

**AI-AUGMENTED ENGINEERING DYNAMICS:
A CONVERSATIONAL APPROACH**

Chapter 5 — Momentum Methods for Particles

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

CHAPTER - 5

PART 3 · ≈ 60 MIN

When Is Momentum Conserved? Choosing the System

Two carts can each gain momentum while the pair gains none. This Part asks which system we chose, which forces are external to it, and what external impulse acts over the interval.

Observe

Idealize

Model

Solve / Analyze

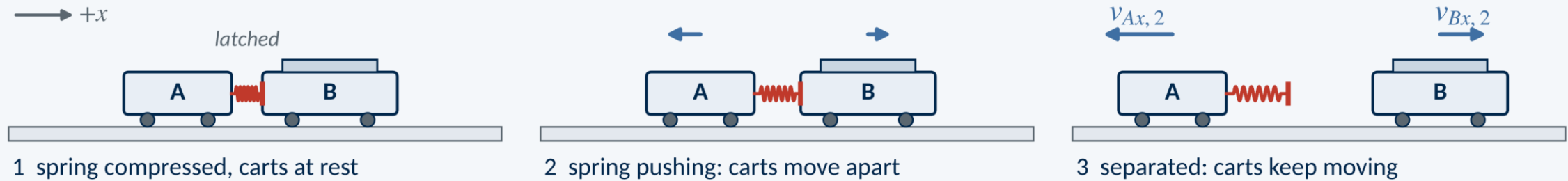
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Where Does the Momentum Come From?

SCHEMATIC SEQUENCE · IDEALIZED, NOT A RECORDING



THE SETUP

- Horizontal, low-friction track; both carts at rest.
- Cart A, 0.50 kg, includes the spring fixed to it.
- Cart B, 0.75 kg (with an add-on mass). The spring, latched compressed, pushes on B when released.

THE QUESTION

How can both carts acquire momentum when the system initially had none?

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Three Predictions

PREDICTION 1

Do the carts leave with equal speeds?

- A** yes
- B** no, A is faster
- C** no, B is faster

No: A, the lighter cart, is faster, 1.5 times B's speed. We derive this without knowing anything about the spring.

PREDICTION 2

Do they leave with equal and opposite momenta?

- A** yes
- B** no
- C** only if the masses are equal

Yes, if the horizontal external impulse on the two-cart system is negligible. The masses do not matter.

PREDICTION 3

Is the spring force internal or external?

- A** internal
- B** external
- C** it depends

It depends on the boundary. External to cart B alone; internal to the two carts together.

Observe

Idealize

Model

Solve / Analyze

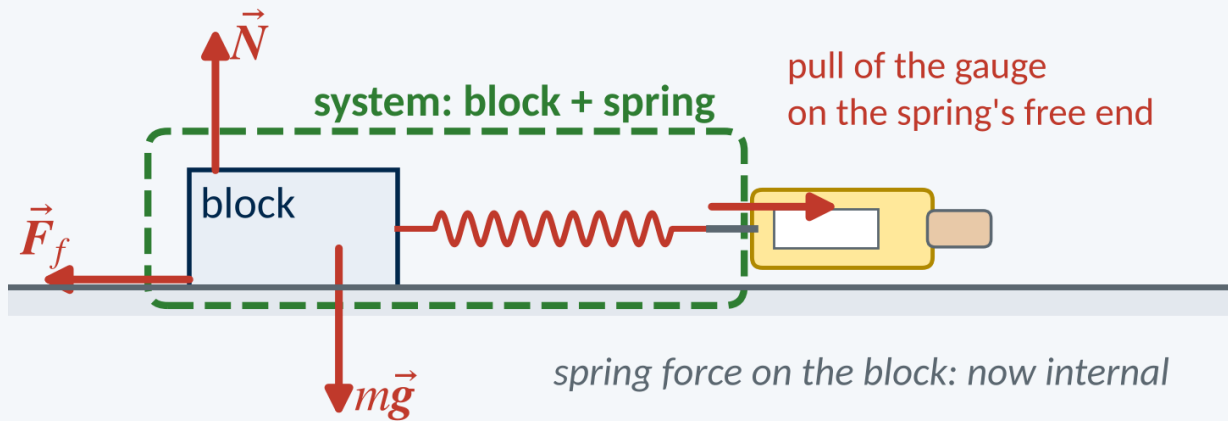
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Block and Spring as One System

THE PART 2 APPARATUS, NEW BOUNDARY



Part 2 asked: what changes if we choose the block and the spring together as our system?

WHAT CHANGES

- Spring force on the block: now **internal**.
- New external force: the pull of the gauge on the spring's free end.
- Friction, N and mg : still external.
- Light spring: its own momentum is negligible.

The motion is the same; the force accounting changes. With a light spring, the gauge's impulse on the spring equals the spring's impulse on the block, so Part 2's numbers do not change. Now: the same idea with two carts.

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One Scene, Four System Boundaries

1 cart B alone



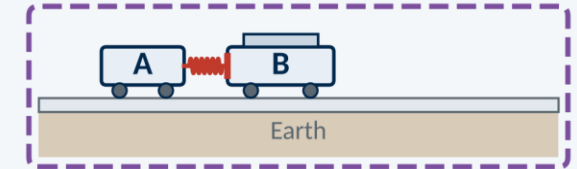
2 cart A with its spring



3 both carts



4 carts + track + Earth



Boundary	A's spring pushes B	B pushes A's spring	$N, mg, \text{friction}$	x -momentum constant during release?
1 cart B alone	external	not on system	external	no
2 cart A with its spring	not on system	external	external	no
3 both carts	internal	internal	external	yes
4 carts + track + Earth	internal	internal	internal	yes*

Classify each force first. Internal or external is a relationship to the boundary, not a label on the force.

*Only because Earth's momentum is included; its change is real but imperceptibly small. Not every system conserves momentum.

Observe

Idealize

Model

Solve / Analyze

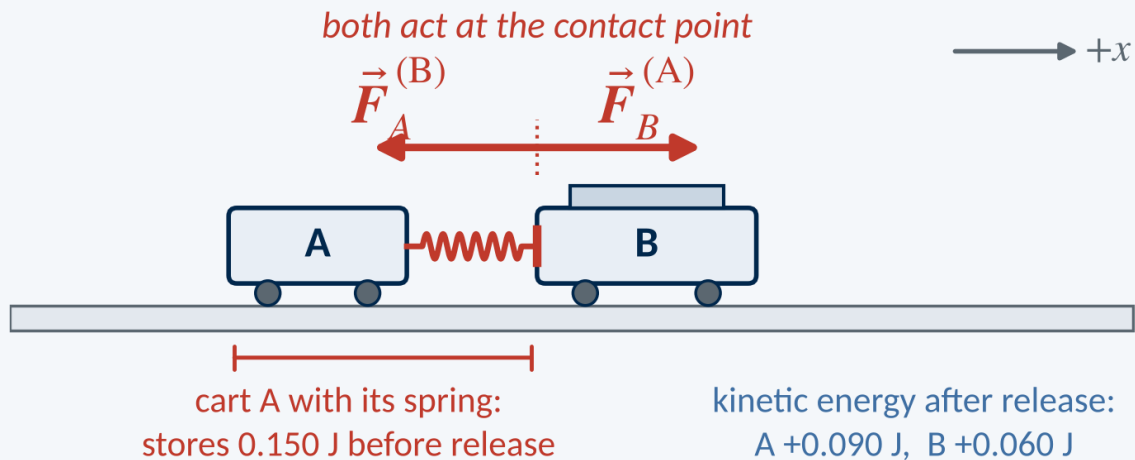
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Internal Impulses Cancel; Internal Energy Can Change Motion

DURING THE RELEASE



IMPULSES: SAME FORCE, SAME TIME

$$\vec{F}_A^{(B)}(t) = -\vec{F}_B^{(A)}(t)$$

$$\vec{I}_A^{(B)} = \int_{t_1}^{t_2} \vec{F}_A^{(B)} dt = -\vec{I}_B^{(A)}$$

$$\vec{I}_A^{(B)} + \vec{I}_B^{(A)} = \vec{0}$$

$\vec{F}_A^{(B)}$: force on A from B, acting at their common contact point.



Internal energy can change the motion. The compressed spring stores 0.150 J of elastic energy inside cart A. On release it becomes kinetic energy: +0.090 J for A, +0.060 J for B, +0.150 J in total. The A–B impulses still cancel, so the total momentum stays zero.

Observe

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The System Impulse–Momentum Equation

ADD THE PARTICLES; ONLY EXTERNAL IMPULSE SURVIVES

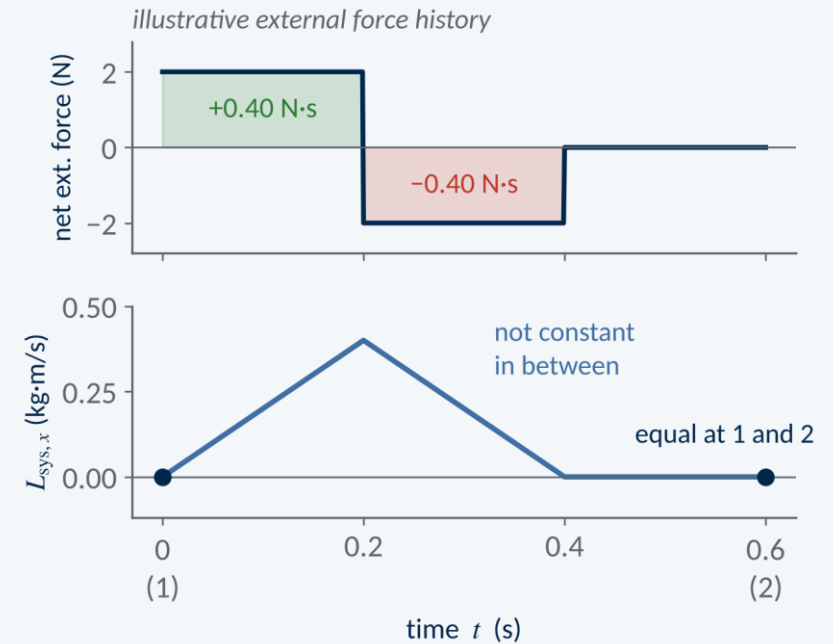
$$\vec{L}_{\text{sys}} = \sum m_i \vec{v}_i = \vec{L}_A + \vec{L}_B$$

$$\vec{L}_{\text{sys},1} + \vec{I}_{1 \rightarrow 2}^{(\text{ext})} = \vec{L}_{\text{sys},2}$$

$$\vec{I}_{1 \rightarrow 2}^{(\text{ext})} = \int_{t_1}^{t_2} \sum \vec{F}^{(\text{ext})} dt$$

CONDITIONS

- Inertial reference frame.
- A fixed set of particles: no mass crosses the boundary.
- Every external impulse identified, in every direction.



Zero net external impulse over the interval: endpoint momenta are equal.

Zero net external force at every instant: momentum is constant throughout.



The Question Determines the Boundary

QUESTION A

Cart B leaves at $+0.40$ m/s. How fast does cart A leave?

BOTH CARTS

Spring push internal; no horizontal external impulse. Use momentum conservation.

QUESTION B

What impulse did the spring give cart B?

CART B ALONE

The spring push is now external. Apply impulse–momentum to cart B.

QUESTION C

Cart A begins against a fixed stop. Is the total momentum of the two carts conserved?

BOTH CARTS

The stop is external and pushes during the release. Not conserved.

Choose the boundary before any equation. Put the forces you do not need inside it; leave outside the one you want.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

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Predict the Velocity Ratio

GIVEN (IDEALIZED)

- $m_A = 0.50$ kg, $m_B = 0.75$ kg
- both at rest at state 1
- system: both carts (green boundary)
- horizontal external impulse negligible during the release

PREDICT

Before any algebra: which cart is faster, and by what factor? You know the masses, and nothing about the spring.

x-COMPONENT, BOTH CARTS

$$m_A v_{Ax,1} + m_B v_{Bx,1} + 0 = m_A v_{Ax,2} + m_B v_{Bx,2} \rightarrow 0 = m_A v_{Ax,2} + m_B v_{Bx,2}$$

$$\frac{v_{Ax,2}}{v_{Bx,2}} = -\frac{m_B}{m_A} = -\frac{0.75}{0.50} = -1.5$$

The lighter cart moves 1.5 times as fast, in the opposite direction.



Predict the Velocities, Infer the Impulses

GIVEN · IDEALIZED

$$v_{Bx,2} = +0.40 \text{ m/s}$$

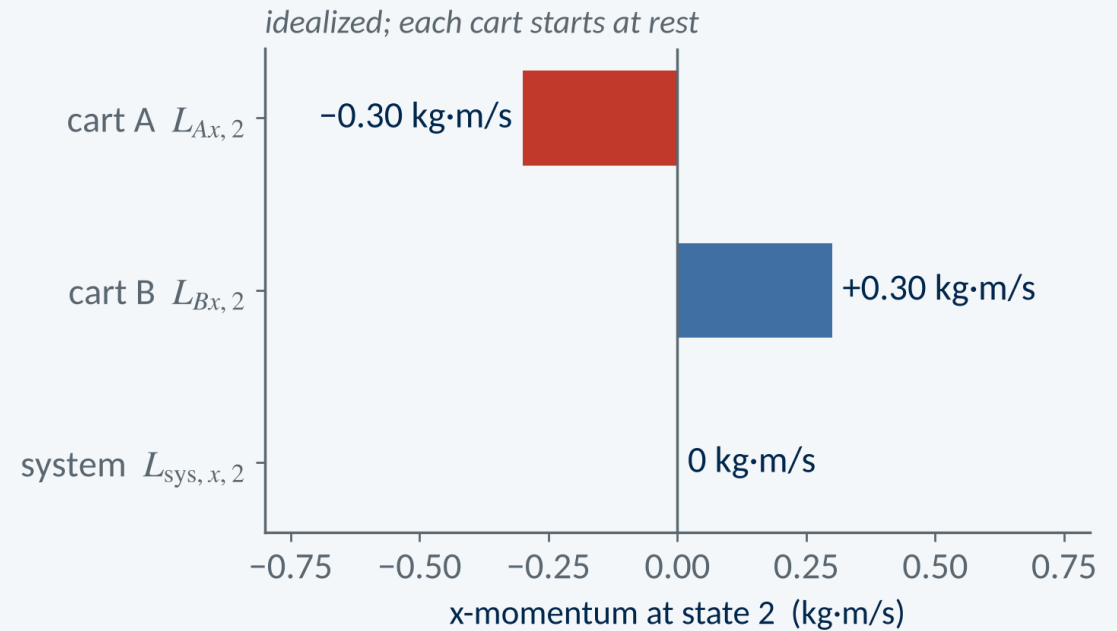
$$v_{Ax,2} = -1.5(+0.40) = -0.60 \text{ m/s}$$

Each cart alone (blue, red boundaries):

$$I_{Bx,1 \rightarrow 2}^{(A)} = m_B(v_{Bx,2} - 0) = +0.30 \text{ N} \cdot \text{s}$$

$$I_{Ax,1 \rightarrow 2}^{(B)} = m_A(v_{Ax,2} - 0) = -0.30 \text{ N} \cdot \text{s}$$

Equal and opposite, as the third law requires.



The impulses were inferred, not assumed. Conservation gave the velocities; each cart's momentum change then gave the impulse it received. Each cart's momentum changed; the system's did not.

Observe

Idealize

Model

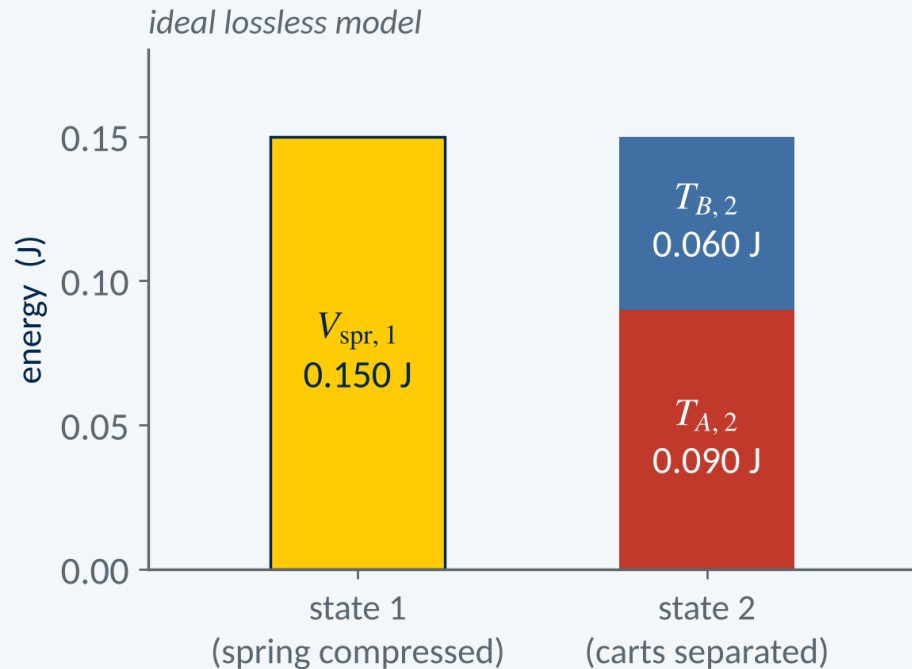
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Momentum Versus Energy



$$T_{A,2} = \frac{1}{2}(0.50)(0.60)^2 = 0.090 \text{ J}$$

$$T_{B,2} = \frac{1}{2}(0.75)(0.40)^2 = 0.060 \text{ J}$$

$$T_2 = 0.150 \text{ J}, \quad T_1 = 0$$

$$V_{\text{spr},1} = \frac{1}{2}k\delta^2 = \frac{1}{2}(750)(0.020)^2 = 0.150 \text{ J}$$

Equal momentum magnitudes, unequal energies. Since $T = L^2/(2m)$, the lighter cart A carries 60% of the kinetic energy.

Momentum is decided by external impulse. The kinetic-energy increase is supplied by released spring energy. The energy result is not a test of momentum conservation.

Observe ›

Idealize ›

Model ›

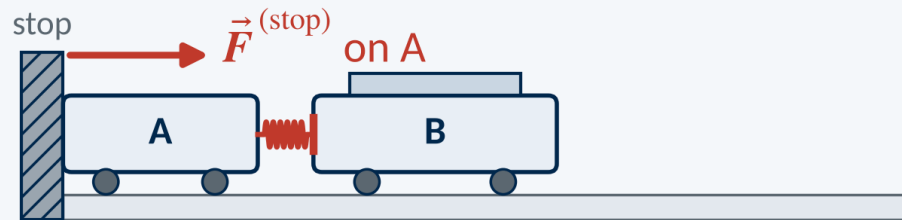
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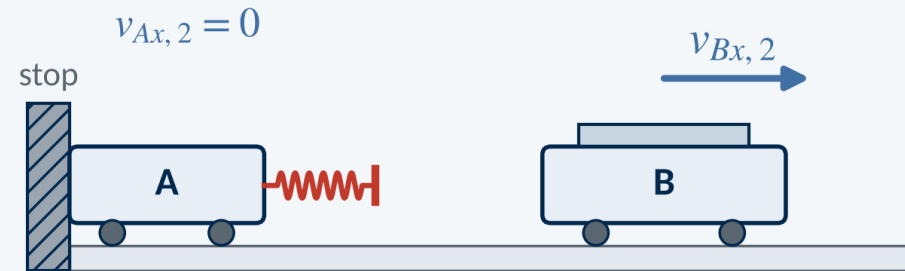
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Cart A Against a Fixed Stop



before: A pressed against the fixed stop



after: A still at rest, B moving

SAME SPRING ENERGY, IDEAL MODEL

$$v_{Bx,2} = (2V_{\text{spr},1} / m_B)^{1/2} = (0.400)^{1/2} \approx +0.632 \text{ m/s}$$

$$L_{\text{sys},x,2} = (0.75)(0.632) \approx +0.474 \text{ kg} \cdot \text{m/s}$$

$$I_{x,1 \rightarrow 2}^{(\text{stop})} \approx +0.474 \text{ N} \cdot \text{s} \text{ (external)}$$



Impulse without work

The stop's contact point does not move, so it does **zero work**, yet it pushes for the whole release: a nonzero impulse.

Idealized: rigid, stationary stop; no losses; same 0.150 J stored.

Observe

Idealize

Model

Solve / Analyze

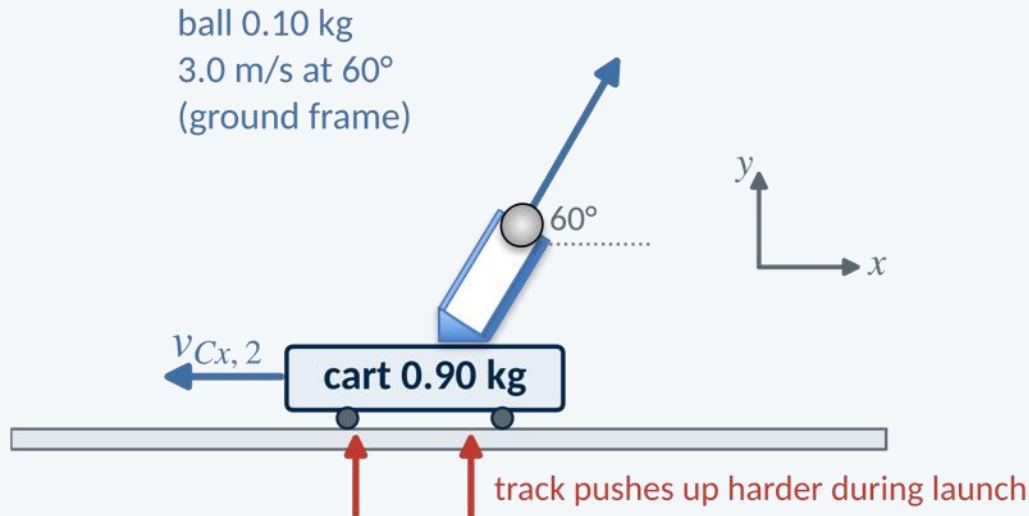
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Conserved in x, Not in y

IDEALIZED: SYSTEM = BALL + CART, AT REST



x: NO HORIZONTAL EXTERNAL IMPULSE

$$0 = (0.10)(3.0 \cos 60^\circ) + (0.90)v_{Cx,2}$$

$$\text{ball: } +0.150 \text{ kg} \cdot \text{m/s} \rightarrow v_{Cx,2} \approx -0.167 \text{ m/s}$$

y: NET EXTERNAL IMPULSE IS NOT ZERO

$$L_{y,2} = (0.10)(3.0 \sin 60^\circ) \approx +0.260 \text{ kg} \cdot \text{m/s}$$

Net external vertical impulse on the ball–cart system $\approx +0.260$ N·s: track normal force and gravity over the short launch, not the normal force alone.

Check each direction separately: momentum can be conserved in x while external forces change it in y .

Observe

Idealize

Model

Solve / Analyze

Validate

Report



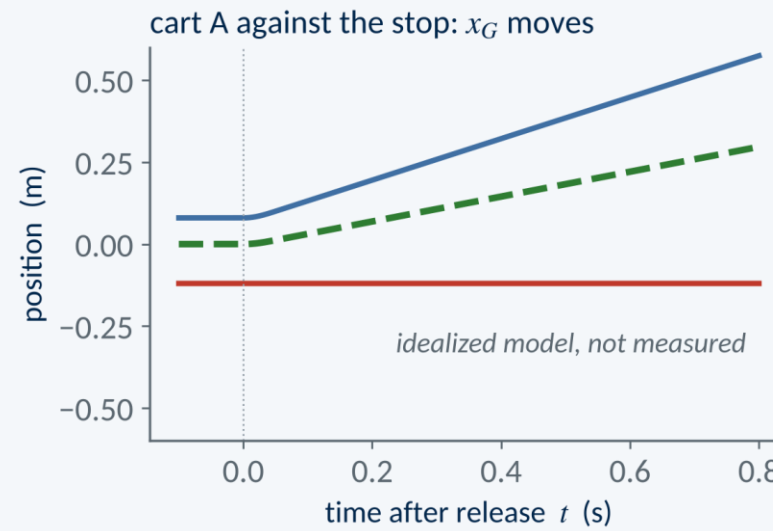
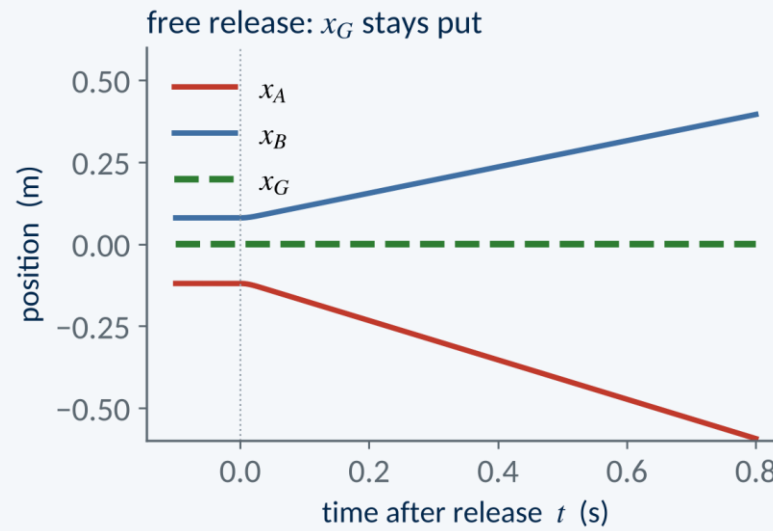
Center of Mass of a Particle System

$$\vec{r}_G = \frac{\sum m_i \vec{r}_i}{m_{\text{sys}}} \quad \vec{v}_G = \frac{\sum m_i \vec{v}_i}{m_{\text{sys}}}$$

$$m_{\text{sys}} = \sum m_i, \quad \vec{L}_{\text{sys}} = m_{\text{sys}} \vec{v}_G$$

$$m_{\text{sys}}(\vec{v}_{G,2} - \vec{v}_{G,1}) = \vec{I}_{1 \rightarrow 2}^{(\text{ext})}$$

Internal forces cannot change the total momentum of a fixed-mass system. Changes in center-of-mass motion are governed by external forces. Stop case: $v_{Gx,2} = 0.474/1.25 \approx 0.38$ m/s.

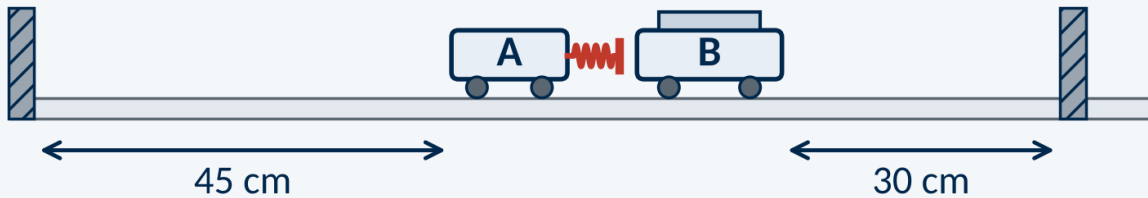




Two Proposed Tests of the Prediction

METHOD 1 · SIMULTANEOUS ARRIVAL

prediction: one click, not two



carts shown just after separation; distances measured from these positions

- Stops 45 and 30 cm beyond the carts' positions just after separation.
- Post-separation travel: $0.45/0.60 = 0.30/0.40 = 0.75$ s. Both arrive together: one click.
- Friction or tilt would make the clicks separate.

METHOD 2 · TRACKER CENTER OF MASS

$$x_G(t) = \frac{m_A x_A(t) + m_B x_B(t)}{m_{\text{sys}}}$$

- Track both carts in one side-on video; compute $x_G(t)$.
- Prediction: flat for an ideal free release.
- A slow drift points to track tilt or other external forces.

Proposed methods, not completed experiments. No cart data exists yet; every number here is a prediction. A 0.5° tilt would move x_G by about 2.4 cm in 0.75 s.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Three Plausible but Wrong Explanations



AI CLAIM 1

“The spring pushes both carts, so each cart conserves its own momentum.”

WHAT IS WRONG

Each cart’s momentum changes (-0.30 and $+0.30$ kg·m/s). The spring’s push is external to B alone, and B’s push back is external to A alone; only the two-cart total is conserved.



AI CLAIM 2

“The carts gain kinetic energy, so total momentum cannot be conserved.”

WHAT IS WRONG

Energy comes from the spring through internal work. Momentum depends on external impulse, still zero: T rises to 0.150 J while L_{sys} stays 0 .



AI CLAIM 3

“The fixed stop does no work, so it cannot change the system’s momentum.”

WHAT IS WRONG

Work and impulse differ. The contact point does not move (zero work), but the force acts over time: $+0.474$ N·s.

Observe ›

Idealize ›

Model ›

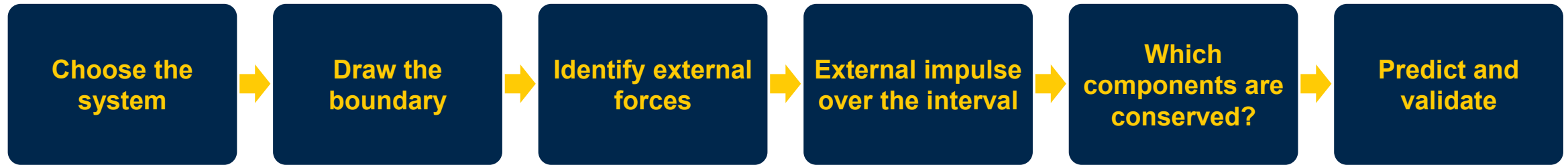
Solve / Analyze ›

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Choosing the System: A Workflow



BACK TO THE OPENING

- Both carts gained momentum because each one, alone, received an external impulse from the other: -0.30 and $+0.30$ N·s.
- The pair gained none because those impulses are internal to the two-cart system and cancel.
- Speeds 0.60 and 0.40 m/s, not equal; kinetic energy from the spring, not from momentum.

Next — Part 4: When two carts collide, momentum conservation gives one equation for two unknown final velocities. What additional information is needed?

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report

CHAPTER - 5 · PART 3

APPENDIX

Appendix — Supplemental Material

A1 three-particle internal impulses · A2 two-dimensional separation · A3
piecewise external force · A4 future two-cart experiment and Tracker · A5
combined energy and momentum · A6 larger systems and subtle boundaries.
Not required for the core.

Observe



Idealize



Model



Solve / Analyze



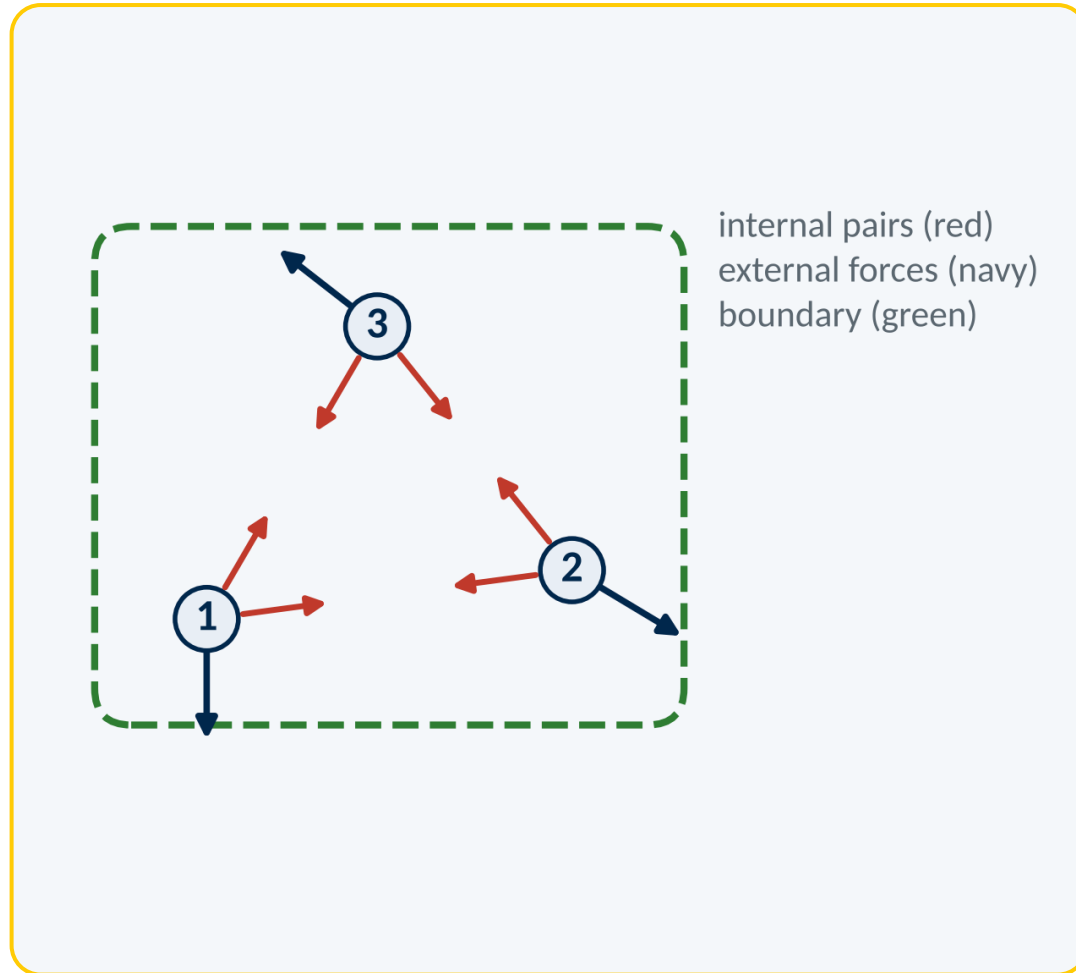
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Internal Pairs Cancel, However Many



SUM OVER PARTICLES 1, 2, 3

$$\vec{F}_1^{(2)} = -\vec{F}_2^{(1)}, \quad \vec{F}_1^{(3)} = -\vec{F}_3^{(1)}, \quad \vec{F}_2^{(3)} = -\vec{F}_3^{(2)}$$

Add Newton's second law for all three particles: the six internal forces form three pairs and sum to zero at every instant.

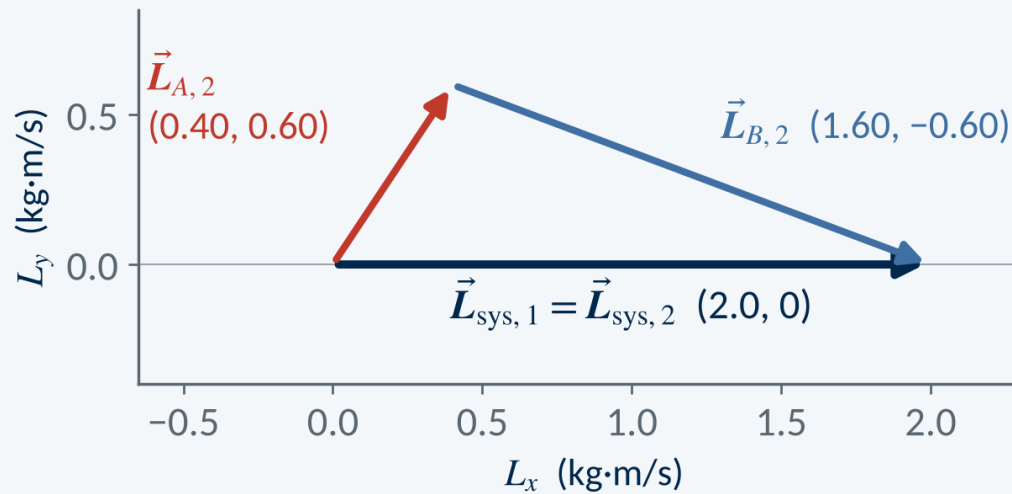
$$\vec{L}_{\text{sys},1} + \vec{I}_{1 \rightarrow 2}^{(\text{ext})} = \vec{L}_{\text{sys},2}$$

With n particles there are $n(n - 1)/2$ pairs. They redistribute momentum among the particles; only external forces change the total.



A Puck Splits on an Air Table

MOMENTUM VECTORS, IDEALIZED



- 1.0 kg puck at (2.0, 0) m/s on a horizontal air table; an internal spring splits it.
- Piece A: 0.40 kg at (1.00, 1.50) m/s. Find piece B (0.60 kg).

$$x: 2.0 = 0.40(1.00) + 0.60v_{Bx,2} \rightarrow +2.67 \text{ m/s}$$

$$y: 0 = 0.40(1.50) + 0.60v_{By,2} \rightarrow -1.00 \text{ m/s}$$

v_G stays (2.0, 0) m/s. Kinetic energy: 2.00 J $\rightarrow$ 3.08 J; the spring supplied about 1.08 J.

Weight and the air-cushion force act along z and cancel, so both horizontal components are conserved.

Observe

Idealize

Model

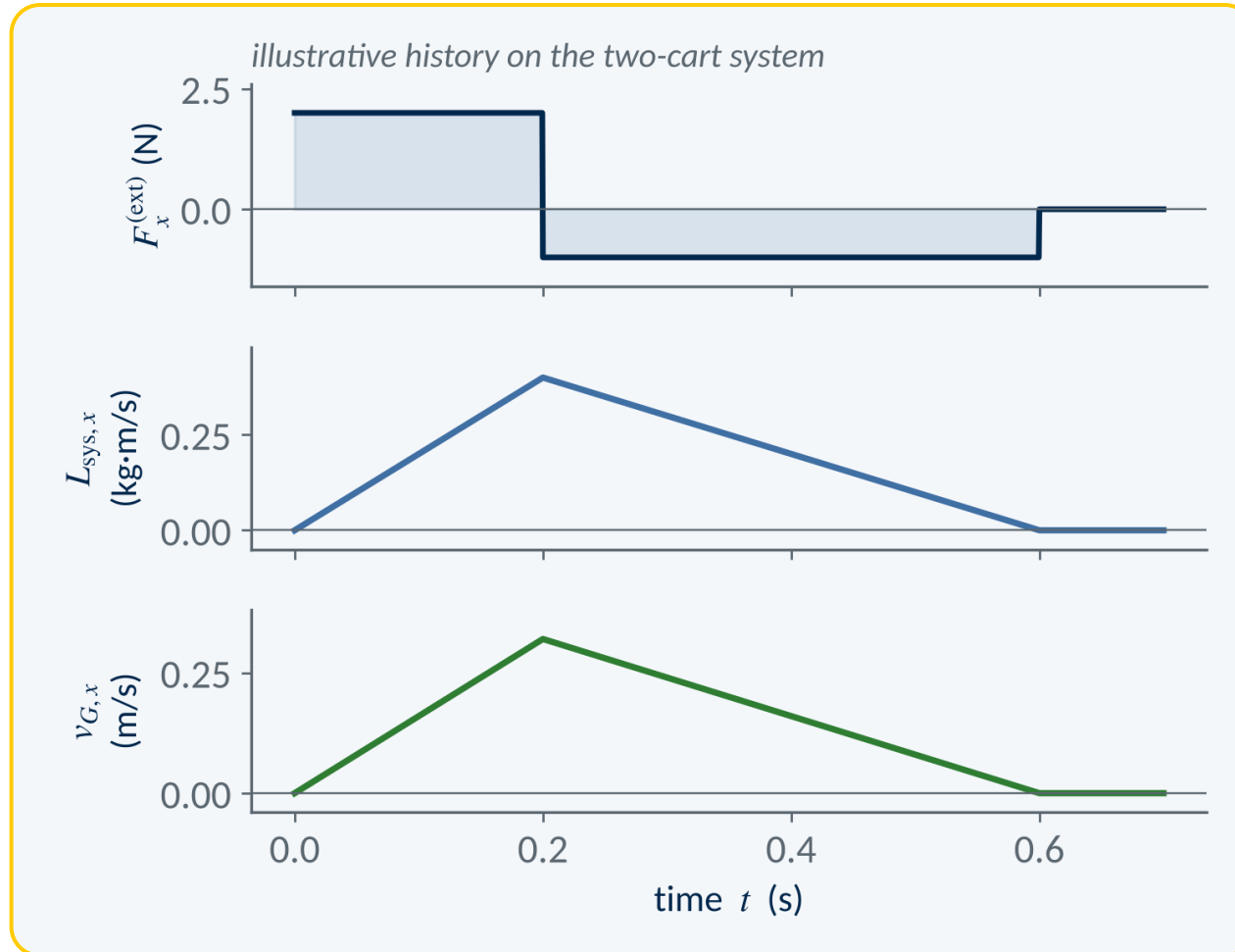
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Equal Endpoints, Not Constant Momentum



TWO-CART SYSTEM, AT REST AT $t = 0$

- +2.0 N for 0.20 s: +0.40 N·s
- -1.0 N for 0.40 s: -0.40 N·s
- Net over 0 to 0.60 s: zero, so $L_{sys,x}$ returns to 0.
- Peak: 0.40 kg·m/s, so $v_{G,x} = 0.32$ m/s at 0.20 s.

The interval decides. Over 0 to 0.60 s the endpoints agree. Over 0 to 0.20 s they do not. “Conserved” always needs a stated system and interval.



A Future Experiment with Tracker

1 Equipment

Two carts, one with a spring plunger; low-friction 1.2 m track with end stops; add-on mass; scale (± 1 g); phone and tripod; metre stick.

2 Prepare

Weigh both carts (spring included in A). Level the track to under about 0.2° ; a gently pushed cart should roll end to end at nearly constant speed.

3 Film

Side view, lens perpendicular, scale in the carts' plane. iPhone slow motion at 240 fps: set the true frame rate in Tracker, not the playback rate.

4 Fit velocities

The release lasts about 31 ms (7–8 frames). Fit each cart's position after separation and extrapolate back to the separation instant.

5 Center of mass

Compute $x_G(t)$ from both tracks; compare it with flat. Repeat with A against a stop.

6 Uncertainty

Masses, scale, frame timing; tilt and friction grow with time; a release push of 0.01 N·s is about 3% of 0.30 N·s.

Proposed, not yet performed. Report measured results only after the experiment has been run and checked.

Observe ›

Idealize ›

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When the Stored Spring Energy Is Known

FREE RELEASE: TWO EQUATIONS, TWO UNKNOWNNS

$$0 = m_A v_{Ax,2} + m_B v_{Bx,2}$$

$$V_{\text{spr},1} = \frac{1}{2} m_A v_{Ax,2}^2 + \frac{1}{2} m_B v_{Bx,2}^2$$

$$v_{Ax,2} = - \left(2V_{\text{spr},1} m_B / (m_A m_{\text{sys}}) \right)^{1/2} = -0.60 \text{ m/s}$$

$$v_{Bx,2} = + \left(2V_{\text{spr},1} m_A / (m_B m_{\text{sys}}) \right)^{1/2} = +0.40 \text{ m/s}$$

	Free release	A against stop
$v_{Bx,2}$	+0.400 m/s	+0.632 m/s
$v_{Ax,2}$	-0.600 m/s	0
$L_{\text{sys},x,2}$	0	+0.474 kg·m/s
T_2	0.150 J	0.150 J
release time	≈ 31 ms	≈ 50 ms

- Speed ratio for B, stop vs free: $\sqrt{m_{\text{sys}}/m_A} = \sqrt{2.5} \approx 1.58$. Same energy, different external impulse.
- Energy plus momentum fixes both speeds only when the stored energy is known and losses are negligible. Momentum alone fixes only the ratio.



Where Should the Boundary Go?



Including the Earth

Carts + track + Earth: the stop's $0.474 \text{ N}\cdot\text{s}$ gives Earth an equal and opposite momentum change, a velocity change of about 10^{-25} m/s ($\approx 8 \times 10^{-26}$). Real, but unmeasurable.



A loose spring with mass

If the spring is a separate body and left out, its pushes on A and on B are external and need not cancel; their sum equals minus the spring's own momentum change.



Cart A alone

The push from B is external, so A's momentum changes by $-0.30 \text{ kg}\cdot\text{m/s}$ during the release. A cart alone never conserves momentum here.



A fixed set of particles

If mass leaves (the launched ball), keep it in the system or account for the momentum it carries across the boundary.

Observe ›

Idealize ›

Model ›

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