

CHAPTER - 4

PART 7 · ≈ 55 MIN

Choosing Between Force–Acceleration and Energy Methods

We have two powerful ways to analyze motion. How do we know which one to use — and when we need both?

Observe

Idealize

Model

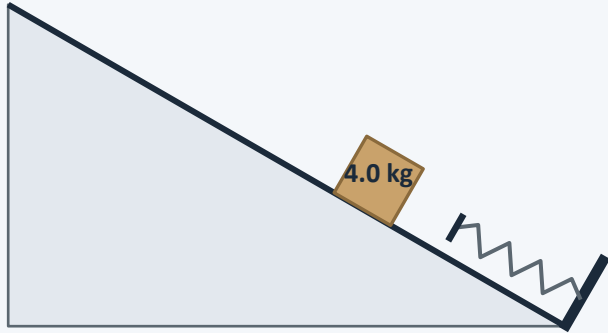
Solve / Analyze

Validate

Report



One Motion, Two Ways to Describe It



4.0 kg · smooth 30° chute · 0.80 m from rest to the spring

BOARD A · FORCE-ACCELERATION

$$a = g \sin 30^\circ = 4.905 \text{ m/s}^2$$

$$v_{\text{sp},2}^2 = v_{\text{sp},1}^2 + 2as$$

$$v_{\text{sp},2} = 2.80 \text{ m/s}$$

BOARD B · ENERGY

$$T_1 + V_1 = T_2 + V_2$$

$$mgh = \frac{1}{2}mv_{\text{sp},2}^2$$

$$v_{\text{sp},2} = 2.80 \text{ m/s}$$

ASK

Both obtained the same speed. **If the next question asks how long the motion takes, which solution would you rather continue?**

Same physical motion. Different mathematical descriptions.

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What Are We Actually Trying to Find?

- 1 What is the speed at first spring contact?
- 2 How long does it take to reach the spring?
- 3 What normal reaction acts while the parcel slides on the straight chute?
- 4 What is the maximum spring compression?
- 5 What is the acceleration at maximum compression?

Force–acceleration

- 1 either
- 2 + constant- a kinematics
- 3 normal to chute

Energy

- 1 either
- 4 spring force varies

Both / more information

- 5 energy $\rightarrow \delta$, then ΣF

Place each card where you would **start**. Then ask: is that start **enough**, or does the question need more than one equation?

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Idealize

Model

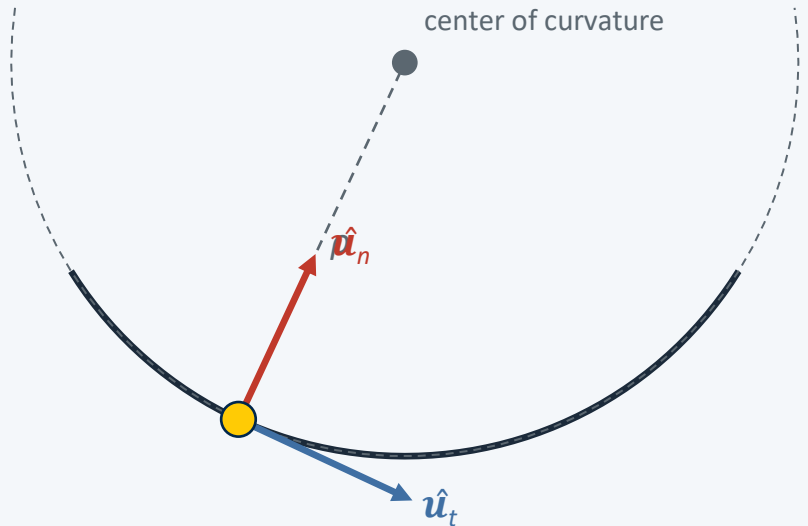
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Two Equations, Two Different Questions



$\hat{u}_n$ points toward the center of curvature. Motion is along $\hat{u}_t$; a fixed smooth path pushes along $\hat{u}_n$ and does no work.

$$\Sigma \vec{F} = m \vec{a} \quad \text{instantaneous} \cdot \text{vector} \cdot \text{every force}$$

ALONG $\hat{u}_t \rightarrow$ ENERGY

$$\Sigma F_t = m a_t$$

$$a_t = v_{sp} \frac{dv_{sp}}{ds}$$

$$\Sigma F_t ds = m v_{sp} dv_{sp}$$

integrate from state 1 to state 2:

$$T_1 + V_1 + U_{1 \rightarrow 2}^{nc} = T_2 + V_2$$

ALONG $\hat{u}_n \rightarrow$ REACTIONS

$$\Sigma F_n = m \frac{v_{sp}^2}{\rho}$$

Contains N and ρ .

Not retained when the tangential equation is integrated — so the energy equation cannot give N directly.



But energy can still help

It supplies v_{sp} , and v_{sp} goes into the normal equation (slide 6).



The Same Chute, Two Descriptions

FORCE-ACCELERATION

$$a = g \sin 30^\circ = 4.905 \text{ m/s}^2$$

$$v_{\text{sp},2} = \sqrt{(2as)} = 2.80 \text{ m/s}$$

$$t = \sqrt{(2s/a)} \approx 0.571 \text{ s}$$

ENERGY

$$mg(s \sin 30^\circ) = \frac{1}{2}mv_{\text{sp},2}^2$$

$$v_{\text{sp},2} = 2.80 \text{ m/s}$$

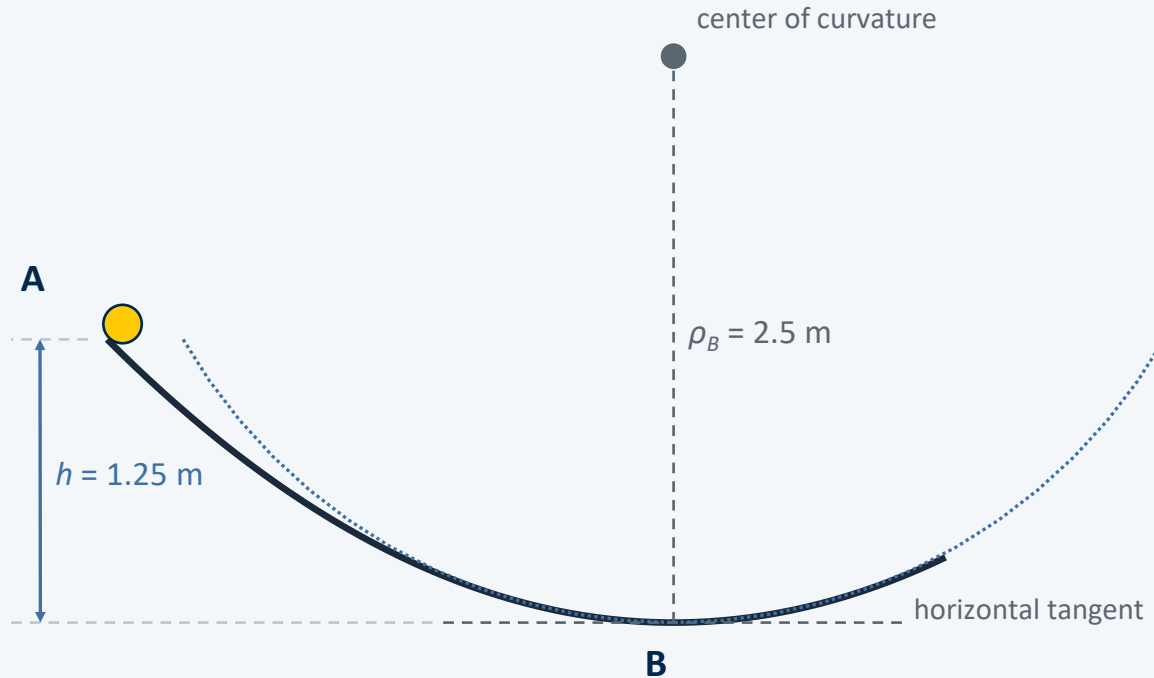
time: not directly

	Force–acceleration	Energy
Needed	Force components; motion assumptions (constant a)	Endpoint states; work and potential changes
Direct result	Acceleration	Speed change
Additional result	Time, with constant- a kinematics	No time directly
After spring contact	a varies: $\Sigma \vec{F} = m\vec{a}$ still valid, but the constant- a formula no longer applies	Still convenient (Part 6: 0.1583 m)



When Energy Alone Is Not Enough

2.0 kg slider · from rest at A · fixed, smooth track, concave up from A to B



PREDICT

At B, is the normal reaction **smaller than, equal to, or greater than** the slider's weight?

THEN

Which equation from slide 3 can determine the normal reaction? What will it need from the energy equation?

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Solve / Analyze

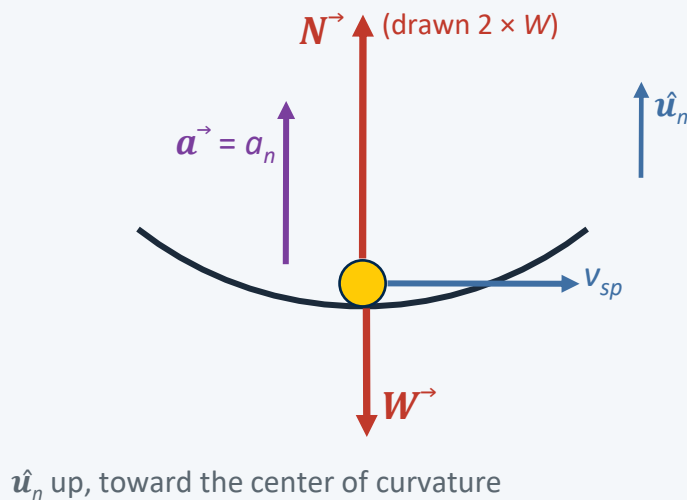
Validate

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Energy Finds Speed; Forces Find the Reaction

FREE-BODY + KINEMATICS AT B



Fastest point, still accelerating

$a_t = 0$ at B, but $a_n = 9.81 \text{ m/s}^2$ up (Part 6: speed $\neq$ acceleration).

STEP 1 · ENERGY DETERMINES SPEED

$$mgh = \frac{1}{2}mv_{\text{sp},B}^2 \Rightarrow v_{\text{sp},B} = \sqrt{2gh} = \sqrt{2(9.81)(1.25)}$$

$$v_{\text{sp},B} = 4.95 \text{ m/s}$$

STEP 2 · NORMAL ACCELERATION (UP, TOWARD THE CENTER)

$$a_n = \frac{v_{\text{sp},B}^2}{\rho_B} = \frac{24.525}{2.5} = 9.81 \text{ m/s}^2$$

STEP 3 · ΣF ALONG $\hat{u}_n$ GIVES THE REACTION

$$N - W = m \frac{v_{\text{sp},B}^2}{\rho_B} \Rightarrow N = mg + m \frac{v_{\text{sp},B}^2}{\rho_B}$$

$$N = 39.24 \text{ N} = 2W$$

$$\frac{N}{W} = 1 + \frac{2h}{\rho_B} \quad \text{checks: } h \rightarrow 0 \text{ or } \rho_B \rightarrow \infty \Rightarrow N \rightarrow W$$

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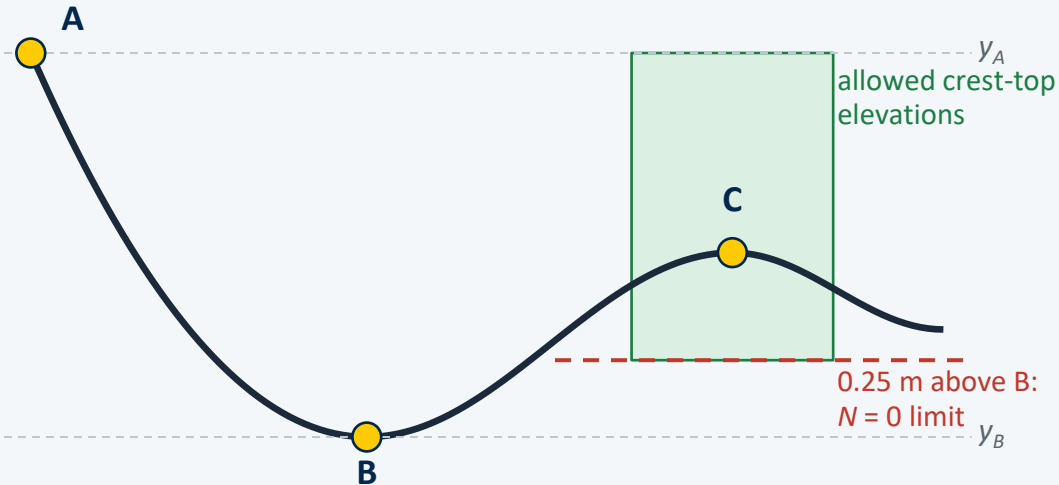
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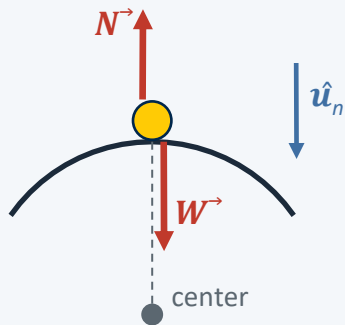


Forces Set the Limit; Energy Finds the Design

Slide 5 track (A → B), continuing over a crest C with $\rho_C = 2.0$ m (not to scale)



at C



ASK

Would a heavier slider need a different limiting crest elevation?

STEP 1 · FORCES SET THE ALLOWABLE SPEED

$$mg - N = m \frac{v_{sp,C}^2}{\rho_C} \quad (\hat{u}_n \text{ down at C})$$

$$N \geq 0 \Rightarrow v_{sp,C}^2 \leq g\rho_C \Rightarrow \boxed{v_{sp,C} \leq 4.43 \text{ m/s}}$$

STEP 2 · ENERGY TURNS IT INTO AN ELEVATION

$$v_{sp,C}^2 = 2g(y_A - y_C) \leq g\rho_C$$

$$\boxed{y_A - y_C \leq \frac{\rho_C}{2} = 1.00 \text{ m}}$$

$$y_A - y_B = 1.25 \text{ m} \Rightarrow \boxed{y_C - y_B \geq 0.25 \text{ m}}$$

A **lower** crest means a **higher** speed at C. At $y_C - y_B = 0.25$ m, $N = 0$: the contact boundary, not a design point. Top-only check: necessary, not sufficient.

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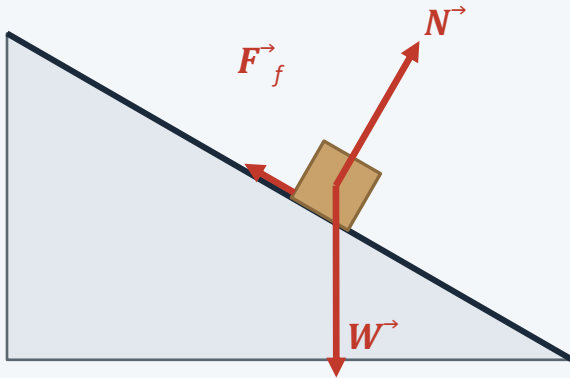
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Does Friction Change Our Choice?



4.0 kg · 30° · 0.80 m from rest · $\mu_k = 0.20$ · arrows to scale

Both methods need N : $\Sigma F_{\perp} = 0 \Rightarrow N = mg \cos 30^\circ = 33.98 \text{ N}$

ENERGY

$$T_1 + V_1 + U_{1 \rightarrow 2}^{(f)} = T_2 + V_2$$

$$U_{1 \rightarrow 2}^{(f)} = -\mu_k N (s_2 - s_1) = -5.44 \text{ J}$$

$$15.70 - 5.44 = \frac{1}{2} (4.0) v_{sp,2}^2$$

$$v_{sp,2} = 2.26 \text{ m/s}$$

FORCE-ACCELERATION

$$a = g(\sin 30^\circ - \mu_k \cos 30^\circ)$$

$$= 3.206 \text{ m/s}^2$$

$$v_{sp,2}^2 = 2as \Rightarrow v_{sp,2} = 2.26 \text{ m/s}$$

$$t = \sqrt{(2s/a)} \approx 0.706 \text{ s}$$

ASK

If the curved sag of slide 6 were rough, could we simply use $F_f = \mu_k W$ everywhere?

No. On a curve N depends on position, curvature and speed (at B, $N = 2W$). The friction work is not known until the force equations are solved too — friction does not make energy invalid, but it can make energy alone insufficient.

Observe

Idealize

Model

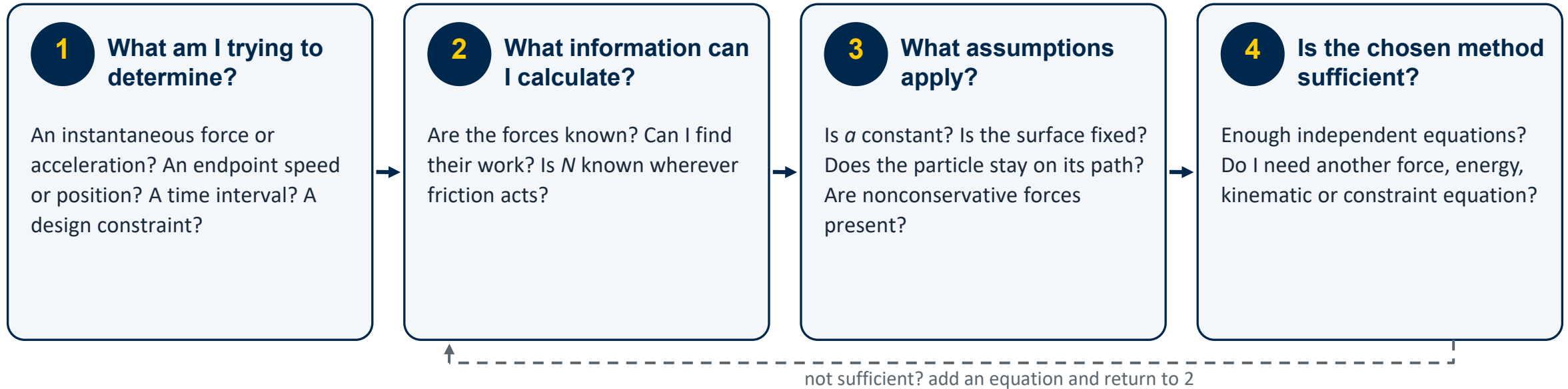
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Four Questions Before You Calculate



VALID

Is the governing law correct?

e.g. $\Sigma \vec{F} = m\vec{a}$ and the work–energy equation: always, for a particle.

APPLICABLE

Are this equation's conditions met?

e.g. $v_{sp}^2 = v_{sp,1}^2 + 2as$ needs constant a .

CONVENIENT

Is it an efficient route?

e.g. Time from $\int ds/v_{sp}$: valid, but slow.

SUFFICIENT

Does it determine the unknowns?

e.g. Energy alone does not give N (slide 6).

Observe

Idealize

Model

Solve / Analyze

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Choose Before You Calculate

For each card: 1 What is requested? 2 What is given?
3 Which method first? 4 Is it sufficient?
5 Which assumptions must be checked?

1 · Highway off-ramp

A 1200 kg car takes a level curve of radius 80 m at a steady 20 m/s. Tire–road $\mu_s = 0.60$. Can it hold the curve without sliding?

Start: ΣF along $\hat{u}_n$. One equation. Static friction.

2 · Spring-launched puck

A 0.20 kg puck is released from a spring ($k = 800 \text{ N/m}$) compressed 0.050 m and glides up a smooth, fixed curved ramp. How high does it rise?

Start: energy. Ramp shape unknown — and not needed.

3 · Fragile parcel

On the Part 6 chute, fragile parcels are rated for at most $5g$. Will the bumper spring protect them?

Both: energy $\rightarrow \delta_{\max}$, then $\Sigma \vec{F} = m\vec{a} \rightarrow \text{peak } a$.

4 · Ramp, then floor

A 2.0 kg box slides from rest down a 2.0 m, 30° ramp onto a level floor; $\mu_k = 0.20$ on both. How far along the floor does it go?

Either; energy is shorter. Smooth transition assumed.

Observe

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Can We Trust the Method We Chose?

A CLASSMATE'S MAXIMUM-COMPRESSION SOLUTION

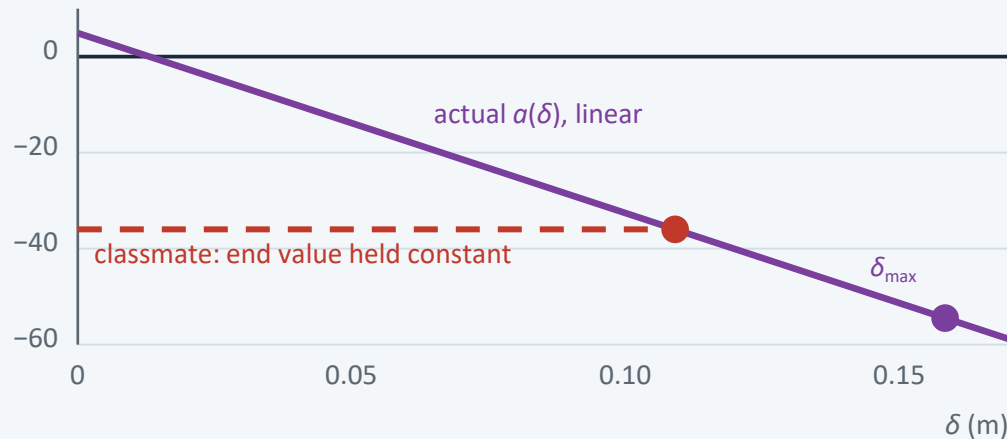
From first contact, $v_{sp,0} = 2.80 \text{ m/s}$; + down the chute:

$$a = 4.905 - 375\delta \quad (\delta \text{ in m, } a \text{ in m/s}^2)$$

$$v_{sp}^2 = v_{sp,0}^2 + 2a\delta \quad \text{with } a \text{ at the final } \delta$$

$$0 = 2.80^2 + 2(4.905 - 375\delta)\delta \Rightarrow \delta_{\max} \approx 0.109 \text{ m}$$

$a \text{ (m/s}^2\text{, + down the chute)}$



ASK

Which equation was used outside its conditions of applicability?

INDEPENDENT ENERGY CHECK AT $\delta = 0.109 \text{ m}$

Gravity released $19.62(0.80 + 0.109) \approx 17.84 \text{ J}$; spring stores $\frac{1}{2}(1500)(0.109)^2 \approx 8.92 \text{ J} \Rightarrow T \approx 8.92 \text{ J left, } v_{sp} \approx 2.11 \text{ m/s. Still moving — not the maximum.}$

DIAGNOSIS

$\Sigma \vec{F} = m\vec{a}$ is valid at every instant. $v_{sp}^2 = v_{sp,0}^2 + 2a\delta$ is **not applicable**: the spring force — and a — change continuously.

THEN, AND ONLY THEN, ASK AN AI

“Which step in this solution assumes constant acceleration, and is that assumption justified?”

$$\delta_{\max} = 0.1583 \text{ m}$$

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The Engineer's Method-Selection Toolkit

STARTING POINTS, NOT RULES

Engineering question	Useful starting point
Instantaneous force or acceleration	Force–acceleration
Speed from position or energy changes	Energy
Maximum spring compression	Energy
Time to reach a state	Equations of motion / kinematics
Speed and reaction at a curved-track point	Energy + force–acceleration
Motion with friction	Work–energy if friction work is known; otherwise additional force equations
Contact or design limit	Force constraint + energy as needed

Valid: Is the governing law correct?

Applicable: Are this equation's conditions met?

Convenient: Is it an efficient route?

Sufficient: Does it determine the unknowns?

Same physical system. Different mathematical descriptions.

1 When does a force do work?

2 Work changes motion

3 Work of common forces

4 Power and efficiency

5 Potential energy

6 Mechanical-energy conservation

7 Choosing a method

A good engineer does not simply know equations. A good engineer knows what each equation can—and cannot—tell us.

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