

CHAPTER - 4

PART 6 · ≈ 56 MIN

When Does Mechanical Energy Stay Constant?

Conservation of mechanical energy: predicting speeds and turning points without time — and knowing when the shortcut fails.

6

Observe



Idealize



Model



Solve / Analyze



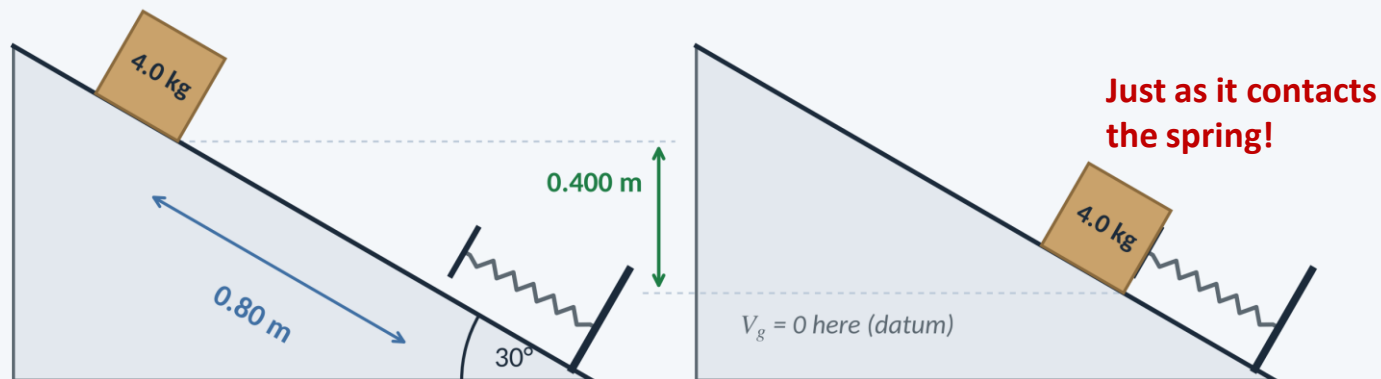
Validate



Report



Where Did the Potential Energy Go?



$V_{spr} = 0$ in both states — the **spring is still undeformed** at first contact.

ENERGY BARS (same scale, J)



OBSERVE

Part 5 asked: if only conservative forces do work, what happens to $T + V$?

What changes between the two states — and what appears to stay the same?

Observe

Idealize

Model

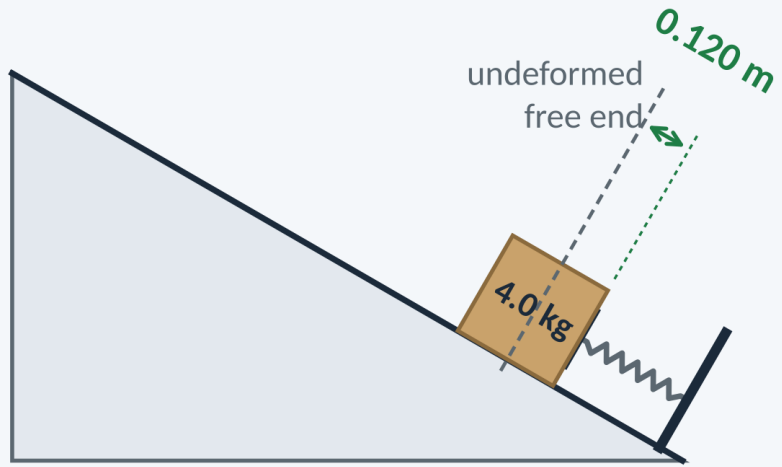
Solve / Analyze

Validate

Report



What Must Stay Constant?



Same chute: 4.0 kg, smooth, 30°, released from rest 0.80 m (along the chute) above the spring; $k = 1500 \text{ N/m}$.

Shown: spring compressed 0.120 m.

1

What is the parcel's speed when it first touches the spring?

2

At 0.120 m compression, is the parcel moving faster, slower, or has it stopped?

3

The chute pushes on the parcel the whole time. Does that normal force stop mechanical energy from being conserved?



Commit first

Write a number or a direction for each. Slides 4 and 7 answer them.

Observe

Idealize

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Conservation Is a Special Case

1

Part 5: the work–energy principle with potential energy

$$T_1 + V_1 + U_{1 \rightarrow 2}^{\text{nc}} = T_2 + V_2$$

2

If the net nonconservative work is zero ...

$$U_{1 \rightarrow 2}^{\text{nc}} = 0$$

3

... the nonconservative-work term drops out

$$T_1 + V_1 = T_2 + V_2$$

4

Define mechanical energy:

$$E_{\text{mech}} = T + V$$

$$E_{\text{mech},1} = E_{\text{mech},2}$$

ASK

Under what condition can we remove the nonconservative-work term?



Forces may act — if their net work is zero

Chute sliding on a smooth surface: $\vec{N} \perp$ motion, zero work.
A motor cable can do zero NET work over an interval — that does not make it conservative.

Statement

Holds

Total energy is conserved

always

$$T_1 + V_1 + U_{1 \rightarrow 2}^{\text{nc}} = T_2 + V_2$$

always
(accounting)

$$E_{\text{mech},1} = E_{\text{mech},2}$$

only if $U_{1 \rightarrow 2}^{\text{nc}} = 0$

Observe

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Gravity Trades Height for Speed

SMOOTH CHUTE $\Rightarrow U_{1 \rightarrow 2}^{nc} = 0$

$$\frac{1}{2}mv_{sp,1}^2 + mgy_1 = \frac{1}{2}mv_{sp,2}^2 + mgy_2$$

$$v_{sp,2}^2 = v_{sp,1}^2 + 2g(y_1 - y_2)$$

$$y_1 - y_2 = 0.80 \sin 30^\circ = 0.400 \text{ m}$$

$$= \sqrt{2(9.81)(0.400)}$$

$$= 2.80 \text{ m/s}$$

INDEPENDENT CHECK — CHAPTER 3

$$a = g \sin 30^\circ = 4.905 \text{ m/s}^2$$

$$v_{sp,2}^2 = 2as = 2(4.905)(0.80) \\ = 7.85 \text{ m}^2/\text{s}^2 \Rightarrow \mathbf{2.80 \text{ m/s}} \text{ — identical.}$$



What did the energy method skip?

No acceleration, no time: only the two states. (Which method to choose, and when, is Part 7.)

Observe ›

Idealize ›

Model ›

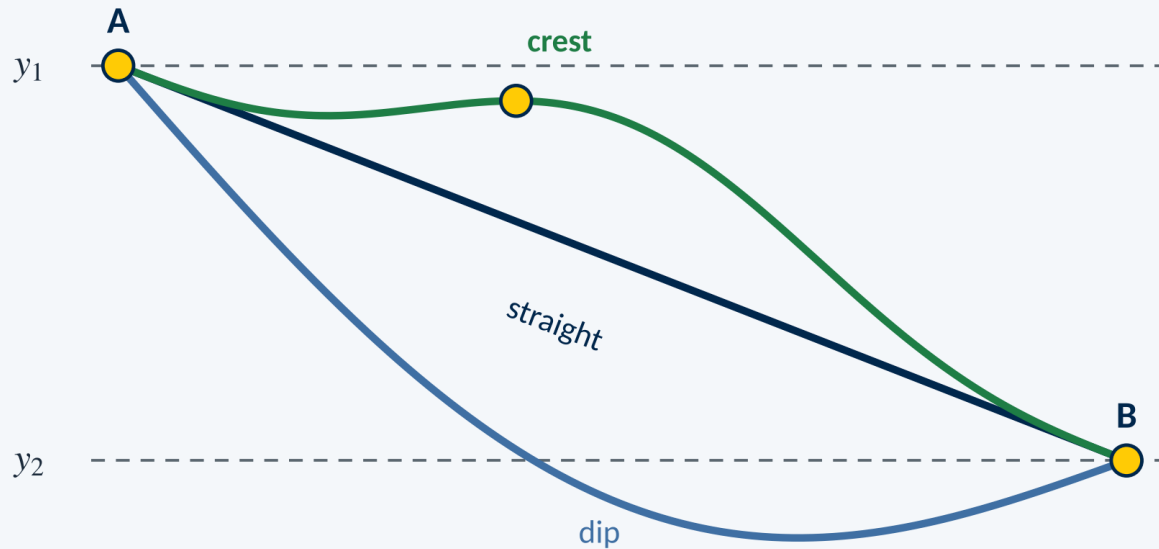
Solve / Analyze ›

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Report



What Does the Track Shape Change?



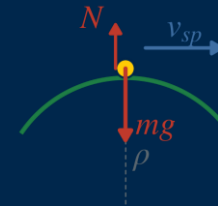
Three smooth fixed tracks, same start speed, same y_1 and y_2 . A separate geometry from the parcel chute.

1. Will the final speed at B be the same on all three?

2. Does that mean the motion is identical?

1. **Yes** — if the path is actually followed and the track does no work.

2. **No**. Shape changes the acceleration, the normal force, the travel time, and whether contact is kept.



AT A LOCALLY HORIZONTAL CREST

$$mg - N = m \frac{v_{sp}^2}{\rho}$$

$$N = 0 \Rightarrow v_{sp} = \sqrt{g\rho}$$

$\rho = 2.0 \text{ m} \Rightarrow$ contact is lost above 4.43 m/s at this crest.

Observe

Idealize

Model

Solve / Analyze

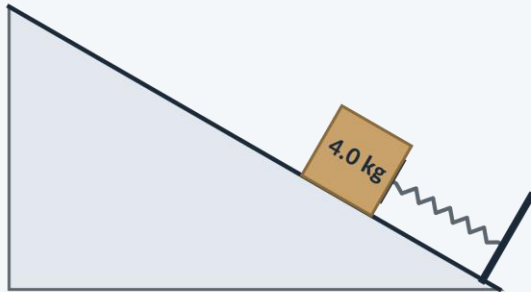
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Report

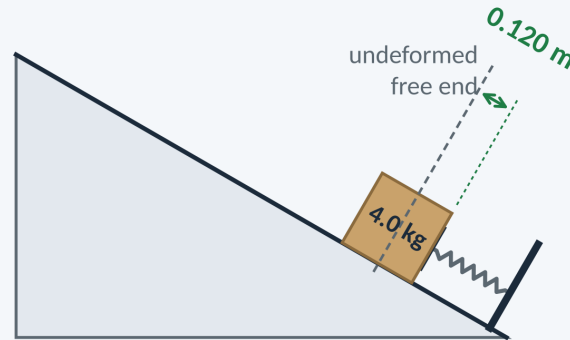


A Spring Can Stop a Moving Object

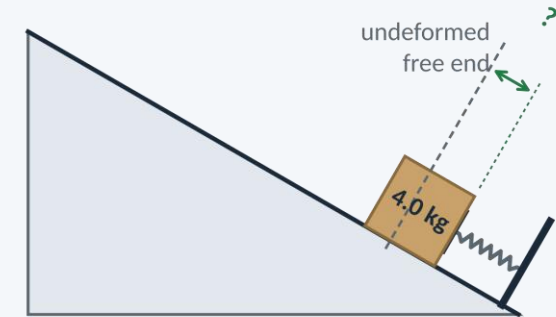
First contact, $\delta = 0$



$\delta = 0.120 \text{ m}$



Maximum compression, $\delta = ?$



TWO COORDINATES — DO NOT MIX THEM

$$x = -\delta$$

x : signed spring deformation (Part 5; compression < 0). δ : compression magnitude > 0 . Parcel travel: $s = 0.80 + \delta$ downhill.

PREDICT

1. Is the parcel moving fastest at first contact with the spring?
2. At maximum compression, is the parcel in equilibrium?

Observe

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From Motion to Maximum Compression

A · AT $\delta = 0.120$ m (PART 5 VALUES)

$$\Delta V = -18.05 + 10.80 = -7.25 \text{ J} \Rightarrow T_2 = 7.25 \text{ J}$$

$$\frac{1}{2}(4.0)v_{\text{sp},2}^2 = 7.25 \Rightarrow \boxed{v_{\text{sp},2} = 1.90 \text{ m/s}}$$

Smooth chute $\Rightarrow U_{1 \rightarrow 2}^{\text{nc}} = 0$; $T_1 = 0$. Still moving downhill.

B · MAXIMUM COMPRESSION ($v_{\text{sp}} = 0$)

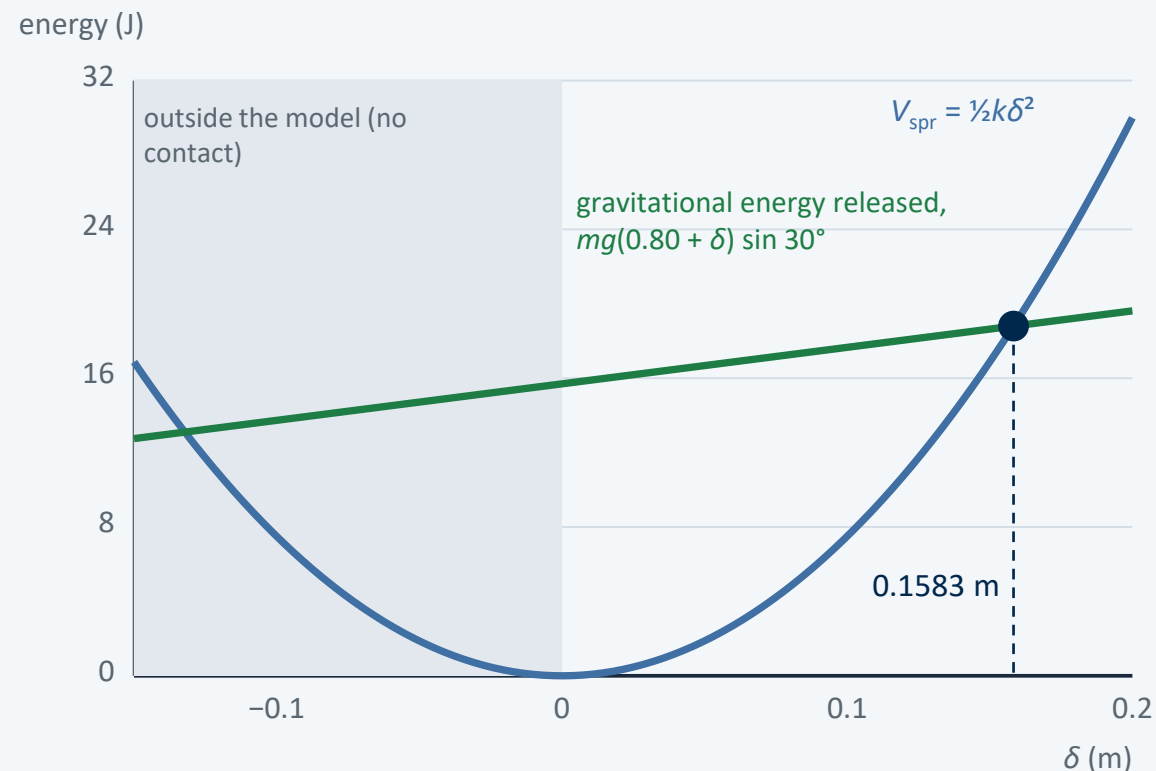
$$\frac{1}{2}k\delta^2 = mg(0.80 + \delta) \sin 30^\circ$$

$$750\delta^2 - 19.62\delta - 15.696 = 0$$

$$\delta = +0.1583 \text{ m} \text{ or } \delta = -0.1322 \text{ m}$$

$$\boxed{\delta_{\text{max}} = 0.1583 \text{ m} \text{ (} x = -0.1583 \text{ m)}}$$

Visual check: where do the two energies meet?



Observe ›

Idealize ›

Model ›

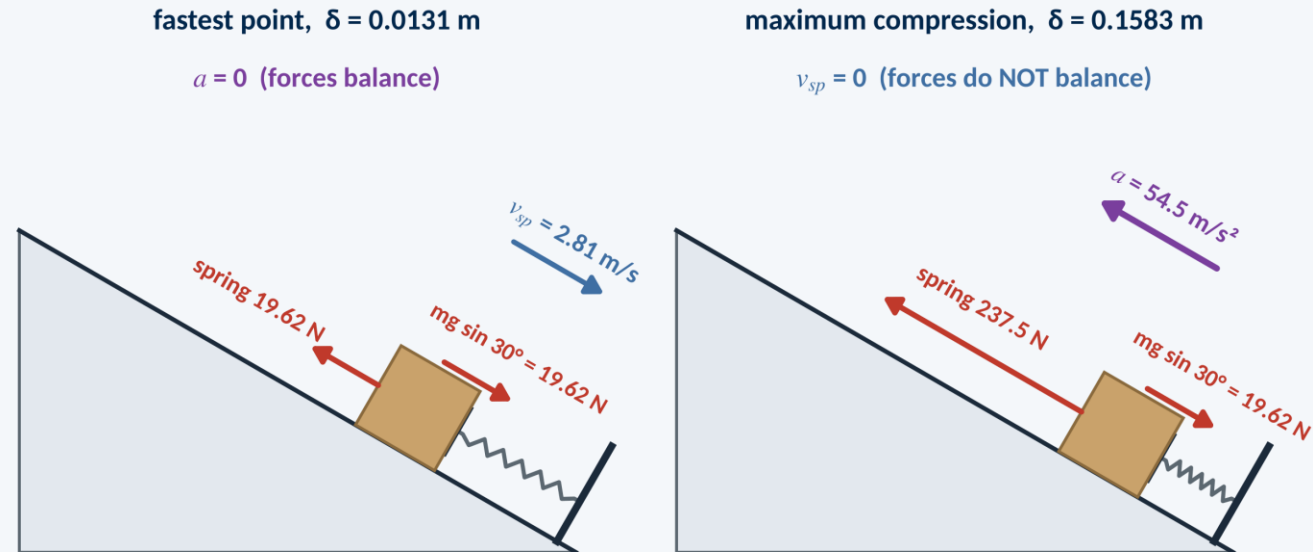
Solve / Analyze ›

Validate ›

Report



Stopped Is Not Balanced



THINK

If the parcel has stopped, why doesn't it stay there?

Condition	Speed	Acceleration
Maximum speed, $\delta = 0.0131$ m	≈ 2.81 m/s	0
Maximum compression, $\delta = 0.1583$ m	0	≈ 54.5 m/s ² uphill

Zero velocity $\neq$ zero acceleration; zero acceleration $\neq$ zero velocity.

Observe

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What Changes When Friction Acts?

REARRANGE THE GENERAL PRINCIPLE

$$T_1 + V_1 + U_{1 \rightarrow 2}^{\text{nc}} = T_2 + V_2$$

$$\boxed{\Delta E_{\text{mech}} = U_{1 \rightarrow 2}^{\text{nc}}}$$

KINETIC FRICTION, STATIONARY SURFACE, CONSTANT N

$$U_{1 \rightarrow 2}^{(f)} = -\mu_k N (s_2 - s_1)$$

Negative friction work lowers E_{mech} .

The energy is not destroyed: it becomes internal (thermal) energy of the parcel and the chute surfaces.

THREE LEVELS — DO NOT CONFUSE THEM

Total energy

always conserved

Mechanical-energy accounting

$\Delta E_{\text{mech}} = U_{1 \rightarrow 2}^{\text{nc}}$ — always valid

Conservation of mechanical energy

special case: $U_{1 \rightarrow 2}^{\text{nc}} = 0$



Avoid “friction destroys energy”

Say: friction transfers mechanical energy into internal (thermal) and possibly other forms.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

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Report

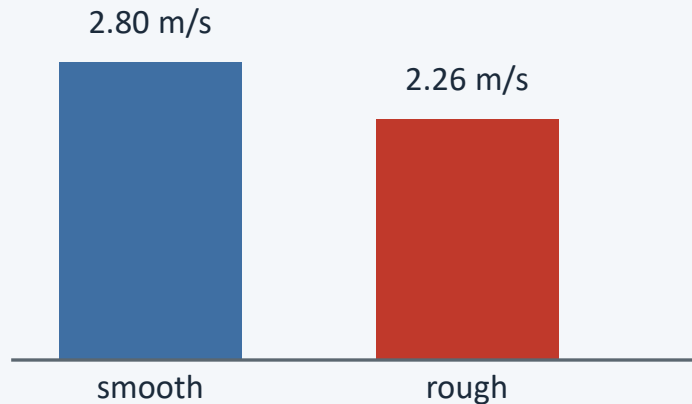


How Much Energy Was Dissipated?

GIVEN

Same parcel and chute, now rough
 $\mu_k = 0.20$; slides from rest ($\mu_s < \tan 30^\circ$)
 Release to contact: $s_2 - s_1 = 0.80$ m

Speed at spring contact



Normal force

$$N = mg \cos 30^\circ = 33.98 \text{ N}$$

Friction work

$$U_{1 \rightarrow 2}^{(f)} = -(0.20)(33.98)(0.80) = -5.44 \text{ J}$$

Gravity releases

$$mg(0.80 \sin 30^\circ) = 15.70 \text{ J}$$

Energy balance

$$15.70 - 5.44 = \frac{1}{2}(4.0)v_{sp,2}^2$$

Result

$$v_{sp,2} = 2.26 \text{ m/s}$$

CHECKS

Chapter 3: $a = g(\sin 30^\circ - 0.20 \cos 30^\circ) = 3.206 \text{ m/s}^2$; $v_{sp,2}^2 = 2(3.206)(0.80) \Rightarrow 2.26 \text{ m/s} \checkmark$
 $\Delta E_{\text{mech}} = -5.44 \text{ J} \rightarrow$ internal (thermal) energy of parcel and chute. Limit: $\mu_k = 0$ gives $2.80 \text{ m/s} \checkmark$

Observe



Idealize



Model



Solve / Analyze



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Can We Trust the Prediction?

CHECK EVERY ENERGY PREDICTION

- 1 Define the physical system.
- 2 Identify the initial and final states.
- 3 Does the conservation condition hold?
- 4 Include nonconservative work when needed.
- 5 State the reference levels.
- 6 Spring: $x = -\delta$, $V_{\text{spr}} = \frac{1}{2}kx^2$.
- 7 Include the FULL vertical displacement.
- 8 Check signs and units.
- 9 Reject physically inadmissible roots.
- 10 Is the assumed path and contact feasible?

DIAGNOSE THIS

A classmate finds the maximum compression with $h = 0.80 \sin 30^\circ$ instead of $h = (0.80 + \delta) \sin 30^\circ$, and gets $\delta \approx 0.1447$ m. Which displacement did they forget?

The parcel keeps descending while it compresses the spring: the missing drop is $\delta \sin 30^\circ$. At $\delta = 0.1447$ m that omits $mg\delta \sin 30^\circ \approx 2.84$ J of released energy — the correct answer is 0.1583 m.



Then — and only then — ask an AI

“Which energy terms did I evaluate at the wrong position?” Verify its answer against your own equations and geometry.

Observe ›

Idealize ›

Model ›

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Validate ›

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What Did the Energy Method Not Tell Us?

ENERGY GAVE US

Speed at spring contact: 2.80 m/s

Speed during compression: 1.90 m/s

Maximum compression: 0.1583 m

Speed on a rough chute: 2.26 m/s

ENERGY ALONE DID NOT GIVE

Time to reach each state

The complete acceleration history

The normal reaction along the path

Whether a contact constraint holds

If we needed these, which equations would we use?

Part 7: when should we choose force–acceleration methods, and when energy methods?

Observe



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What Have We Learned?

ALWAYS (ACCOUNTING)

$$T_1 + V_1 + U_{1 \rightarrow 2}^{\text{nc}} = T_2 + V_2$$

SPECIAL CASE

$$U_{1 \rightarrow 2}^{\text{nc}} = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$E_{\text{mech}} = T + V$. Total energy is always conserved; E_{mech} only when $U_{1 \rightarrow 2}^{\text{nc}} = 0$.

- 1 Mechanical energy combines kinetic and potential: $E_{\text{mech}} = T + V$.
- 2 Conservation requires zero NET nonconservative work.
- 3 Forces can act while doing zero work (smooth normal force).
- 4 Gravity increases speed as height decreases: 2.80 m/s.
- 5 A spring stores energy while the parcel keeps moving (1.90 m/s).
- 6 Maximum compression (0.1583 m) is a turning point, not equilibrium.
- 7 Zero acceleration and zero velocity are different conditions.
- 8 Friction lowers E_{mech} (−5.44 J); total energy is not destroyed.
- 9 Energy predictions must satisfy physical constraints.
- 10 Energy gives some quantities efficiently — not every aspect of motion.

Part 7: choosing between force–acceleration and energy methods.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report