

**AI-AUGMENTED ENGINEERING DYNAMICS:
A CONVERSATIONAL APPROACH**

Chapter 5 — Momentum Methods for Particles

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

CHAPTER - 5

PART 2 · ≈ 60 MIN

How Do We Predict Motion Using Impulse and Momentum?

A spring can pull on a block without changing its momentum. This Part turns impulse into a prediction tool: add the impulses of every force, then predict the final velocity.

2

Observe

Idealize

Model

Solve / Analyze

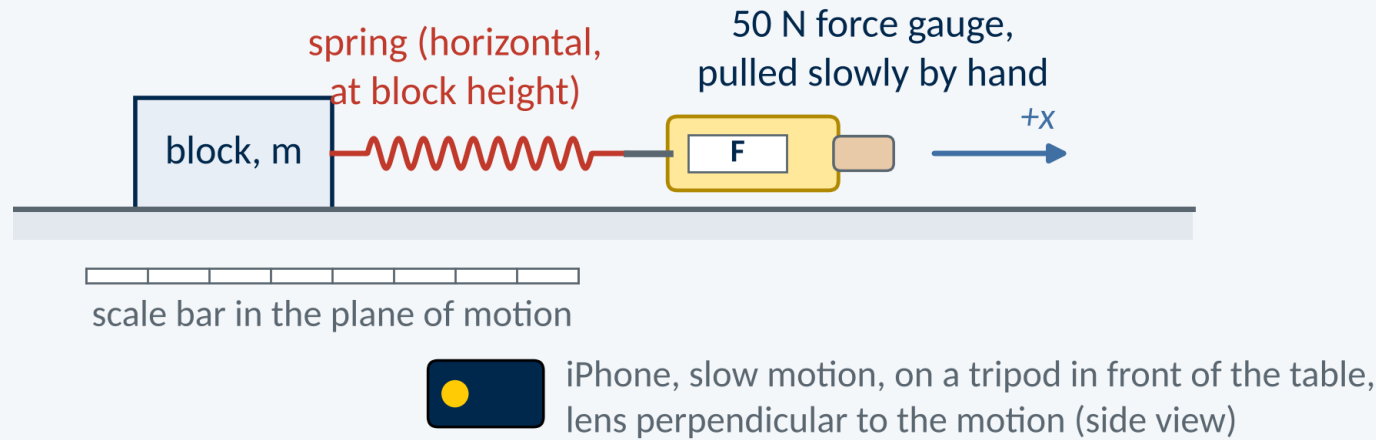
Validate

Report



Pulling Harder, Going Nowhere?

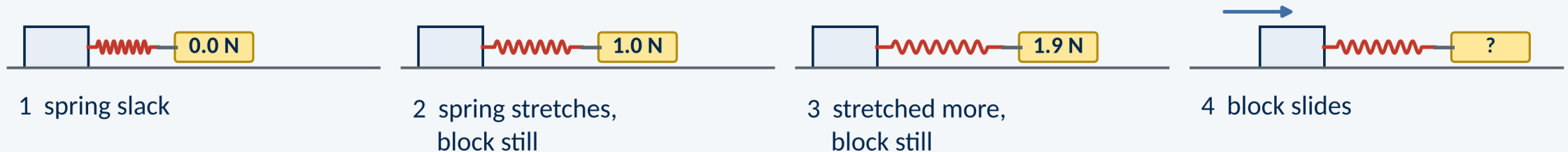
THE APPARATUS



THE ENGINEERING QUESTION

How can we use the impulses of multiple forces to predict a particle's final velocity?

WHAT WE OBSERVE · SCHEMATIC, ILLUSTRATIVE READINGS

**Observe**

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Three Predictions

PREDICTION 1

While the block is still, the net impulse on it is...

- A** zero
- B** positive
- C** impossible to tell

Zero. The block stays at rest, so $L^{\rightarrow}$ does not change. Static friction cancels the spring force.

PREDICTION 2

Once it slides, must its speed keep increasing?

- A** yes
- B** no
- C** only if the gauge reading rises

No. It speeds up only while $F_{spr} > F_k$. If the spring force drops below kinetic friction, it slows.

PREDICTION 3

Can the gauge reading alone give the change in momentum?

- A** yes
- B** no

No. One reading is one force at one instant. We need the force history and every other force.

Observe

Idealize

Model

Solve / Analyze

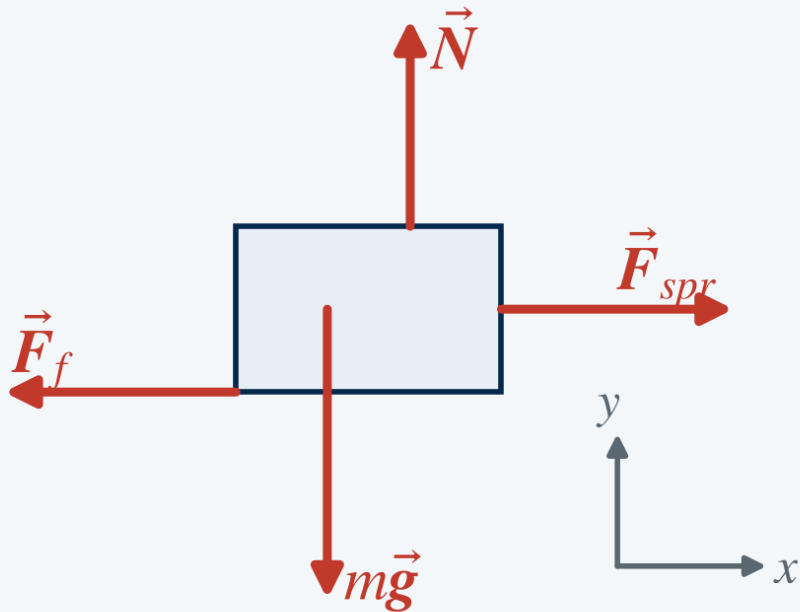
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Free-Body Diagram: Who Pushes, Who Resists

THE BLOCK AS A PARTICLE



Spring horizontal and attached at block height, so $N = mg$ under this model.

STATIC FRICTION · block at rest

$$F_f = F_{spr} \text{ as long as } F_{spr} \leq \mu_s N$$

Static friction adjusts to match the pull, up to its limit. It is a response, not a fixed value.

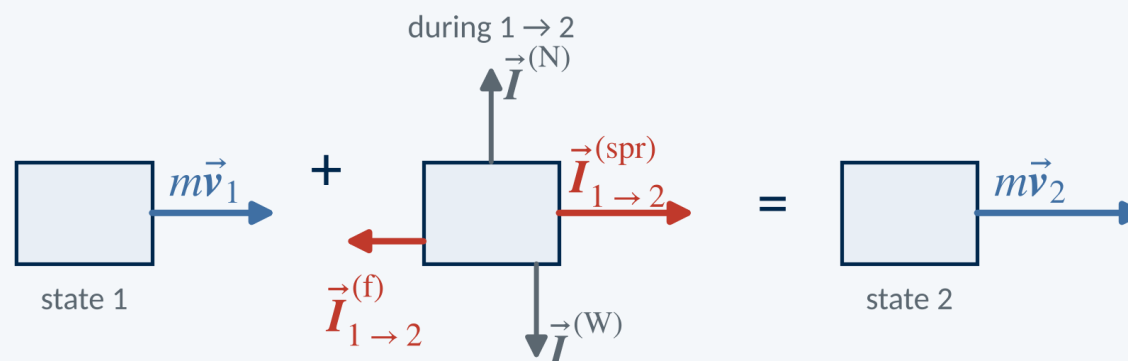
KINETIC FRICTION · block sliding

$$F_f = F_k = \mu_k N$$

Magnitude set by the surfaces; direction always opposite to the sliding. Breakaway happens when F_{spr} reaches $\mu_s N$.



Before, During, After



ADD EVERY IMPULSE, THEN COMPONENTS

$$\vec{L}_1 + \vec{I}_{1 \rightarrow 2} = \vec{L}_2$$

$$\vec{I}_{1 \rightarrow 2} = \vec{I}_{1 \rightarrow 2}^{(\text{spr})} + \vec{I}_{1 \rightarrow 2}^{(\text{f})} + \vec{I}_{1 \rightarrow 2}^{(\text{N})} + \vec{I}_{1 \rightarrow 2}^{(\text{W})}$$

$$mv_{x,1} + I_{x,1 \rightarrow 2}^{(\text{spr})} + I_{x,1 \rightarrow 2}^{(\text{f})} = mv_{x,2}$$

In y : the impulses of N and mg cancel.

SIX-STEP PROCEDURE

- 1 Identify the particle and the time interval.
- 2 Establish coordinate directions.
- 3 Draw the free-body diagram.
- 4 Find each external force's impulse.
- 5 Apply impulse–momentum by component.
- 6 Interpret and validate the result.

Observe ›

Idealize ›

Model ›

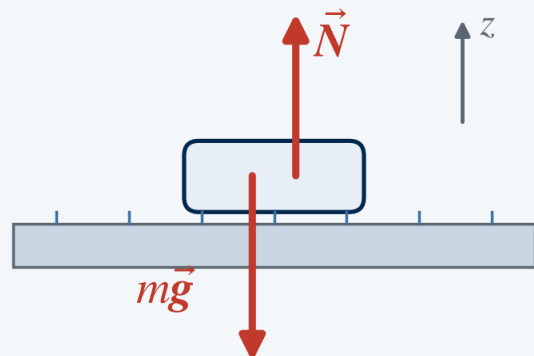
Solve / Analyze ›

Validate ›

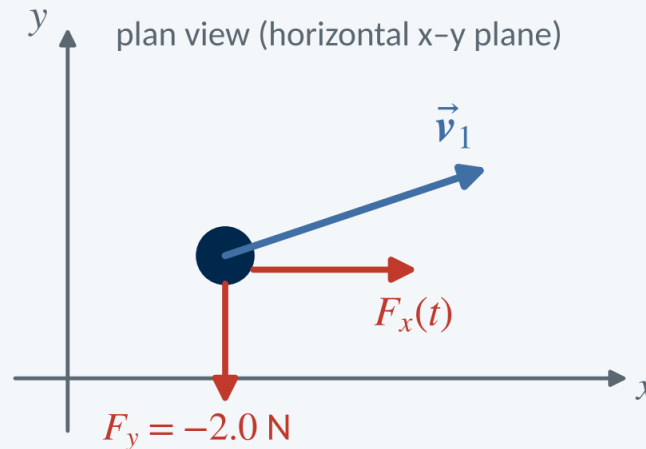
Report



An Air-Table Glider Under Two Forces



side view: air cushion, no friction;
N and mg cancel along z



GIVEN

- $m = 2.0 \text{ kg}$
- $v_{x,1} = 3.0 \text{ m/s}, v_{y,1} = 1.0 \text{ m/s}$
- $0 \leq t \leq 0.40 \text{ s}$
- $F_x(t) = 10t \text{ N}$ (t in s)
- $F_y = -2.0 \text{ N}$ (constant)

PREDICT

The y -force pushes against the glider's y -velocity for the whole 0.40 s . Will v_y reverse sign? Will the glider end up faster or slower?

Observe ›

Idealize ›

Model ›

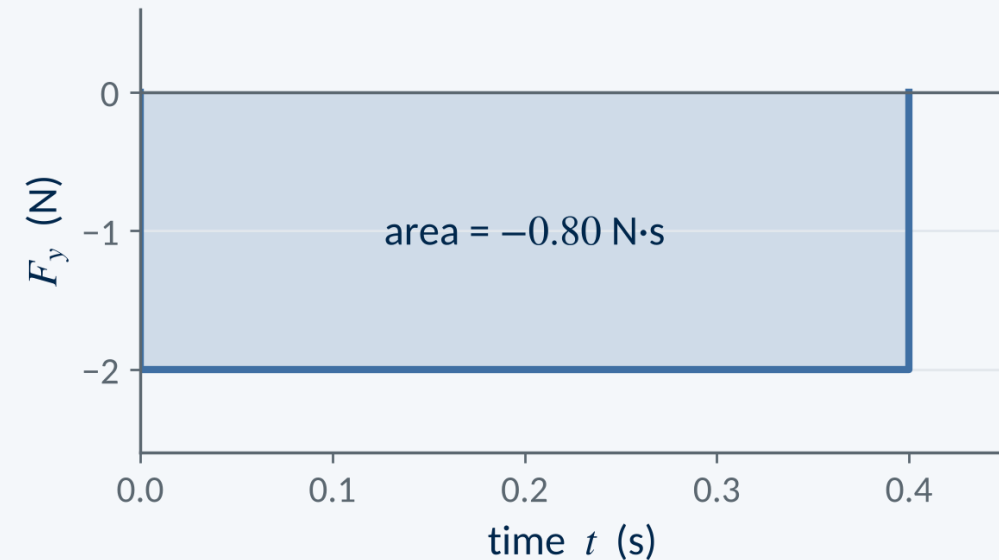
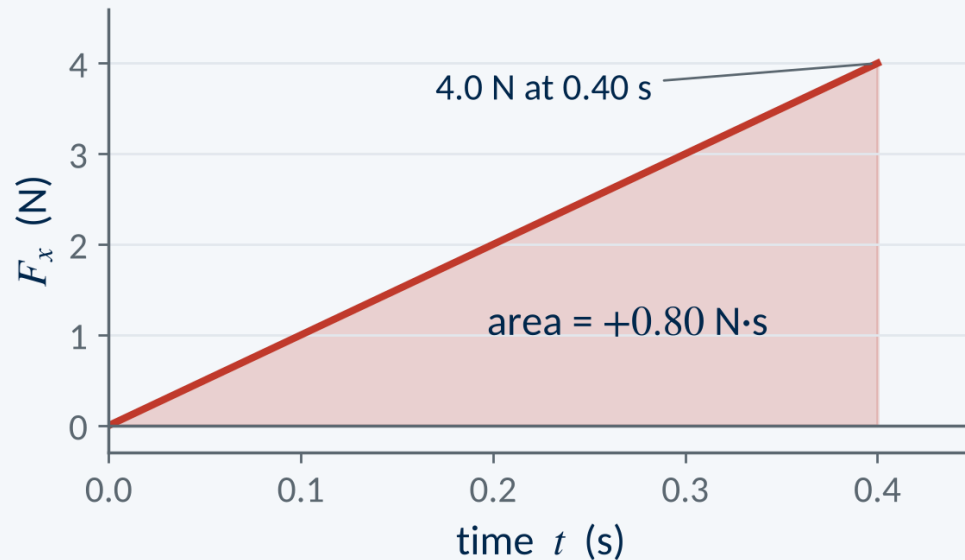
Solve / Analyze ›

Validate ›

Report



Read the Impulses from the Force Histories



$$\begin{aligned} I_{x,1 \rightarrow 2} &= \int_0^{0.40} 10t \, dt = 5(0.40)^2 \\ &= +0.80 \, \text{N} \cdot \text{s} \\ I_{y,1 \rightarrow 2} &= (-2.0 \, \text{N})(0.40 \, \text{s}) = -0.80 \, \text{N} \cdot \text{s} \end{aligned}$$

Triangle for F_x : area $\frac{1}{2}(4.0 \, \text{N})(0.40 \, \text{s})$. Rectangle for F_y : area below the axis, so the impulse is negative. Signs come from the axes, not from words like “push”.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

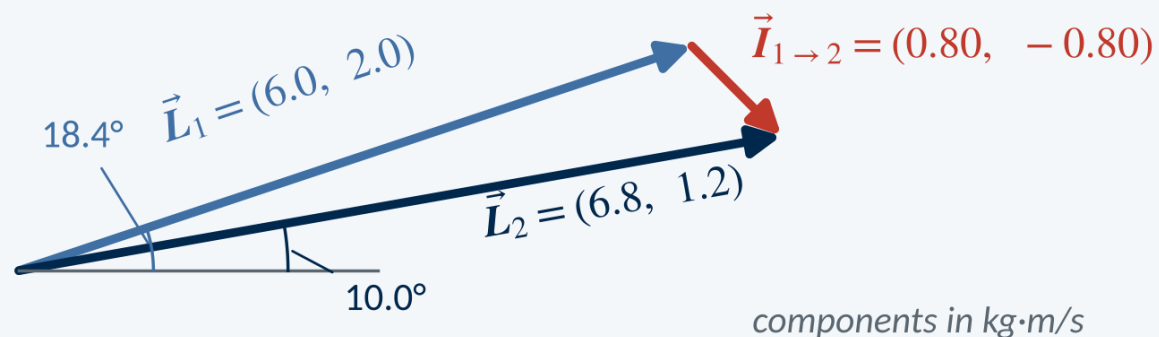
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From Momentum to Final Velocity

MOMENTUM VECTORS, TIP TO TAIL



$$\vec{L}_1 = m\vec{v}_1 = (6.0 \hat{i} + 2.0 \hat{j}) \text{ kg} \cdot \text{m/s}$$

$$\vec{L}_2 = \vec{L}_1 + \vec{I}_{1 \rightarrow 2} = (6.8 \hat{i} + 1.2 \hat{j}) \text{ kg} \cdot \text{m/s}$$

$$\vec{v}_2 = \frac{\vec{L}_2}{m} = (3.4 \hat{i} + 0.6 \hat{j}) \text{ m/s}$$

$$v_{sp,2} = \sqrt{(3.4^2 + 0.6^2)} \approx 3.45 \text{ m/s}$$

Why v_y does not reverse: the y -impulse ($-0.80 \text{ N} \cdot \text{s}$) is smaller than the initial y -momentum ($+2.0 \text{ kg} \cdot \text{m/s}$). It reduces L_y to $+1.2 \text{ kg} \cdot \text{m/s}$ without reversing it.

Direction: from 18.4° to 10.0° above $+x$. The glider turns toward x and speeds up from 3.16 to 3.45 m/s .

Observe

Idealize

Model

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Validate

Report



Does Work–Energy Agree?

FROM IMPULSE–MOMENTUM

$$T_1 = \frac{1}{2}(2.0)(3.0^2 + 1.0^2) = 10.00 \text{ J}$$

$$T_2 = \frac{1}{2}(2.0)(3.4^2 + 0.6^2) = 11.92 \text{ J}$$

$$\Delta T = 1.92 \text{ J}$$

FROM THE FORCES (CHAPTER 4)

$$U_{1 \rightarrow 2}^{(\text{net})} = \int_0^{0.40} \vec{F} \cdot \vec{v} dt$$

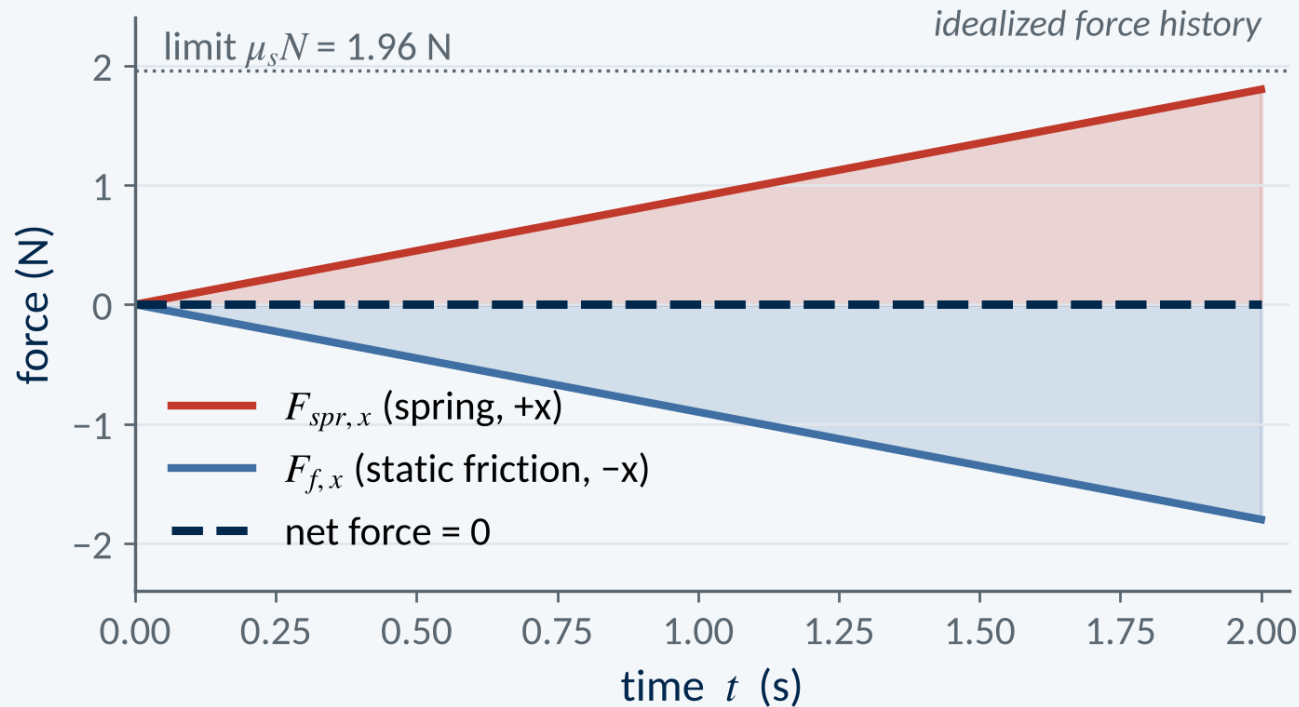
work in x : +2.56 J work in y : -0.64 J

$$U_{1 \rightarrow 2}^{(\text{net})} = +1.92 \text{ J}$$

$T_1 + U_{1 \rightarrow 2}^{(\text{net})} = T_2$: 1.92 J both ways. Two principles, two different integrals, one answer: a cross-method check of the areas, the vector sum and $\vec{v}_2$. A force missing from the model would fool both methods.



Stationary Phase: Big Pull, Zero Net Impulse



WHILE THE BLOCK IS STILL

$$F_{f,x} = -F_{spr,x}$$

$$I_{x,1 \rightarrow 2}^{(f)} = -I_{x,1 \rightarrow 2}^{(spr)}$$

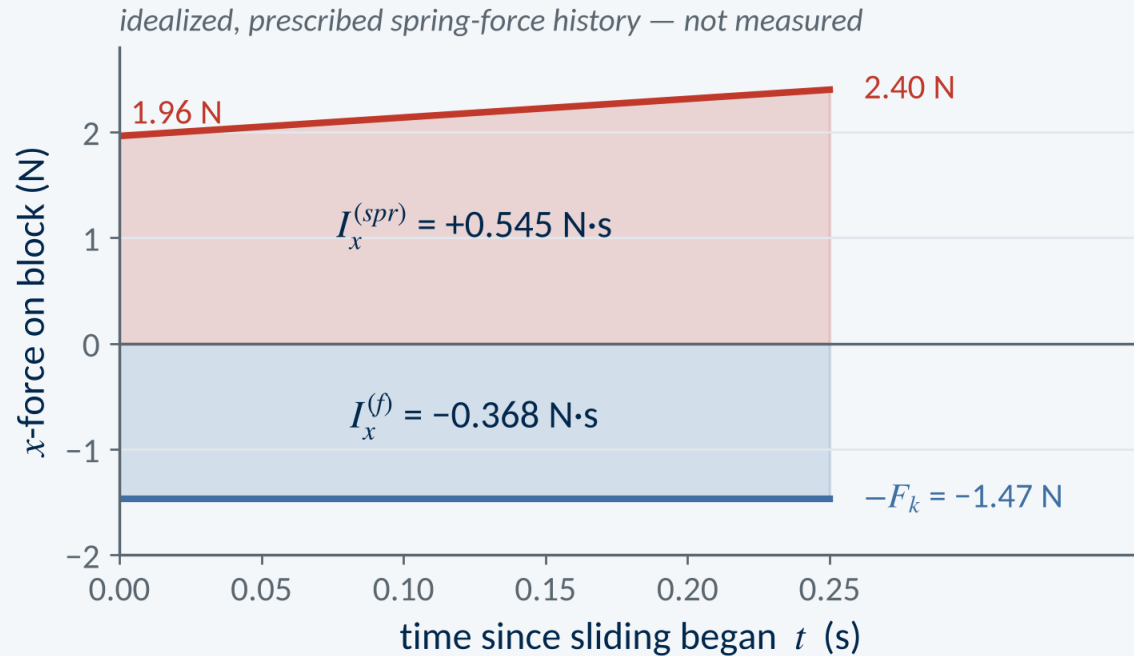
$$I_{x,1 \rightarrow 2} = 0 \rightarrow L_{x,2} = L_{x,1} = 0$$

The spring impulse grows the whole time. The net impulse stays zero because static friction cancels it.

An individual force can deliver a large impulse while the net impulse is zero.



Predicting the Velocity While It Slides



IDEALIZED EXAMPLE

$m = 0.50 \text{ kg}$, $\mu_s = 0.40$, $\mu_k = 0.30$. Breakaway at 1.96 N. Then 0.25 s of continuous sliding in $+x$, starting from rest.

$$I_{x,1 \rightarrow 2}^{(spr)} = \frac{1}{2}(1.96 + 2.40)(0.25) = +0.545 \text{ N}\cdot\text{s}$$

$$I_{x,1 \rightarrow 2}^{(f)} = -(0.30)(0.50)(9.81)(0.25) = -0.368 \text{ N}\cdot\text{s}$$

$$v_{x,2} = \frac{0.545 - 0.368}{0.50} \approx +0.355 \text{ m/s}$$



Qualifications: the linear rise in spring force is a prescribed, idealized history, not an automatic result of pulling the gauge at constant speed. The gauge-end motion and the block motion together set the extension. The result holds only while the block keeps sliding in $+x$.

Observe

Idealize

Model

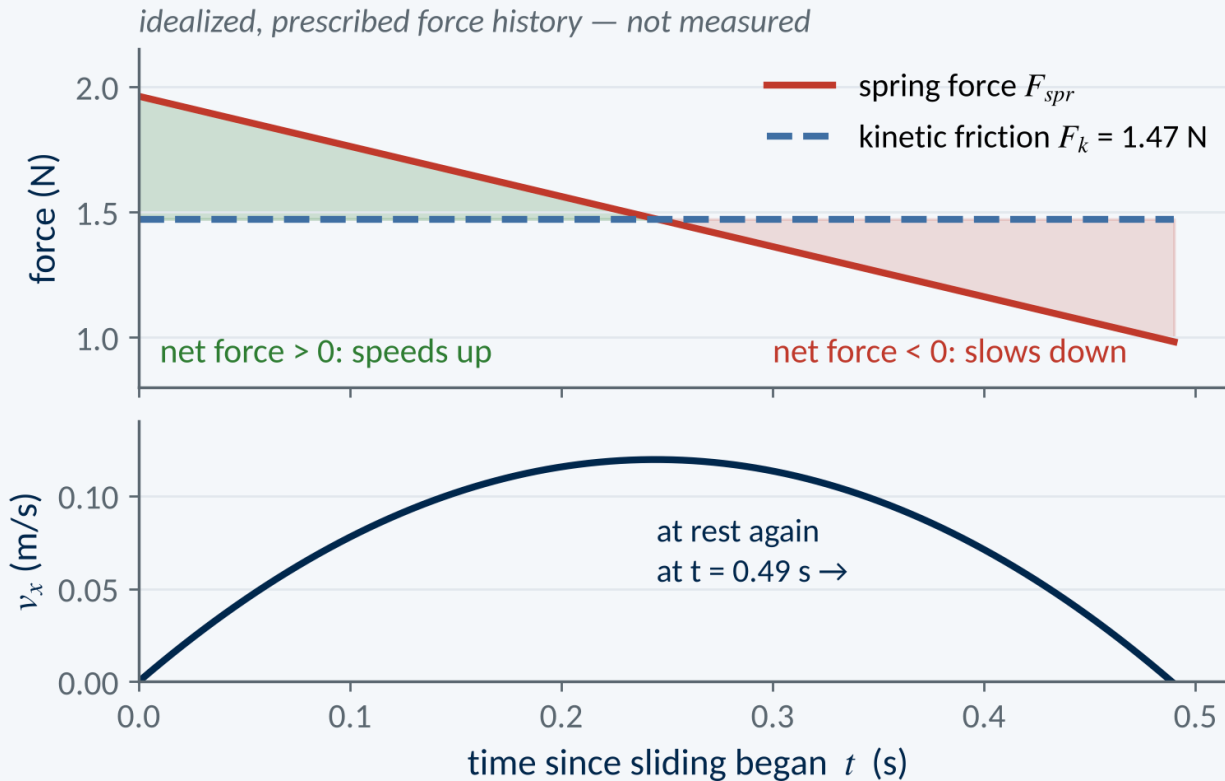
Solve / Analyze

Validate

Report



Speeding Up, Slowing Down, Stopping



Sign of the net force

While $F_{spr} > F_k$, the net x -force is positive and the speed rises. When the spring relaxes below F_k , the block slows.



Impulse view

$v_x(t) = (I^{(spr)} + I^{(f)}) / m$, both accumulated from the start of sliding; $I^{(f)} < 0$.



When it stops

Friction becomes static again. The kinetic-friction model ends at that instant.



Rest to Rest: What Can We Infer?

ONE SLIP, REST TO REST

- The block starts at rest, slides continuously in $+x$, and stops.
- No stationary phase inside the interval, so friction is kinetic throughout.
- Start and end at rest: $\Delta L_x = 0$.

THINK

If the net impulse is zero, the spring impulse must balance the friction impulse. Suppose we could measure the spring impulse, the mass and Δt . What unknown could we infer?

BALANCE OF IMPULSES

$$I_{x,1 \rightarrow 2}^{(\text{spr})} = \mu_k mg \Delta t \rightarrow \mu_k = \frac{I_{x,1 \rightarrow 2}^{(\text{spr})}}{mg \Delta t}$$

Idealized slip of slide 13: spring impulse 0.722 N·s, $\Delta t = 0.49$ s, so $\mu_k = 0.722 / (0.50 \times 9.81 \times 0.49) \approx 0.30$.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



Measured, Modeled, or Inferred?

FORCE GAUGE CAN

- measure quasi-static spring force
- help calibrate spring stiffness
- give the approximate breakaway force (peak hold, slow loading)
- help estimate kinetic friction (steady sliding pull)

GAUGE CANNOT BE ASSUMED TO

- record an accurate high-speed force history
- measure impulse directly
- give the net force when spring and friction both act

iPHONE + TRACKER CAN

- measure spring extension $\delta(t)$
- measure block position $x(t)$
- estimate velocity from fitted positions
- support spring-force reconstruction after calibration

SPRING CALIBRATION MODEL

$$F_{spr} = F_0 + k\delta$$

δ = spring extension relative to a defined reference length.

F_0 accounts for possible initial tension. It may be ≈ 0 for some springs, but not for all extension springs.

Observe ›

Idealize ›

Model ›

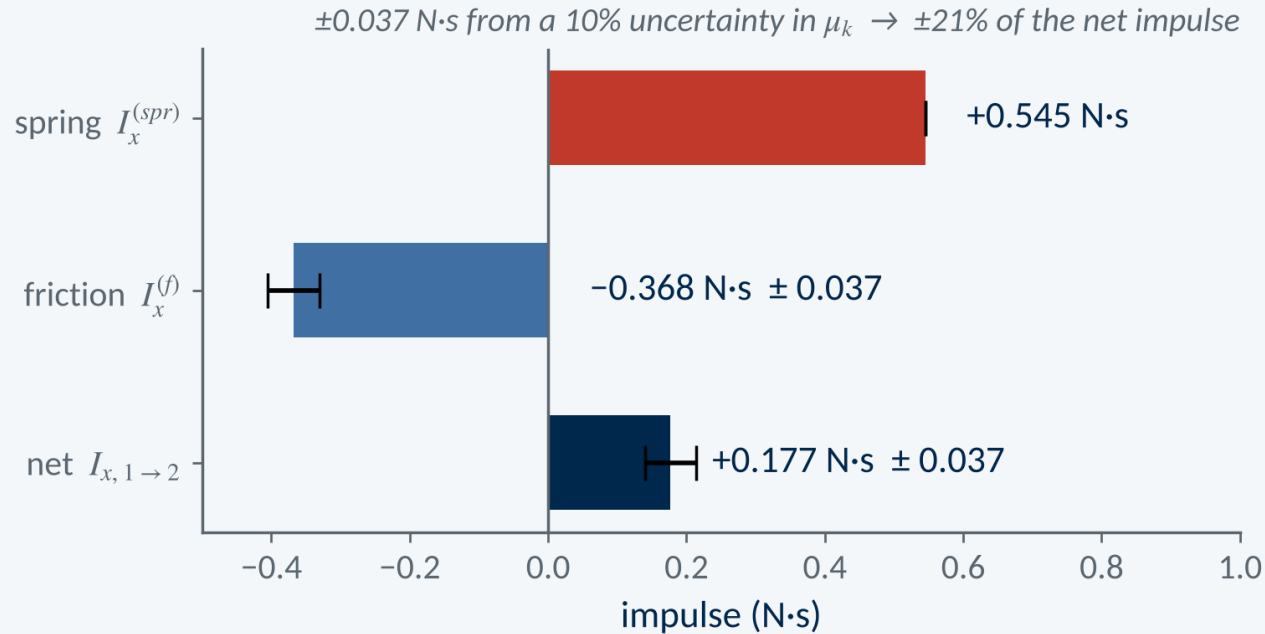
Solve / Analyze ›

Validate ›

Report



Small Differences of Big Numbers



THE IDEALIZED EXAMPLE

Spring impulse $\approx 0.545 \text{ N}\cdot\text{s}$; friction impulse magnitude $\approx 0.368 \text{ N}\cdot\text{s}$.

Net impulse $\approx 0.177 \text{ N}\cdot\text{s}$, only a third of the spring impulse.

A 10% uncertainty in μ_k shifts the friction impulse by $0.037 \text{ N}\cdot\text{s}$, which is about 21% of the net impulse.

Subtracting two large, uncertain impulses leaves a small, much less certain result. The full uncertainty budget belongs to the lab (Appendix A5).



Three Checks and One AI Claim

1

Can a nonzero spring impulse produce zero momentum change?

Yes. Static friction can deliver an equal and opposite impulse (slide 12).

2

Can two pulls with equal impulses produce different momentum changes?

Yes, when other forces act: the net impulse includes friction and every other force.

3

Does a 12 N gauge reading represent a 12 N·s impulse?

No. A force is not an impulse: it must be integrated over time, and the units differ.



AI-GENERATED CLAIM

“The force gauge reads 12 N, so the block receives 12 N·s of impulse.”

WHAT IS WRONG

- No time integration: impulse is the area under the force history, not one reading.
- Wrong units: N is not N·s.
- Friction omitted: the gauge reports the spring force only, not the net force on the block.

Diagnose the AI claim first (pairs, 2 min), then reveal.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



The Impulse–Momentum Workflow



YOU SHOULD NOW BE ABLE TO

- draw the free-body and before / during / after impulse–momentum diagrams for a particle;
- separate individual impulses from the net impulse and apply $\vec{L}_1 + \vec{I}_{1 \rightarrow 2} = \vec{L}_2$ by component;
- check a prediction with an independent method, and state what an experiment can and cannot establish.

Next — Part 3: What changes if we choose the block and the spring together as our system?

CHAPTER - 5 · PART 2

APPENDIX

Appendix — Supplemental Material

A1 inclined spring–block · A2 static-to-kinetic transition · A3 Tracker data reduction · A4 spring and friction calibration · A5 numerical integration and uncertainty · A6 Phyphox extension. Not required for the Part 2 learning objectives.

Observe



Idealize



Model



Solve / Analyze



Validate

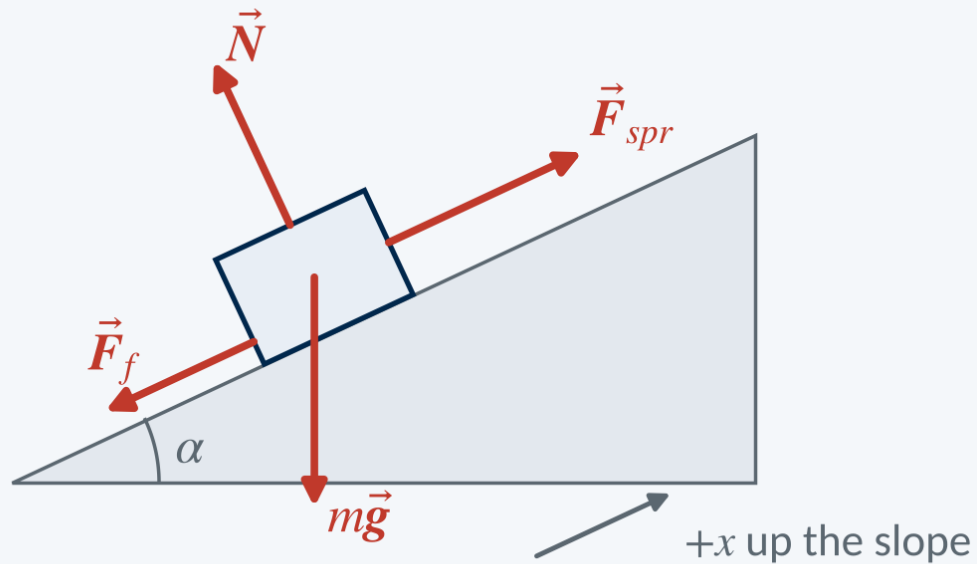


Report



The Same Method on a Slope

block sliding up the slope



+x UP THE SLOPE, SLIDING UPHILL

$$m(v_{x,2} - v_{x,1}) = \int_{t_1}^{t_2} (F_{spr} - F_k - mg \sin \alpha) dt$$

$$N = mg \cos \alpha, \quad F_k = \mu_k N$$

Gravity now has a component along the motion.



When $N = mg \cos \alpha$ holds

The spring acts parallel to the incline and no other force acts normal to it.



Friction direction

Set by the actual or impending motion. If the block slides down, friction acts up the slope and the sign changes.

Observe

Idealize

Model

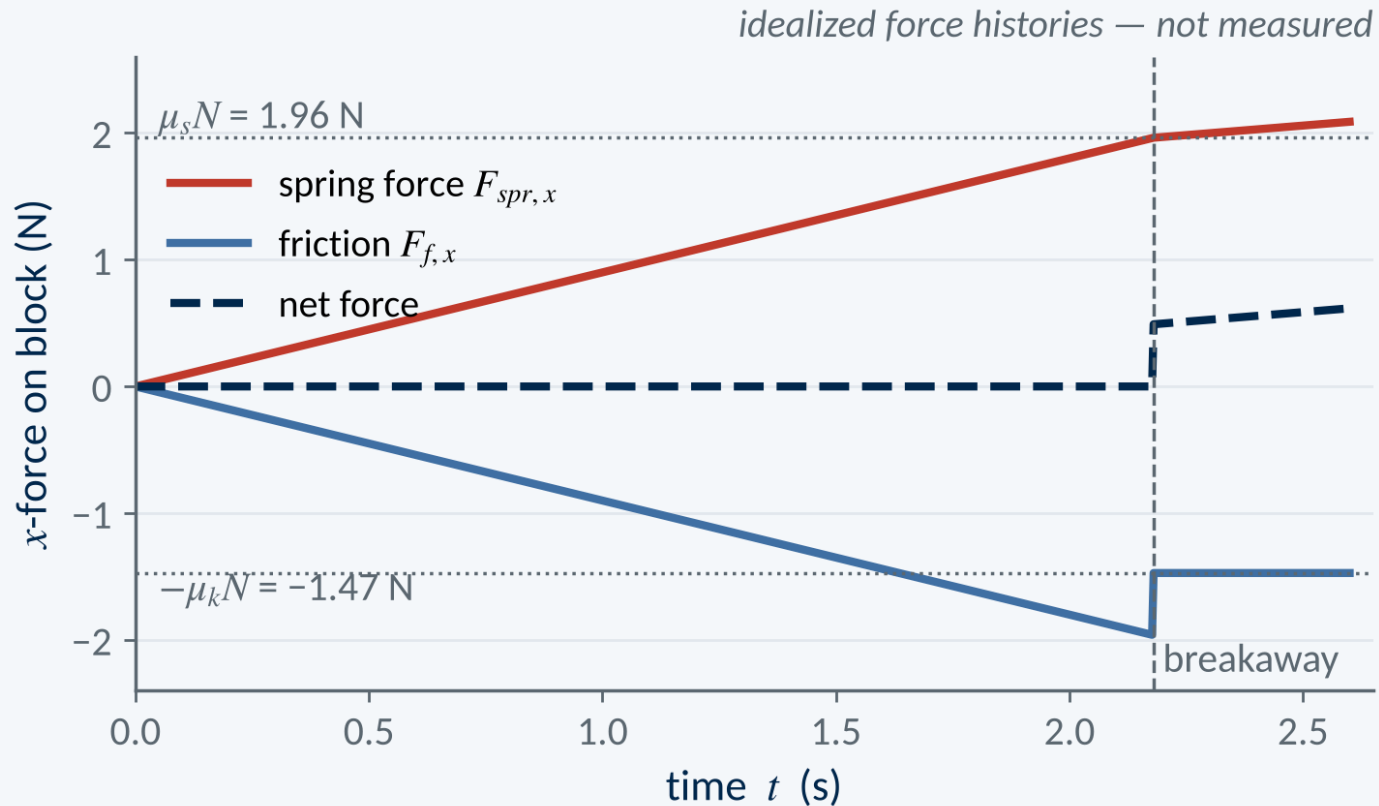
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Report



What Happens at Breakaway



Before breakaway

Static friction mirrors the spring force; net force is zero.



At breakaway

Friction drops from $\mu_s N$ to $\mu_k N$. The net force jumps, so the block lurches.



Real surfaces

Friction can depend on speed and surface condition. Stick-slip cycles are common.

Observe

Idealize

Model

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Report



From Slow-Motion Video to Extension and Position

1 Scale calibration

Put a scale bar in the plane of motion; calibrate Tracker's length scale on it.

2 Correct time base

iPhone slow motion records at 120 or 240 fps (check the file metadata) but plays back slower. Set the true frame rate in Tracker.

3 Slow-motion export

Slow-motion clips can contain speed-ramped sections. Export the whole clip at the slow rate, or the timing is wrong.

4 Points to track

Gauge hook, block hook (for δ = separation – reference length) and a block marker (for x).

5 Velocity from local fits

Fit position over several frames. With about 0.5 mm per pixel, frame-to-frame differences scatter by about 0.17 m/s; a fit over 12 frames, about 0.01 m/s.

6 Parallax and lighting

Keep the lens perpendicular to the motion. Some LED lights flicker at high frame rates; use daylight or flicker-free lighting.

Observe ›

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Model ›

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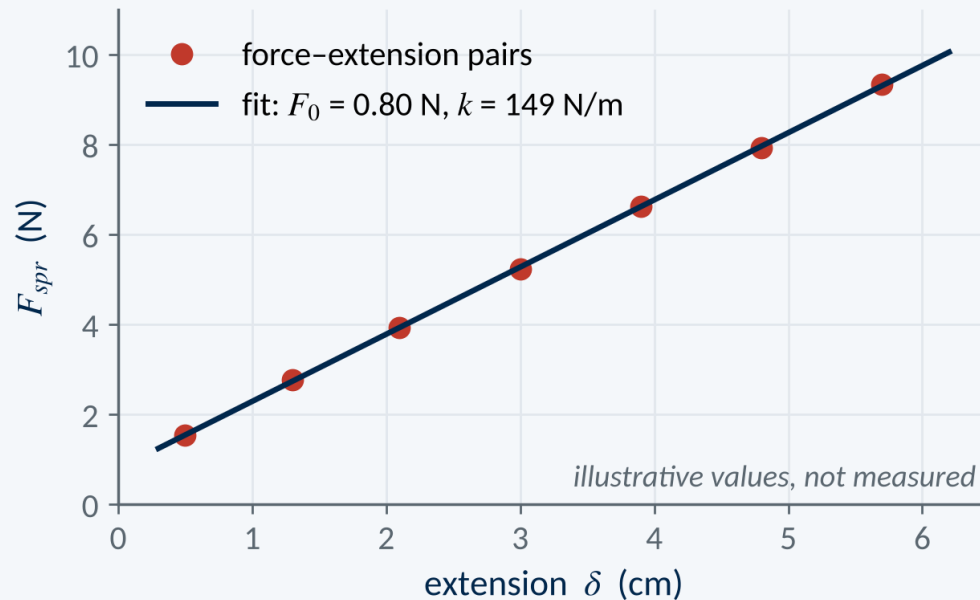
Validate ›

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Calibrate Before You Predict

LINEAR FIT WITH INTERCEPT



PROCEDURE

- Record 6–8 force–extension pairs under slow loading, all with $\delta > 0$ (an extension spring does not extend until $F > F_0$).
- Fit $F_{spr} = F_0 + k\delta$; inspect residuals and loading/unloading hysteresis.
- Breakaway: load slowly with peak hold, so $\mu_s \approx F_{spr,max} / (mg)$. Repeat several times.
- Kinetic friction: hook the gauge directly to the block (no soft spring, which invites stick–slip), pull at a slow, steady speed checked on video; $\mu_k \approx (\text{average reading}) / (mg)$.



Gauge limitations: peak hold is valid only for slow loading. The dynamic response is unverified, so never use the gauge as a force-history or impulse sensor. Keep loads well under 50 N.

Observe ›

Idealize ›

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Predicted Net Impulse vs Measured Momentum Change

- 1 Reconstruct** $F_{spr}(t) = F_0 + k\delta(t)$ from the tracked spring ends.
- 2 Integrate** Trapezoidal sum of F_{spr} over the continuous-sliding interval gives $I^{(spr)}_{x,1 \rightarrow 2}$.
- 3 Friction** $I^{(f)}_{x,1 \rightarrow 2} = -\mu_k mg \Delta t$, with μ_k from the steady-pull test (A4).
- 4 Compare** Predicted net impulse vs $m(v_{x,2} - v_{x,1})$ from fitted video velocities, each with an uncertainty range.

UNCERTAINTY BUDGET

- k and F_0 (fit uncertainty)
- μ_k (often the largest: $\pm 10\text{--}20\%$)
- frame timing and pixel scale
- mass of the block
- choice of the interval endpoints

Rest-to-rest strategy: choose a slip that starts and ends at rest. Then $\Delta L_x = 0$, no velocity differentiation is needed, and the spring impulse yields μ_k directly. Compare it with the steady-pull value. **Near-cancellation:** the net impulse is a small difference; report it with its range.

Observe ›

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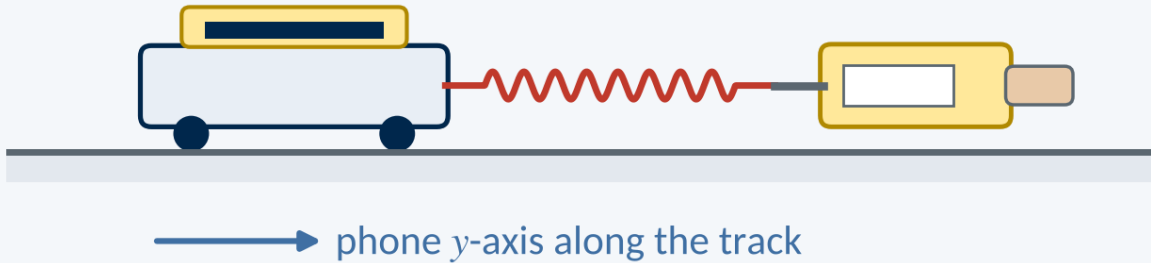
Report



Acceleration on a Low-Friction Cart

SPRING-PULLED CART

phone in padded holder



Coordinate orientation

Phyphox axes: x across the screen, y along the long edge, z out of the screen. Align y with the track.



Gravity correction

“Acceleration with g ” includes gravity on the vertical axis; “without g ” removes it. A tilt mixes gravity into a_y .



Sampling and integration

Check the sampling rate; integrate a_y with trapezoids to get Δv_y . Offsets accumulate.

Mount the phone on a low-friction cart, not on the sliding block. It would change the friction contact and the system studied.

Optional enrichment, not a required classroom activity.

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