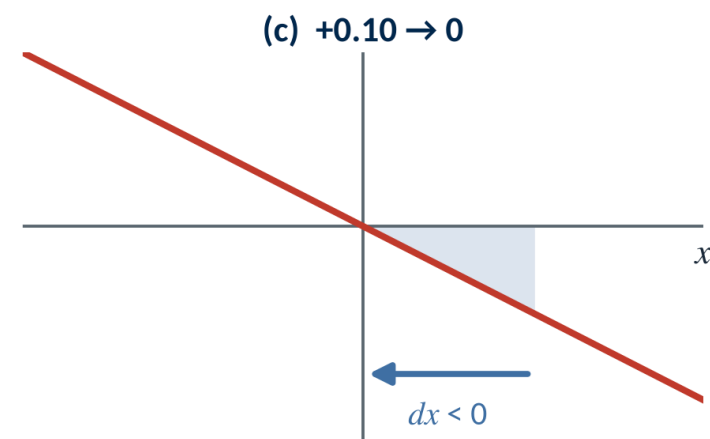
$$U_{1 \rightarrow 2}^{(spr)} = \int_{x_1}^{x_2} (-kx) dx = \boxed{\frac{1}{2} k (x_1^2 - x_2^2)}$$



(a) compressed $\rightarrow$ undeformed: work +, +9.0 J



(b) undeformed $\rightarrow$ stretched: work -, -4.0 J



(c) stretched $\rightarrow$ undeformed: work +, +4.0 J

$k = 800 \text{ N/m}$ in all three. x is a signed coordinate, so dx can be negative. (Parts 1–2 used path length, $ds > 0$, with the sign carried by F_t .)

(c): area below the axis, yet positive work — because $dx < 0$. Read the arrow, not just the region.

Observe

Idealize

Model

Solve / Analyze

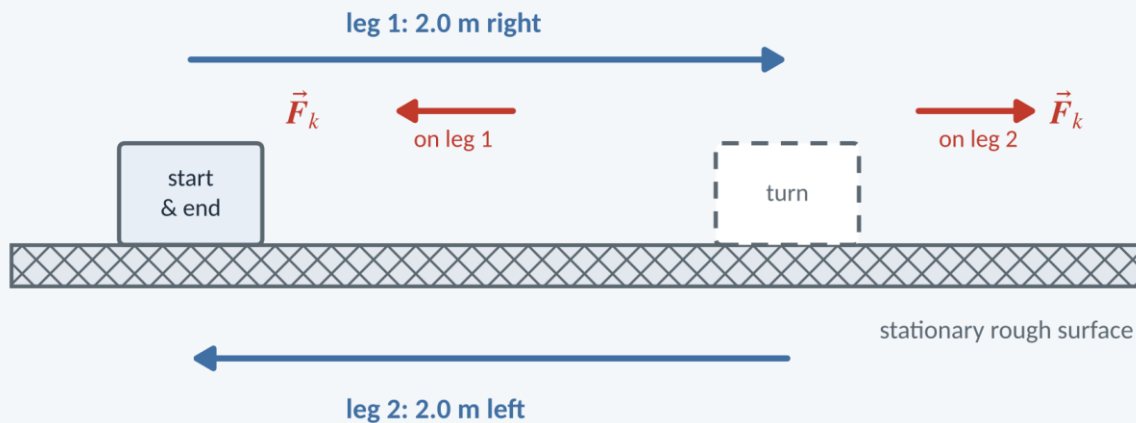
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Friction: Which Kind, and Which Distance?

A BLOCK SLIDES OUT AND BACK ON A STATIONARY FLOOR



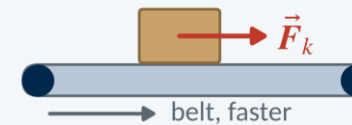
Displacement: 0. Total distance slid: 4.0 m. Friction's work: ?

$\vec{F}_k$ reverses with the motion, so its work is negative on **both** legs:

$$U_{1 \rightarrow 2}^{(f)} = -\mu_k N(4.0 \text{ m}) - \text{not zero.}$$

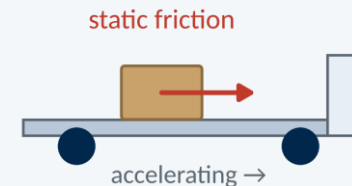
NOT ALWAYS NEGATIVE — STATE THE FRAME

Kinetic, faster belt



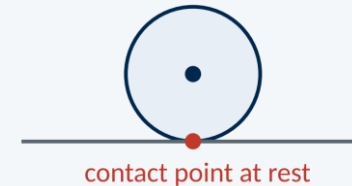
Parcel on a faster belt (ground frame): friction points along its motion → **positive** work.

Static, no slip



Box on an accelerating truck bed (ground frame): static friction speeds it up → **positive** work.

Static, rolling



Ideal wheel rolling without slip on fixed ground: contact point at rest → **zero** work.

Observe

Idealize

Model

Solve / Analyze

Validate

Report



Work Done by Kinetic Friction

DERIVATION

$$F_t = -F_k = -\mu_k N$$

$$dU_{1 \rightarrow 2}^{(f)} = -\mu_k N ds, \quad ds > 0$$

$$U_{1 \rightarrow 2}^{(f)} = -\int_{s_1}^{s_2} \mu_k N ds$$

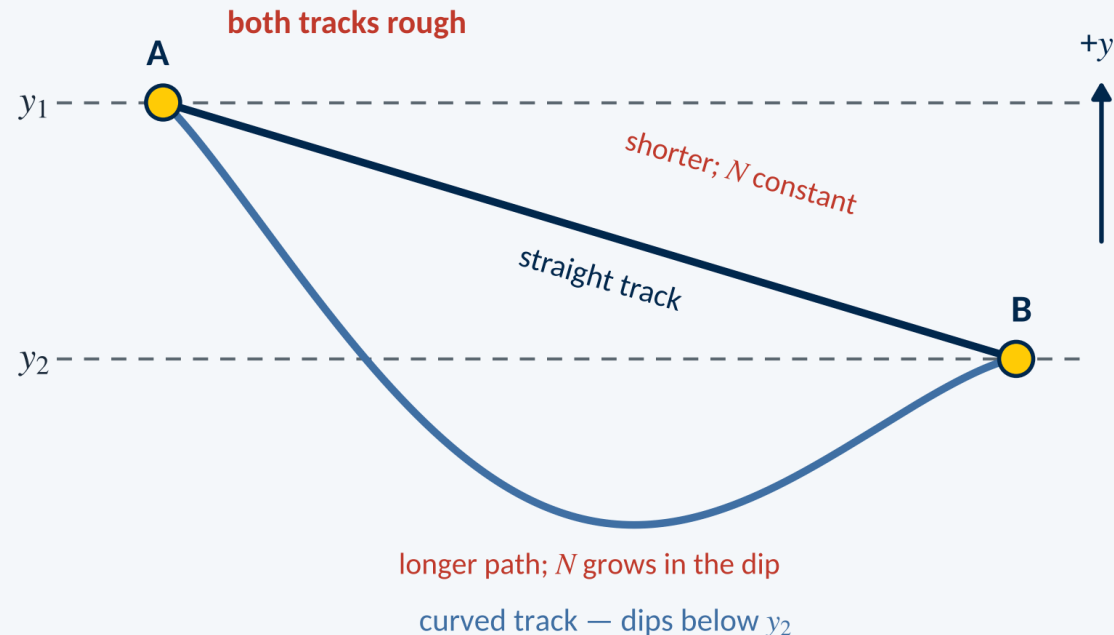
$$U_{1 \rightarrow 2}^{(f)} = -\mu_k N (s_2 - s_1)$$

arc length traveled,

boxed form: only if N is constant along the path

Graph F_t against s : a line at $-\mu_k N$. Because s only increases, the area — and the work — is always negative here.

The longer path and potentially larger normal force can increase the magnitude of friction work. The correct general expression remains $U^{(f)} = -\int \mu_k N ds$.



Same endpoints — different friction work.

Crate (slide 11): $U_{1 \rightarrow 2}^{(f)} = -(0.30)(301.4 \text{ N})(3.0 \text{ m}) = -271.3 \text{ J}$

Observe

Idealize

Model

Solve / Analyze

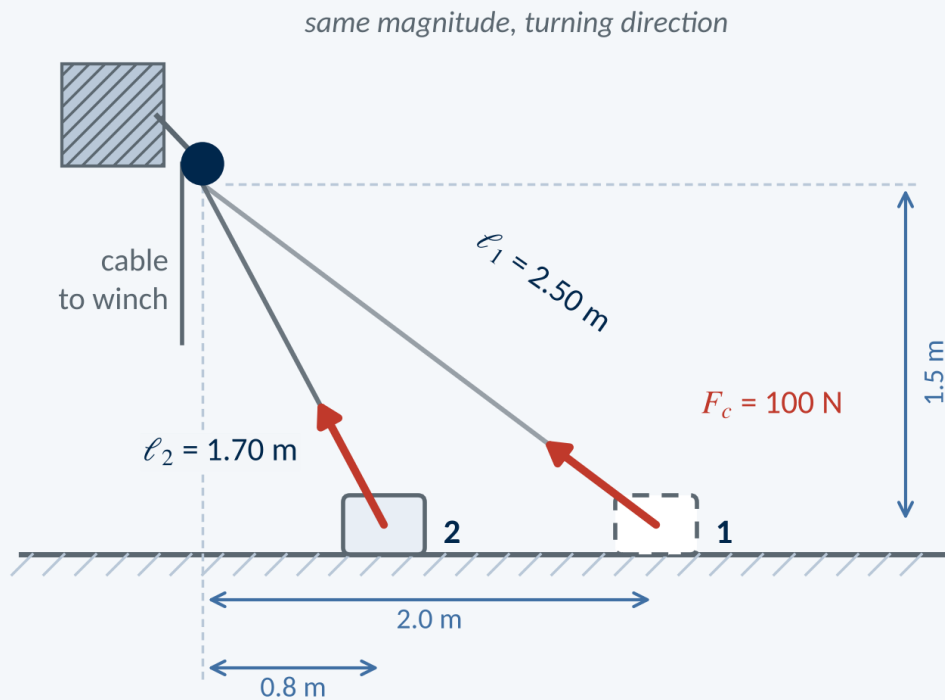
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Applied Forces and Changing Directions

Assumptions: a taut cable, ideal fixed pulley, constant 100 N tension, and fixed pulley position. Clarify that ℓ is the block-to-pulley segment, not the entire cable length



A winch keeps $F_c = 100$ N while the block slides from 1 to 2 on the floor (1.2 m). Ideal fixed pulley, taut cable. ℓ = cable length from block to pulley.

PREDICT

Constant 100 N over 1.2 m. Is the cable's work 120 J?

$$\vec{F}_c \cdot d\vec{r} = -F_c d\ell$$

$$U_{1 \rightarrow 2}^{(c)} = F_c(\ell_1 - \ell_2)$$

$$= (100 \text{ N})(2.50 \text{ m} - 1.70 \text{ m}) = 80.0 \text{ J}$$

Attempt	Result	What went wrong
100×1.2	120 J	treats the cable as horizontal
$100 \times (2.0/2.50) \times 1.2$	96 J	freezes the starting angle

Observe

Idealize

Model

Solve / Analyze

Validate

Report

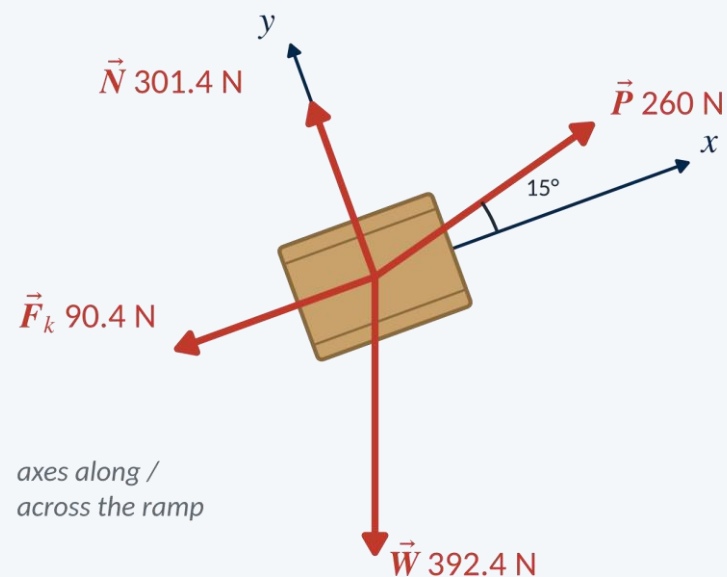


A Crate on an Inclined Plane

System: crate (particle). 1 → 2: 3.0 m up the ramp.

$P = 260 \text{ N}$ at 15° · $\mu_k = 0.30$ · $v_{sp,1} = 0.50 \text{ m/s}$

Assume the cable stays at 15° to the ramp (winch far away).



① NORMAL FORCE

② WORK LEDGER

Force	Work over $(s_2 - s_1) = 3.0 \text{ m}$	Work (J)
Cable pull	$P \cos 15^\circ (3.0)$	+753.4
Weight	$-mg(3.0 \sin 20^\circ)$	-402.6
Normal	perpendicular to motion	0
Friction	$-\mu_k N(3.0)$	-271.3
Net	$U_{1 \rightarrow 2}^{\text{net}}$	+79.5

③ WORK-ENERGY

$$T_1 = 5.00 \text{ J}, \quad T_2 = 5.00 + 79.5 = 84.5 \text{ J}, \quad v_{sp,2} = 2.06 \text{ m/s}$$

④ Chapter 3 check:

$$\Sigma F_x = 26.5 \text{ N}, \quad a = 0.6625 \frac{\text{m}}{\text{s}^2}, \quad v_{sp,2}^2 = 0.50^2 + 2(0.6625)(3.0) \rightarrow \mathbf{2.06 \text{ m/s} \checkmark}$$

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



A Spring Launches a Slider



1 released from rest, $x_1 = -0.150$ m



2 separates at $x = 0$ — spring work stops



3 stops at $x = ?$

$m = 2.0$ kg · $k = 800$ N/m · released from rest at $x_1 = -0.150$ m · **not attached** · $\mu_k = 0.20$ everywhere · spring region drawn enlarged

Q1 · Speed at separation (1 → 2)?

$$U_{1 \rightarrow 2}^{(spr)} = \frac{1}{2}(800)(0.150^2) = +9.000 \text{ J}$$

$$U_{1 \rightarrow 2}^{(f)} = -(0.20)(2.0)(9.81)(0.150) = -0.589 \text{ J}$$

$$T_2 = 0 + 8.411 \text{ J} \rightarrow v_{sp,2} = \mathbf{2.90 \text{ m/s}}$$

Q2 · Where does it stop (2 → 3)?

Spring no longer touches it. Only friction works:

$$0 = 8.411 \text{ J} - (3.924 \text{ N})(s_3 - s_2) \rightarrow 2.144 \text{ m}$$

Stops at $x = \mathbf{+2.144 \text{ m}}$ (2.294 m slid in all)

Observe

Idealize

Model

Solve / Analyze

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Can We Trust Our Work Ledger?

CHECK EVERY LEDGER

- 1 Every force on the FBD has a row — Account for every force on the FBD, including those that do zero work.
- 2 Each sign matches the direction of the force relative to the motion.
- 3 Gravity used the height change, friction the distance slid.
- 4 Every row is in joules; $T_2 \geq 0$.
- 5 A limiting case behaves: $\mu_k = 0$ must give a larger speed (crate: 4.22 m/s).
- 6 An independent method agrees (slide 11, step ④).

DIAGNOSE THIS

A classmate redoes the slide 11 crate with $N = mg \cos 20^\circ = 368.7 \text{ N}$ and finds $v_{sp,2} = 1.09 \text{ m/s}$. Which check catches it, and what is wrong?

The cable's 15° tilt lifts the crate, so $N = 301.4 \text{ N}$. Using 368.7 N makes the friction row -331.9 J instead of -271.3 J . Check 1 (a fresh FBD) finds the missing $P \sin 15^\circ$; check 6 (Newton, from that FBD) then disagrees.



Then — and only then — ask an AI

“Here is my FBD and ledger. Which force could be missing? Which sign looks wrong?” Treat its answer as a second opinion: verify each claim against your own FBD.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



What Have We Learned?

Force	Its work, $1 \rightarrow 2$	Depends on ...	Path matters?
Gravity (near Earth)	$mg(y_1 - y_2)$	the height change only	No
Linear spring	$\frac{1}{2}k(x_1^2 - x_2^2)$	the end deformations only	No
Kinetic friction (fixed surface)	$-\mu_k N(s_2 - s_1)$	N and the total distance slid	Yes
Normal force (fixed surface)	0	zero: always perpendicular to motion	—
Cable over a fixed pulley	$F_c(\ell_1 - \ell_2)$	the cable length reeled in	No (ideal case)
Any force	$\int F_t ds$	the force along the whole path	In general, yes



Same ledger as Part 2

Add the rows that act: $T_1 + U_{1 \rightarrow 2}^{\text{net}} = T_2$.



Look at the last column

Some forces' work needs only the endpoints. Remember which — Part 5 returns to it.

Next (Part 4): we know how much work a force does. How fast does it do it?

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report