

CHAPTER - 4

PART 4 · ≈ 56 MIN

How Fast Is Work Done? Power and Efficiency

Two hoists do the same work on the same pallet. Why does one of them need a much bigger motor?

4

Observe

Idealize

Model

Solve / Analyze

Validate

Report



Same Work, Different Time



THE SITUATION

Two hoists lift identical 500 kg pallets.
Both lift 6.0 m, starting and ending at rest.
Hoist A takes 12.0 s. Hoist B takes 6.0 s.

COMPARE

What is the same about these two lifts?
What is different?



The question this Part answers

Two machines can do the same work. What else must an engineer know to choose the right one?

Observe

Idealize

Model

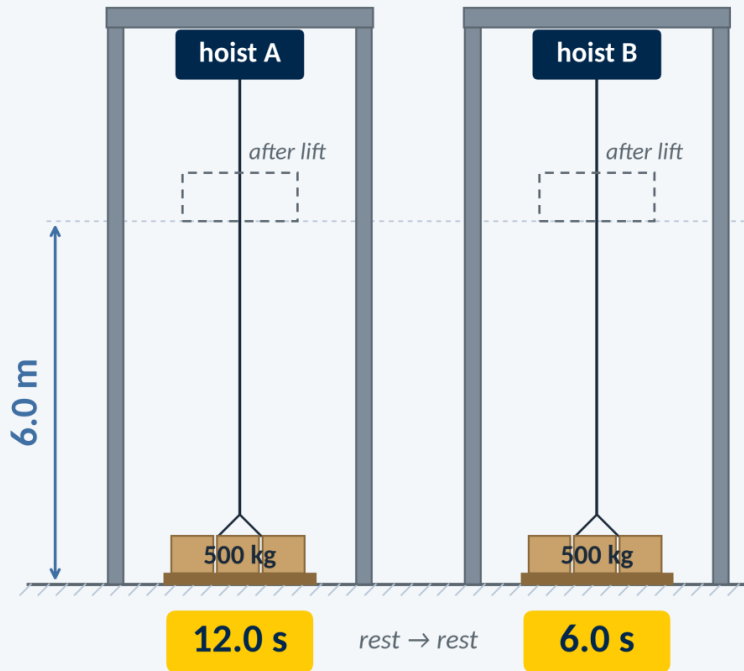
Solve / Analyze

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Which Hoist Needs More Power?



500 kg · 6.0 m · rest to rest

A: 12.0 s B: 6.0 s

1

Does the cable do more work on the pallet in hoist A, in hoist B, or the same in both?

2

How does B's average cable power compare with A's?

3

During B's lift, is the cable power the same at every instant? Sketch your guess of power against time.



Commit first

Write your three answers and one reason each. We test all three on the next slides.

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Work per Unit Time

AVERAGE POWER OF ONE FORCE

$$P_{\text{avg}} = \frac{U_{1 \rightarrow 2}}{t_2 - t_1}$$

$$1 \text{ W} = 1 \text{ J/s} \quad 1 \text{ kW} = 1000 \text{ J/s}$$

CABLE WORK, FROM PART 2

$$T_1 + U_{1 \rightarrow 2}^{(c)} + U_{1 \rightarrow 2}^{(g)} = T_2$$

$$0 + U_{1 \rightarrow 2}^{(c)} - mgh = 0 \Rightarrow U_{1 \rightarrow 2}^{(c)} = mgh$$

Practical feel: 1 kW lifts about 100 kg at 1 m/s.

A kilowatt-hour (3.6 MJ) is energy, not power.

THE TWO HOISTS ($m = 500 \text{ kg}$, $h = 6.0 \text{ m}$)

Quantity	Hoist A	Hoist B
Cable work $U_{1 \rightarrow 2}^{(c)}$	29.43 kJ	29.43 kJ
Lift time	12.0 s	6.0 s
Average cable power	2.45 kW	4.91 kW
Net work $U_{1 \rightarrow 2}^{net}$	0	0
Average net power	0	0



Whose power?

The cable delivers 4.91 kW on average in hoist B, yet the average NET power on the pallet is zero. Always say which force — or the net force — you mean.

Observe



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Power at an Instant

1

Part 1: work of a force over a small displacement

$$dU = \vec{F} \cdot d\vec{r}$$

2

Divide by dt : the rate of doing work (Power- P)

$$P = \frac{dU}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

3

θ = angle between $\vec{F}$ and $\vec{v}$

$$P = F v_{sp} \cos \theta = F_t v_{sp}$$

4

Undo the rate: work from a power history

$$U_{1 \rightarrow 2} = \int_{t_1}^{t_2} P dt$$

NET POWER (PART 2)

$$\Sigma P = \frac{dT}{dt}$$

Sum the power of every force: you get the rate of change of kinetic energy. Integrate it: $T_1 + U_{1 \rightarrow 2}^{\text{net}} = T_2$.



Can a constant force deliver changing power?

Predict before slide 5.

Hint: what else is in $P = \vec{F} \cdot \vec{v}$?

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

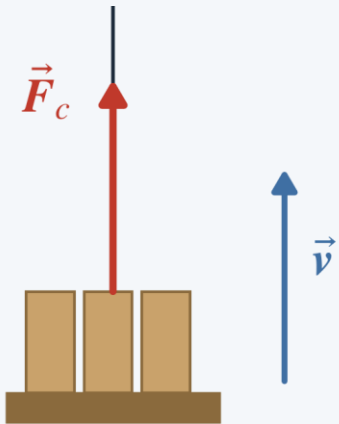
Validate ›

Report



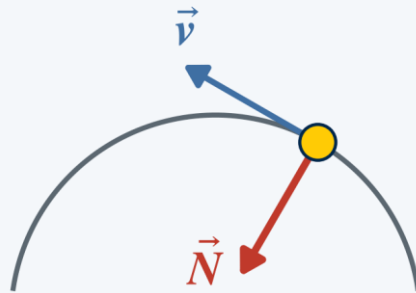
Force, Velocity, and the Sign of Power

Cable lifting



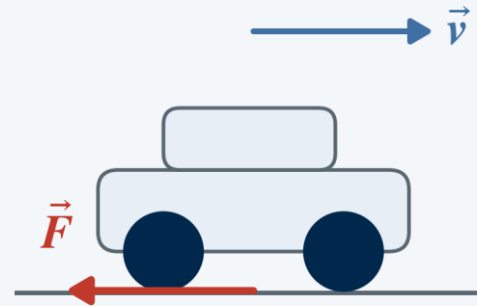
$$\theta = 0^\circ \quad P > 0$$

Normal force on a moving bead



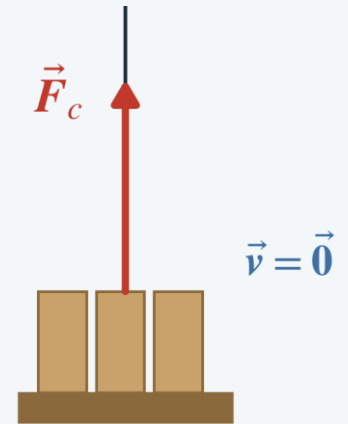
$$\theta = 90^\circ \quad P = 0$$

Braking force on a car



$$\theta = 180^\circ \quad P < 0$$

Cable holding a load still



$$v_{sp} = 0 \quad P = 0$$

Power needs motion along the force: the 4.9 kN cable force on a pallet held still delivers zero mechanical power.

Observe



Idealize



Model



Solve / Analyze



Validate

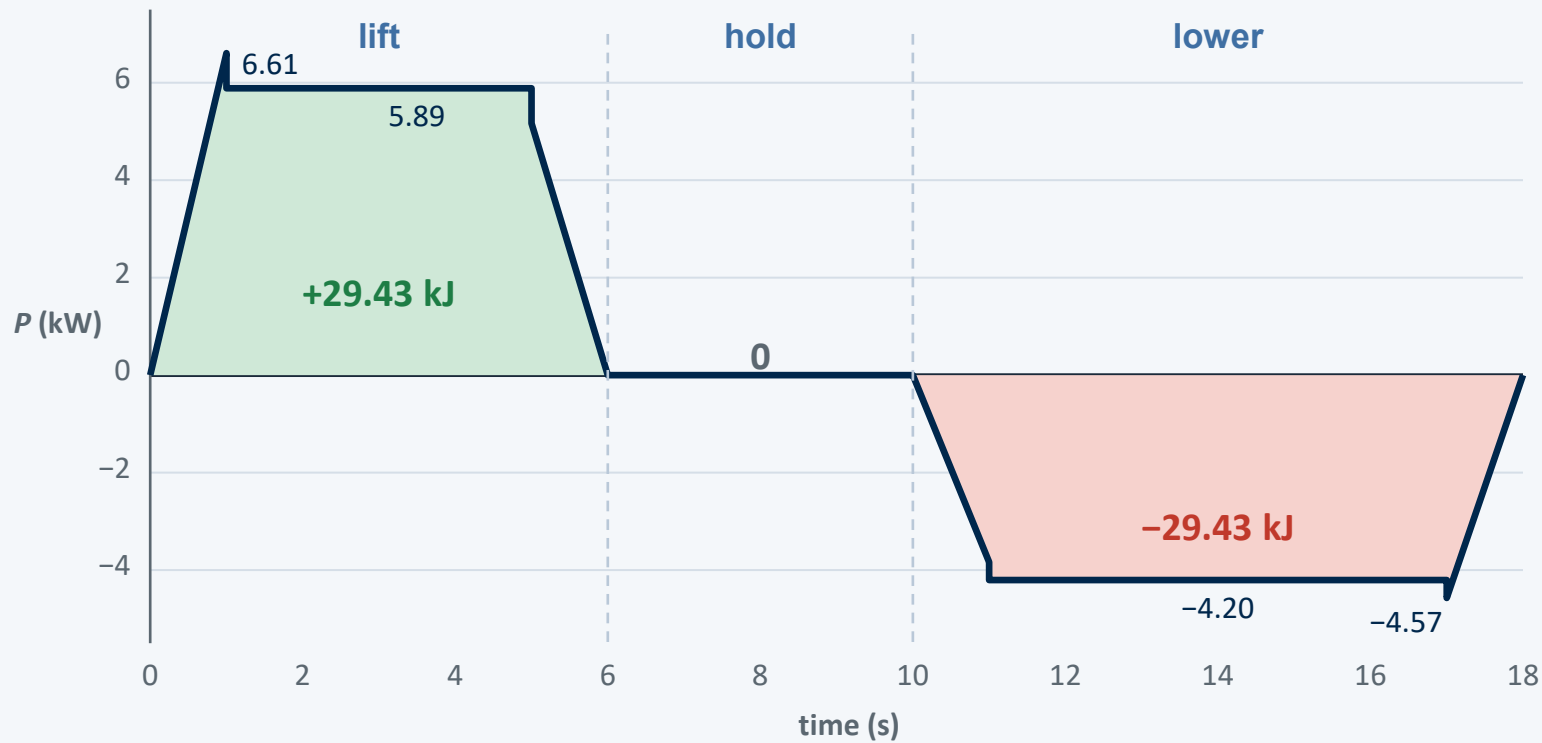


Report



Reading a Power–Time Graph

Power of the cable force on the pallet



READ IT

1. Label each interval +, 0 or -. Why?
2. What does each shaded area measure?
3. What is the cable's work over the whole cycle?

$$U_{1 \rightarrow 2}^{(c)} = \int_{t_1}^{t_2} P \, dt$$

$$U_{\text{cycle}}^{(c)} = 29.43 \, \text{kJ} - 29.43 \, \text{kJ} = 0$$

Area above the axis: positive work. Below: negative work. Hold: big force, zero area.

Observe ›

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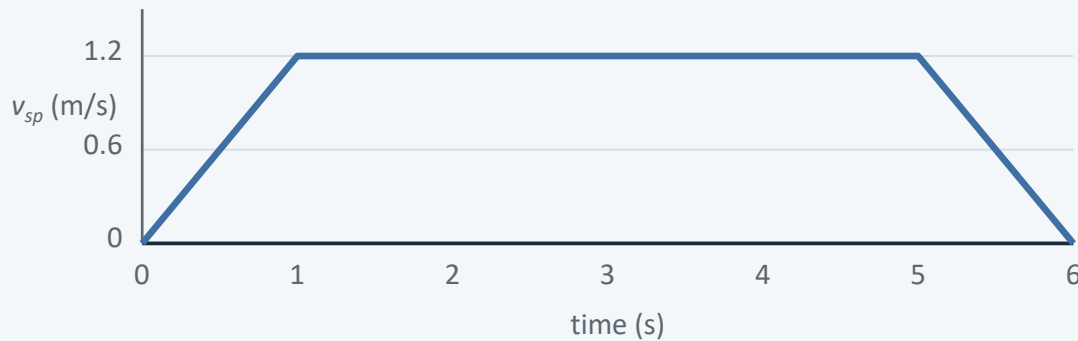
Validate ›

Report



How Much Power Does the Hoist Need?

Speed profile (given): 1 s up to 1.20 m/s, 4 s cruise, 1 s down



$$(+y \text{ up}) \quad F_c - mg = ma \Rightarrow F_c = m(g + a)$$

PREDICT

Where in the lift is the cable power largest?

Peak **6.61 kW**, at the end of the acceleration

Cruise **5.89 kW** · Average **4.91 kW**

Peak ÷ average = 1.35 · Hoist A peak 3.03 kW

Phase	a (m/s ²)	$F_c = m(g + a)$	$P = F_c v_{sp}$	Work
Accelerate, 0–1 s	+1.20	5505 N	0 → 6606 W	3303 J
Cruise, 1–5 s	0	4905 N	5886 W	23 544 J
Decelerate, 5–6 s	–1.20	4305 N	5166 → 0 W	2583 J
Whole lift				29 430 J = mgh ✓

Observe



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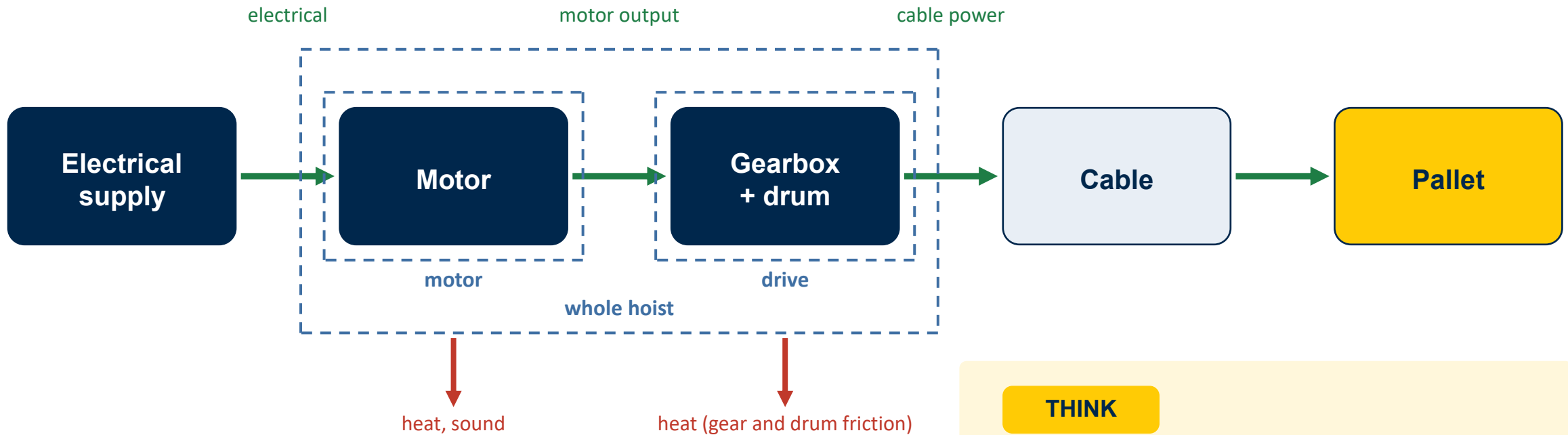


Report



Where Does the Input Power Go?

Green arrows: useful mechanical or electrical power flow. Red arrows: losses leaving the machine.



At hoist B's cruise: cable 5.89 kW; drive efficiency 0.90 $\Rightarrow$ motor output 6.54 kW;
0.65 kW lost in the drive.

THINK

Which boundary has the highest efficiency? The lowest?
The whole-hoist efficiency is no greater than either individual stage efficiency, but we cannot identify which individual stage is most efficient from the given data.

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What Does Efficiency Really Measure?

AT A STATED OPERATING POINT

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

POWER BALANCE

$$P_{\text{in}} = P_{\text{out}} + P_{\text{loss}}$$

OVER A WHOLE OPERATING INTERVAL

Energy-based efficiency = useful work out ÷ energy in, over the interval. When power varies, it is not the simple average of the instantaneous efficiencies.

Efficiency often decreases substantially at light loads; the actual efficiency depends on the motor and operating point.



Same efficiency at 10% load?

Usually not: a motor rated 90% at full load is much less efficient at 10% load. A nameplate value applies at the rated point.



Do losses destroy energy?

No. They leave the useful path as heat and sound. Energy is conserved; efficiency is still below 1.



What if the cable power is negative?

Lowering sends energy back into the hoist. It may be regenerated or dissipated; η as defined does not describe that flow.

Observe ›

Idealize ›

Model ›

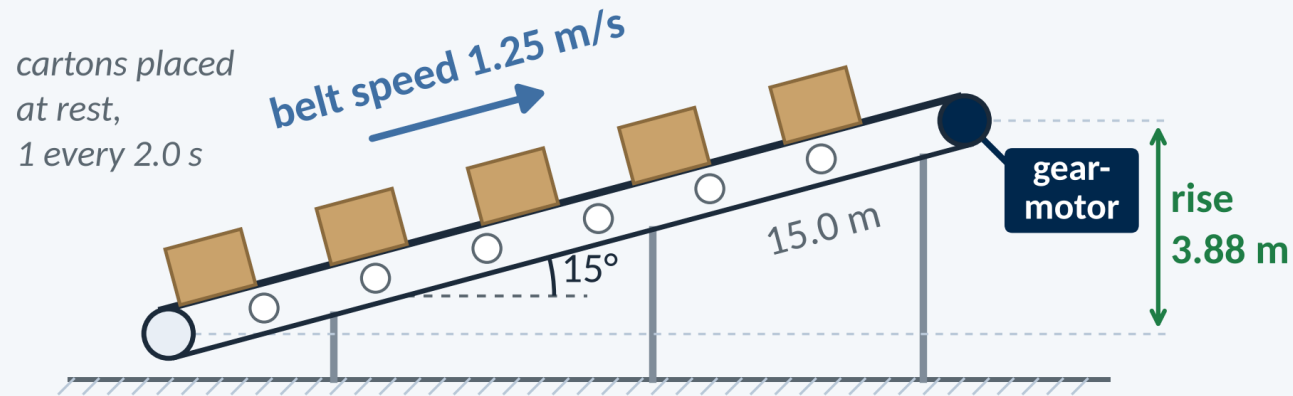
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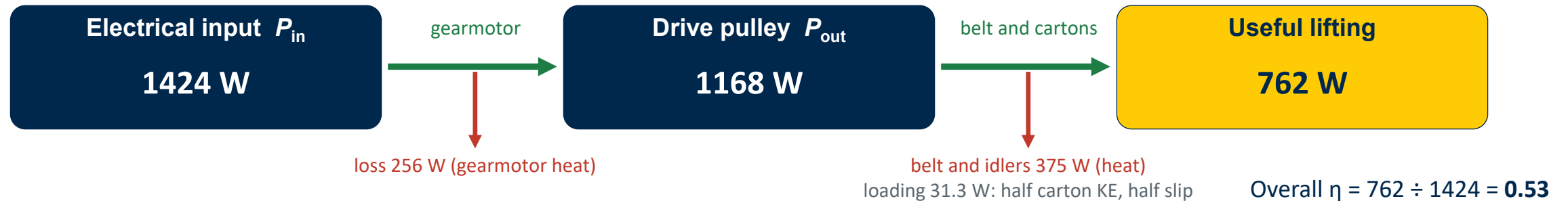


A Motor-Driven Conveyor



GIVEN

15.0 m belt at 15°, speed 1.25 m/s
 40 kg cartons, placed at rest, 1 every 2.0 s
 Average load on the incline: 240 kg
 Belt and idler resistance (measured): 300 N
 Gearmotor efficiency here: 0.82
 Steady throughput, time-averaged model



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Can the Motor Handle the Demand?

Hoist B: motor output needed = cable power ÷ 0.90

motor output (kW)



TWO CANDIDATE MOTORS

Option A: 5.5 kW continuous rating

Option B: 7.5 kW continuous rating

Average demand is 5.45 kW, below 5.5 kW. Is Option A adequate for that reason alone?

No — the average cannot decide. Cruise runs 19% and the peak 33% above 5.5 kW. Check short-time overload capability, duty cycle and the maker's data. Option B covers the whole profile continuously.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Can We Trust the Power Estimate?

CHECK EVERY POWER ESTIMATE

- 1 Whose power? Which force, which boundary?
- 2 Units: W (rate) vs J or kWh (amount).
- 3 Signs match the force–velocity geometry.
- 4 Average vs peak: which one sizes the machine?
- 5 $\text{Input} = \text{output} \div \eta$, never $\text{output} \times \eta$.
- 6 Limiting case: gentler acceleration $\rightarrow$ peak approaches cruise.
- 7 Plausible: about 1 kW per 100 kg lifted at 1 m/s.

DIAGNOSE THIS

A classmate sizes hoist B's motor: $4.91 \text{ kW} \times 0.90 = 4.42 \text{ kW}$, "so a 4.5 kW motor is enough." Find two errors.

(1) The average was used as the demand (check 4). (2) Efficiency was multiplied, not divided (check 5). The motor-output demand peaks at 7.34 kW — 1.7 times the classmate's figure.



Then — and only then — ask an AI

"Here is my demand history and my motor choice. Which demand did I size for, at which boundary?" Verify every claim against your own profile and numbers.

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What Have We Learned?

Quantity	Definition	Unit	Answers the question ...
Work	$U_{1 \rightarrow 2} = \int \vec{F} \cdot d\vec{r}$	J	how much energy a force transfers
Kinetic energy	$T = \frac{1}{2} m v_{sp}^2$	J	how much motion a particle has
Power	$P = \vec{F} \cdot \vec{v} = F v_{sp} \cos \vartheta$	W	how fast a force transfers energy
Average power	$P_{avg} = U_{1 \rightarrow 2} \div (t_2 - t_1)$	W	how fast, on average, over an interval
Efficiency	$\eta = P_{out} \div P_{in}$	—	what fraction reaches the output, at a boundary



Name the force and the boundary

Cable, net, motor output and electrical input powers are different numbers.



Size machines for the demand history

Peak, duration and duty cycle — not just the average.

Lowering: the cable did -29.43 kJ of work on the pallet, exactly undoing the lift. Where was that energy while the pallet waited at the top? (Part 5)

Observe ›

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