

CHAPTER - 4

PART 5 · ≈ 54 MIN

Where Is the Energy Stored?

Potential Energy and Conservative Forces

Gravity and springs do work we can account for without integrating every time. What makes them special — and which forces are not?

Observe

Idealize

Model

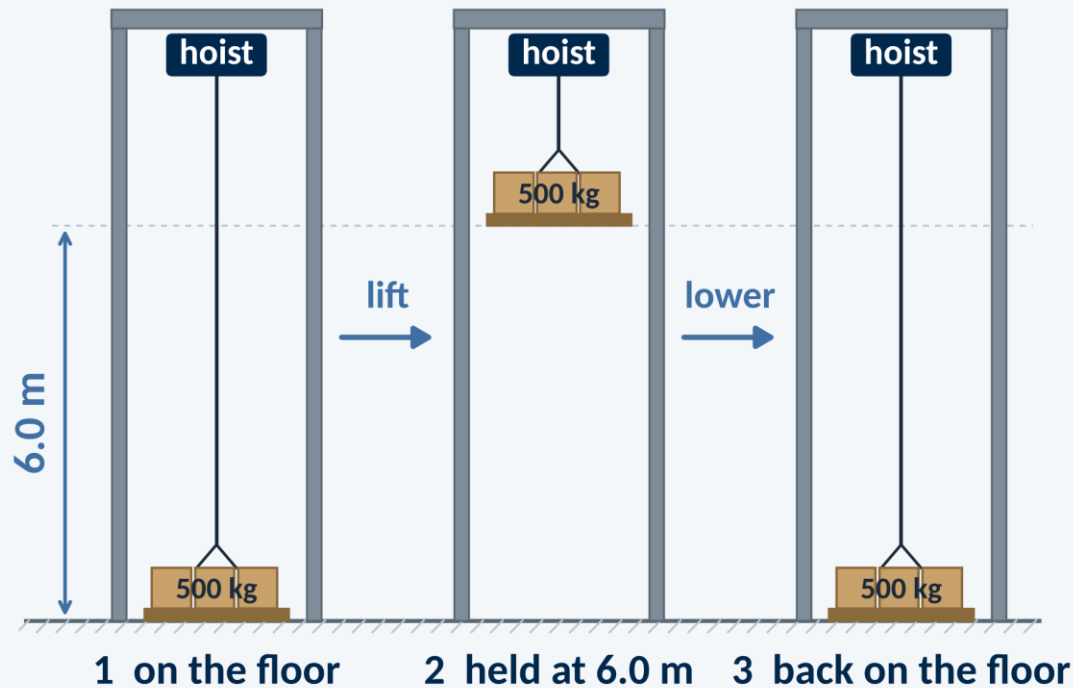
Solve / Analyze

Validate

Report



Where Was the Energy While the Pallet Waited?



Cable work on the pallet	Lift	Hold	Lower
$U_{1 \rightarrow 2}^{(c)}$	+29.43 kJ	0	-29.43 kJ

FROM PART 4

Same 500 kg pallet, same 6.0 m lift.

These are CABLE-work values — not the work of gravity.

THINK

The pallet is stationary at the top, yet lowering it can transfer energy back through the cable.

What changed in the physical system during the lift?

Observe

Idealize

Model

Solve / Analyze

Validate

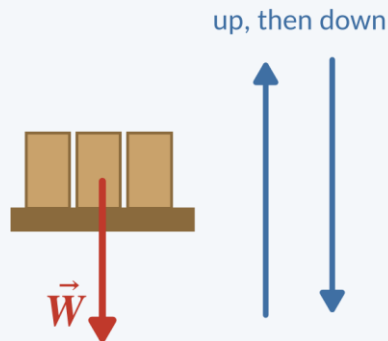
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Which Forces Care Only About Endpoints?

Gravity

pallet up, then down

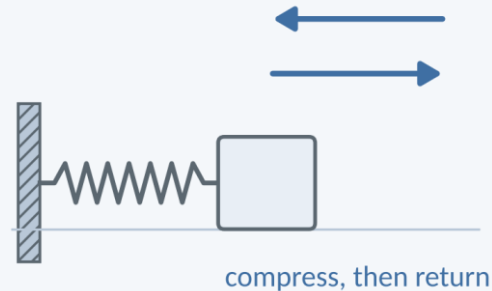


Net work, whole round trip?

0

Ideal spring

compressed, then returned

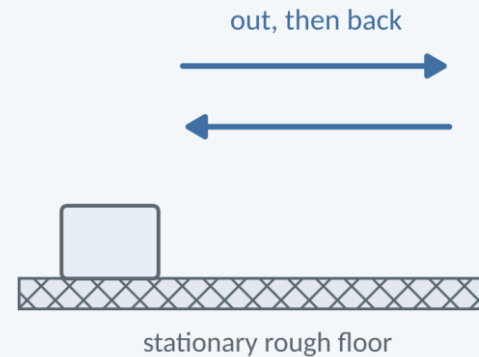


Net work, whole round trip?

0

Kinetic friction

out and back on a stationary rough floor

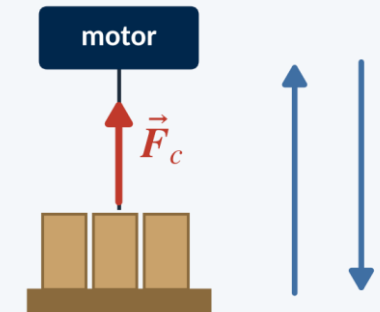


Net work, whole round trip?

negative

Motor-driven cable

load lifted, then lowered



Net work, whole round trip?

depends on the force history (0 here: rest to rest)

Reversing the motion does not by itself make the cable's net work zero — the motor sets the force. Here it is 0 only because the pallet starts and ends at rest.

Observe

Idealize

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A Different Way to Account for Work

For a conservative force:

$$\Delta V = V_2 - V_1$$

WORK LEDGER

ENERGY FORM

Gravity $U_{1 \rightarrow 2}^{(g)}$



$$-\Delta V_g$$

Spring $U_{1 \rightarrow 2}^{(spr)}$



$$-\Delta V_{spr}$$

Friction $U_{1 \rightarrow 2}^{(f)}$



in $U_{1 \rightarrow 2}^{nc}$

Cable $U_{1 \rightarrow 2}^{(c)}$



in $U_{1 \rightarrow 2}^{nc}$

Part 2: the work-energy principle

$$T_1 + U_{1 \rightarrow 2}^{net} = T_2$$

Split the work: conservative + all the rest

$$T_1 + U_{1 \rightarrow 2}^{cons} + U_{1 \rightarrow 2}^{nc} = T_2$$

Replace the conservative work by $V_1 - V_2$

$$T_1 + V_1 + U_{1 \rightarrow 2}^{nc} = T_2 + V_2$$

Each conservative force appears ONCE: as its work, or through its ΔV — never both.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



Gravitational Potential Energy

DERIVATION (+y UPWARD, UNIFORM g)

$$U_{1 \rightarrow 2}^{(g)} = mg(y_1 - y_2)$$

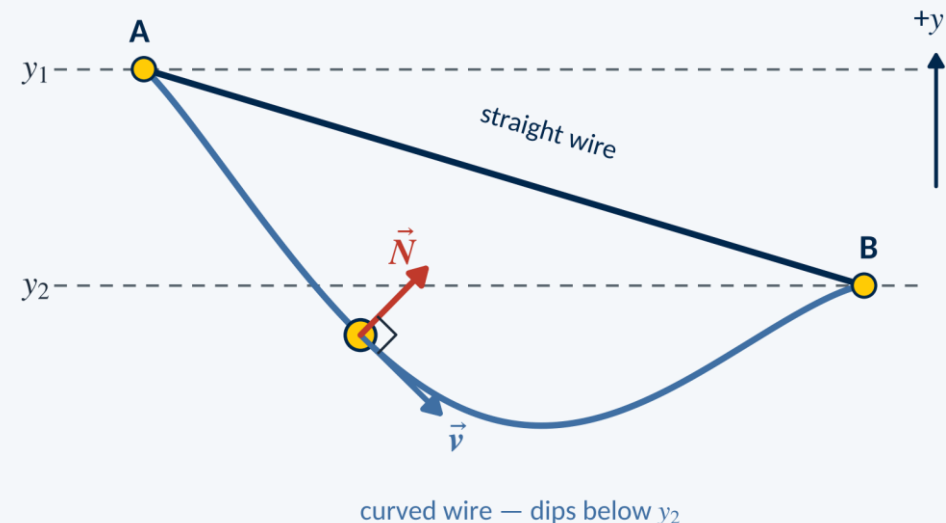
$$U_{1 \rightarrow 2}^{(g)} = -(mgy_2 - mgy_1)$$

$$V_g = mgy + C$$

$$\boxed{V_g = mgy}$$

with $V_g = 0$ at $y = 0$ (a chosen reference level)

Check: $U_{1 \rightarrow 2}^{(g)} = -\Delta V_g = -(mgy_2 - mgy_1) \checkmark$ — the constant C always cancels.



Whose energy?

Gravitational potential energy belongs to the Earth–object interaction. Near Earth, “the bead’s V_g ” is acceptable shorthand once that system is named.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

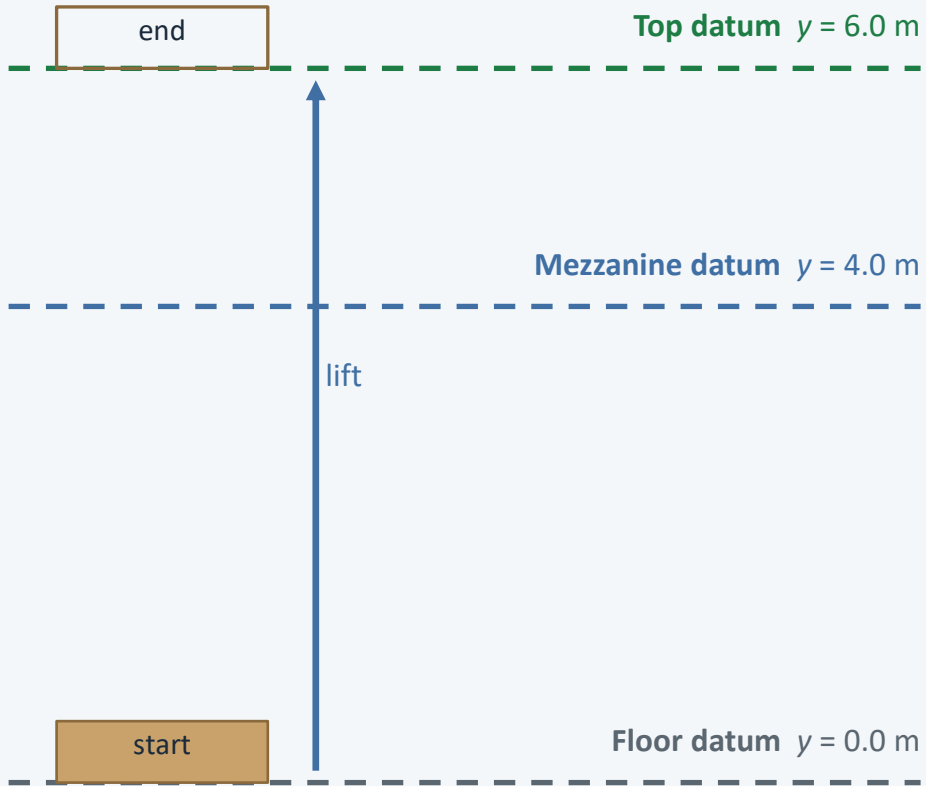
Validate ›

Report



Where Is Zero Potential Energy?

Pallet $m = 500$ kg, raised from $y = 0$ to 6.0 m



THINK

Which reference level gives the correct potential-energy change?

Datum	$V_{g,1}$	$V_{g,2}$	ΔV_g
Floor, $y = 0.0$ m	0	+29.43 kJ	+29.43 kJ
Mezzanine, $y = 4.0$ m	-19.62 kJ	+9.81 kJ	+29.43 kJ
Top, $y = 6.0$ m	-29.43 kJ	0	+29.43 kJ

$$\Delta V_g = +29.43 \text{ kJ} \Rightarrow U_{1 \rightarrow 2}^{(g)} = -29.43 \text{ kJ}$$

All three. V_g itself depends on the datum and can be negative; ΔV_g does not. Gravity's work has the opposite sign of ΔV_g .

Observe

Idealize

Model

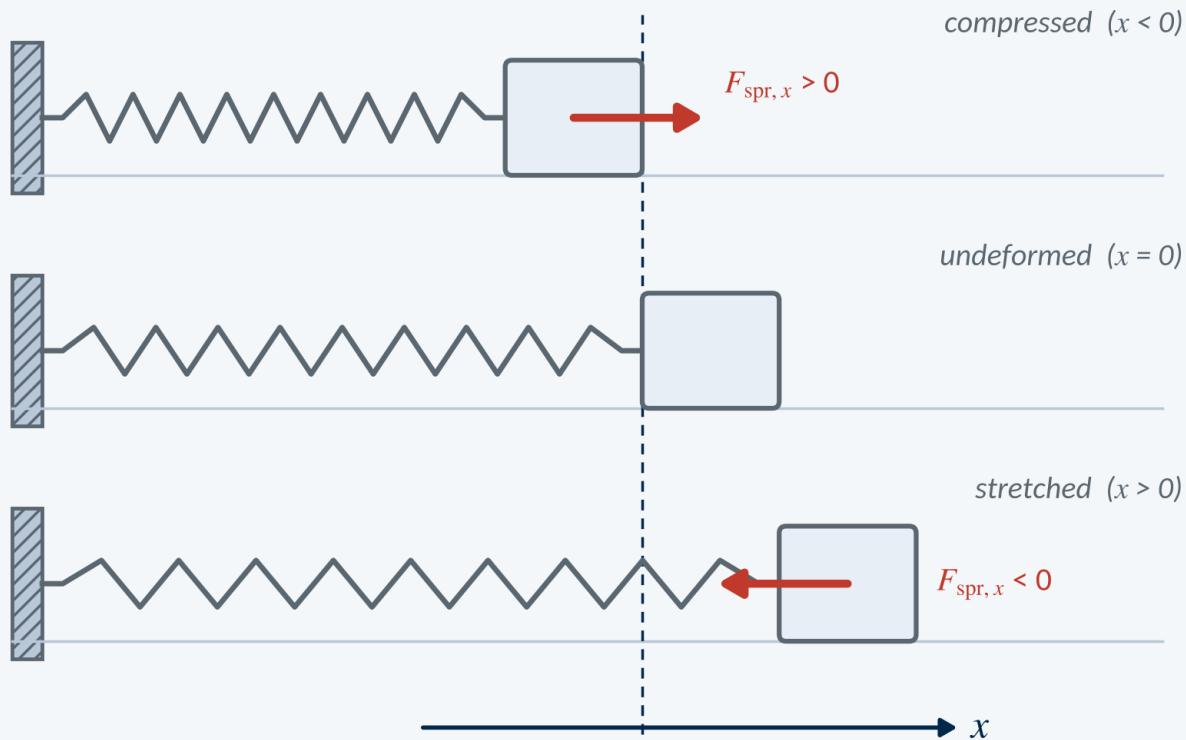
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A Spring Stores Energy Too



One ideal linear spring, one signed x axis: $x = 0$ at the undeformed length.

PREDICT

A spring is compressed by 0.15 m, or stretched by 0.15 m.

Which state has more elastic potential energy? Is either value negative?

A

compressed has more

B

stretched has more

C

equal



Think about how you got there

In each case, what work did your hand do on the spring to deform it from $x = 0$?

Observe

Idealize

Model

Solve / Analyze

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Elastic Potential Energy

DERIVATION

$$F_{\text{spr},x} = -kx$$

$$U_{1 \rightarrow 2}^{(\text{spr})} = \int_{x_1}^{x_2} (-kx) dx$$

$$= \frac{1}{2}k(x_1^2 - x_2^2) = -\Delta V_{\text{spr}}$$

$$V_{\text{spr}} = \frac{1}{2}kx^2$$

x : signed deformation from the undeformed length. x^2 makes $V_{\text{spr}} \geq 0$ for compression and extension alike.

PART 3 SPRING, $k = 800 \text{ N/m}$, BLOCK ATTACHED THROUGHOUT

	State 1	State 2
Deformation x	-0.150 m (compressed)	+0.100 m (stretched)
Spring force $F_{\text{spr},x} = -kx$	+120 N	-80 N
$V_{\text{spr}} = \frac{1}{2}kx^2$	9.00 J	4.00 J

$$\Delta V_{\text{spr}} = 4.00 - 9.00 = -5.00 \text{ J}$$

$$U_{1 \rightarrow 2}^{(\text{spr})} = -\Delta V_{\text{spr}} = +5.00 \text{ J}$$

FOUR DIFFERENT THINGS

Quantity	How to read it
Deformation x	signed: < 0 compressed, > 0 stretched
Force direction	always toward $x = 0$
Spring work	signed: depends on force AND motion
ΔV_{spr}	signed: from the change in x^2

Observe



Idealize



Model



Solve / Analyze



Validate



Report



What Makes a Force Conservative?

1

Work depends only on the endpoints

the path between states 1 and 2 does not matter

2

Work over every closed path is zero

every loop — not just one convenient loop

3

A potential-energy function exists

$U_{1 \rightarrow 2} = V_1 - V_2$ for every pair of states

In one dimension:

$$F_x = -\frac{dV}{dx}$$

PASS THE TESTS (IDEAL MODELS)

Gravity near Earth: $V_g = mgy$

Ideal linear spring: $V_{\text{spr}} = \frac{1}{2}kx^2$

THINK

A force does zero work around ONE closed path. Is it conservative?



Not proven

Every closed path must give zero. A real spring with internal damping is not perfectly conservative.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



Can Every Force Have Potential Energy?

Plan view of a level rough floor (to scale)



Detour: $2.0 + 3.0 + 2.0 = 7.0$ m. Closed trip: detour out, direct back = 10.0 m.

GIVEN

2.0 kg block · $\mu_k = 0.30$ · $g = 9.81$ m/s²

$F_k = \mu_k mg = 5.886$ N, always opposite the sliding

Route	Gravity work	Kinetic-friction work
Direct, 3.0 m	0	-17.66 J
Detour, 7.0 m	0	-41.20 J
Closed trip, 10.0 m	0	-58.86 J



Path-dependent $\Rightarrow$ no potential energy

Same endpoints, different friction work; a closed trip gives -58.86 J, not 0. No position-only V can represent kinetic friction on a stationary surface.

Observe



Idealize



Model



Solve / Analyze



Validate

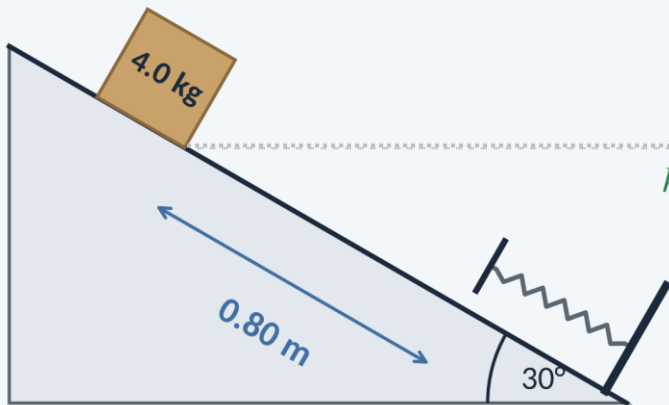


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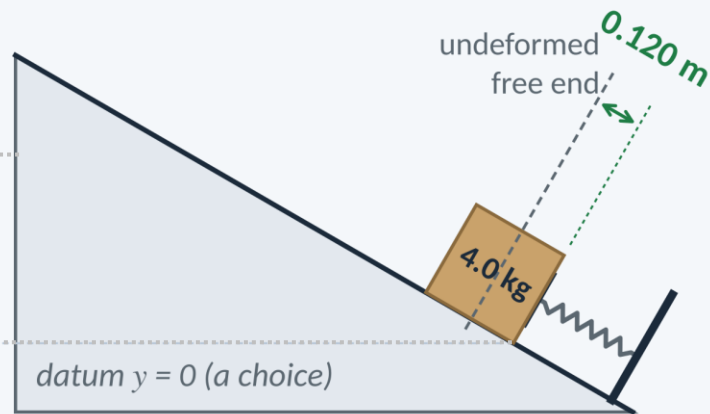


Gravity and Spring Together

State 1: released



State 2: spring compressed



GIVEN

4.0 kg parcel, smooth 30° chute

Gap to spring: 0.80 m

$k = 1500 \text{ N/m}$, undeformed at 1

Compression at 2: 0.120 m

Find $U_{1 \rightarrow 2}^{\text{cons}}$. No speeds.

Travel along chute

$$s = 0.80 + 0.120 = 0.920 \text{ m}$$

Height drop

$$h = 0.920 \sin 30^\circ = 0.460 \text{ m}$$

Gravity

$$\Delta V_g = -(4.0)(9.81)(0.460) = -18.05 \text{ J}$$

Spring

$$\Delta V_{\text{spr}} = \frac{1}{2}(1500)(0.120)^2 - 0 = +10.80 \text{ J}$$

Total

$$\Delta V = -7.25 \text{ J} \Rightarrow U_{1 \rightarrow 2}^{\text{cons}} = +7.25 \text{ J}$$

CHECK WITH PART 3 WORK

$$U_{1 \rightarrow 2}^{(g)} = +18.05 \text{ J}$$

$$U_{1 \rightarrow 2}^{(\text{spr})} = -10.80 \text{ J}$$

$$U_{1 \rightarrow 2}^{\text{cons}} = +7.25 \text{ J} \checkmark$$

Observe

Idealize

Model

Solve / Analyze

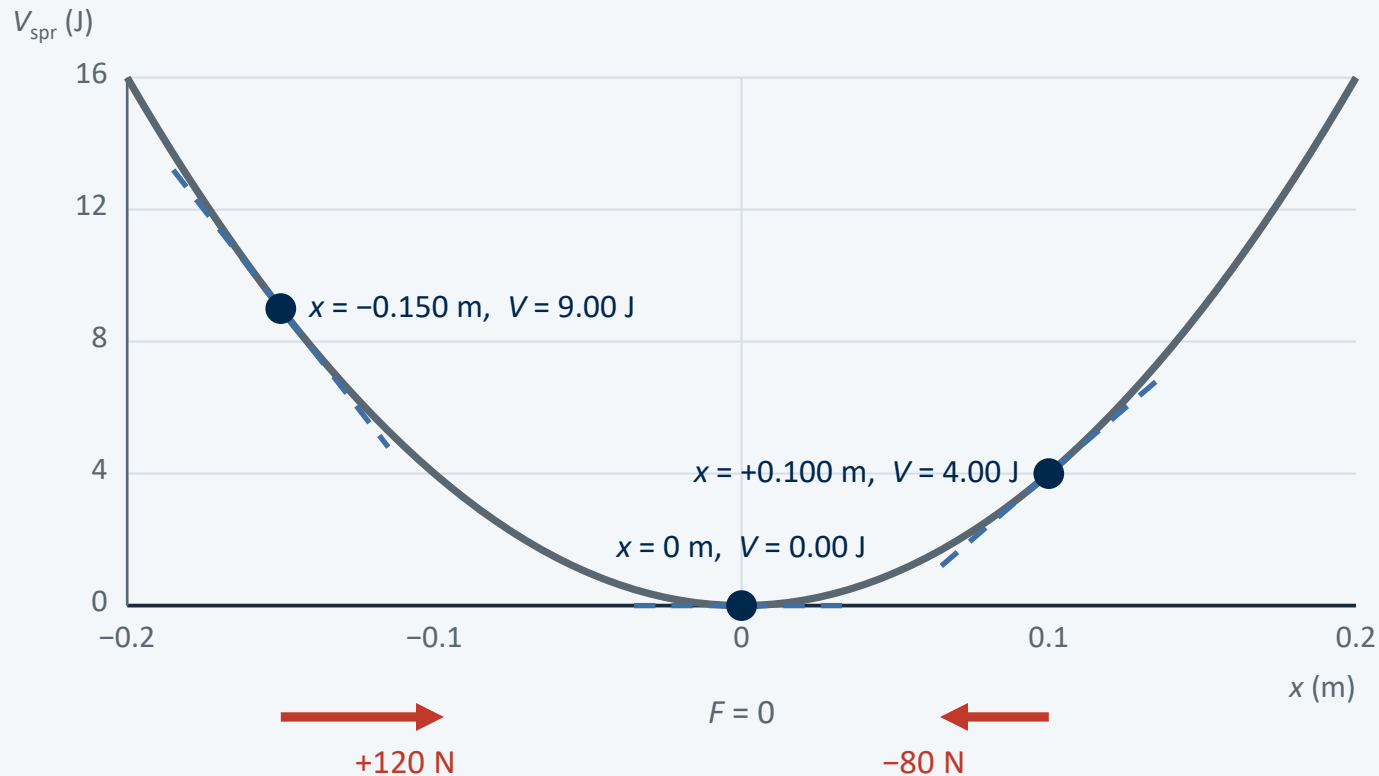
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What Does the Potential-Energy Curve Tell Us?

$V_{\text{spr}} = \frac{1}{2}kx^2$ with $k = 800 \text{ N/m}$ (slide 7 spring)



At each marked point, which way does the spring force point? Can you tell from the curve?

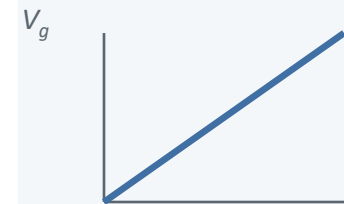
$$F_{\text{spr},x} = -\frac{dV_{\text{spr}}}{dx}$$

slope $> 0 \rightarrow$ force toward $-x$

slope $< 0 \rightarrow$ force toward $+x$

slope $= 0 \rightarrow$ no force

steeper $\rightarrow$ larger force, not higher V



$V_g = mgy$: constant slope $\Rightarrow$ constant force, $-mg$ (uniform g).

Observe

Idealize

Model

Solve / Analyze

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Can We Trust the Energy Model?

CHECK EVERY ENERGY ACCOUNT

- 1 Identify the physical system.
- 2 Sort the forces: conservative vs all others.
- 3 Choose and state every reference level.
- 4 Measure spring deformation from the undeformed length.
- 5 Check signs: positive work $\Leftrightarrow \Delta V < 0$.
- 6 Check units: every term in joules.
- 7 Count each conservative interaction ONCE.
- 8 Validate with an independent Part 3 work calculation.

DIAGNOSE THIS (SLIDE 10)

A classmate writes $U_{1 \rightarrow 2}^{(g)} + U_{1 \rightarrow 2}^{(\text{spr})}$ AND $-\Delta V_g - \Delta V_{\text{spr}}$ as separate contributions, and gets 14.50 J instead of 7.25 J. What went wrong?

The same two conservative interactions were counted twice: $(18.05 - 10.80) + (18.05 - 10.80) = 14.50$ J. Check 7 catches it; check 8 confirms 7.25 J.



Then — and only then — ask an AI

“Examine my work ledger. Which physical interactions have I represented more than once?” Verify its answer against your own equations.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



What Have We Learned?

Force	Round-trip work	Potential energy	Conservative?
Gravity (uniform g)	0	$V_g = mgy$	Yes
Ideal linear spring	0	$V_{\text{spr}} = \frac{1}{2}kx^2$	Yes
Kinetic friction (stationary surface)	negative	none — path-dependent	No
Normal force (fixed smooth surface)	0 — no work at all	not needed	does no work
Applied cable force	depends on the machine	none in general	No

V represents a conservative interaction (gravity, ideal spring) — not a force · $U_{1 \rightarrow 2}^{\text{cons}} = -\Delta V$
 datum is a free choice; ΔV is not · $F_x = -dV/dx$ · count each conservative interaction once



The work–energy principle, rewritten

$T_1 + V_1 + U_{1 \rightarrow 2}^{\text{nc}} = T_2 + V_2$ — the same principle as Part 2, with conservative forces in the V column.

Part 6: if only conservative forces do work, what happens to $T + V$?

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report