

**AI-AUGMENTED ENGINEERING DYNAMICS:
A CONVERSATIONAL APPROACH**

Chapter 5 — Momentum Methods for Particles

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

CHAPTER - 5

PART 4 · ≈ 60 MIN

What Happens During Impact?

Momentum conservation relates the velocities after an impact, but it cannot fix both of them. This Part adds the missing relationship, restitution, and asks what impact does to kinetic energy and force.

4

Observe

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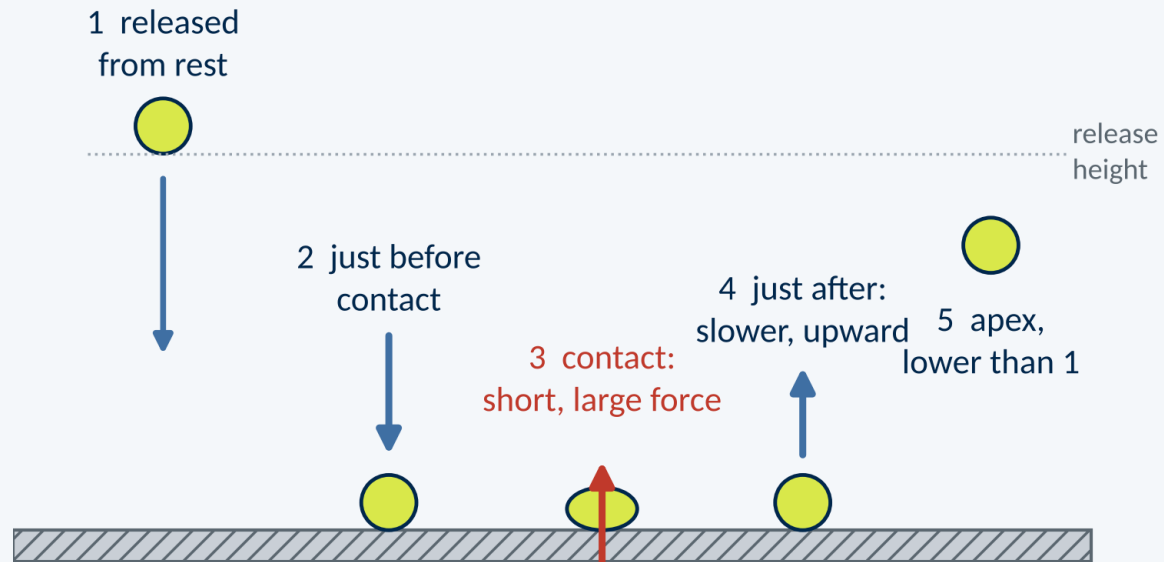
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Why Does the Ball Come Back Lower?

SCHEMATIC SEQUENCE · NOT A RECORDING



LIVE DEMO

- Drop a tennis ball from about 1 m onto the floor. Watch, do not measure.
- Contact lasts only milliseconds.
- It comes back slower and lower, and its momentum reverses direction.

From Part 3: momentum conservation alone does not determine both final velocities in a two-body collision. **What else do we need?**

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Three Predictions About the Bounce

PREDICTION 1

During contact, is the ball's momentum conserved?

- A** yes
- B** no
- C** only if $e = 1$

No. The floor pushes the ball, an external force on the ball alone. Its momentum reverses.

PREDICTION 2

During contact, is the momentum of ball + Earth conserved?

- A** yes
- B** no
- C** only for a hard floor

Yes, to excellent accuracy: the contact force is internal. Earth's share is real but imperceptible.

PREDICTION 3

Just after contact, does the ball have the kinetic energy it had just before?

- A** yes
- B** no, less
- C** no, more

No, less. Some becomes sound, vibration and heat. Energy is converted, not destroyed.

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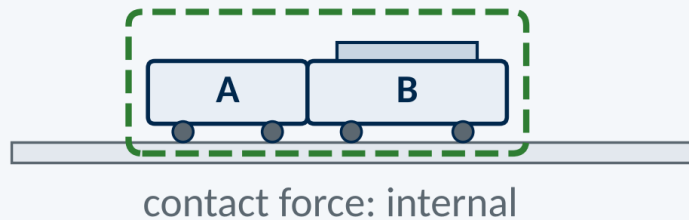
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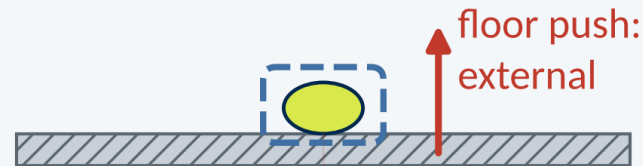


Carts, Ball Alone, Ball and Earth

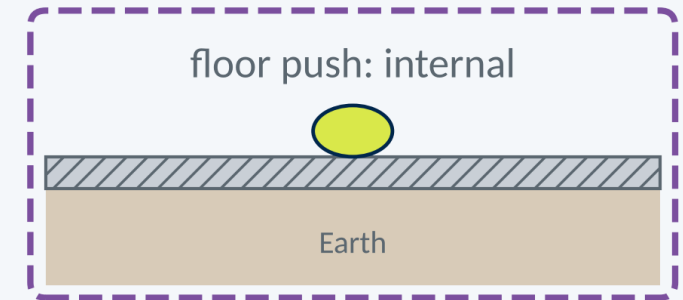
1 both carts



2 ball alone



3 ball + Earth



System	Contact impulse is	Momentum during contact
1 both carts	internal	conserved (friction impulse $\approx 10^{-3}$ N·s, negligible)
2 ball alone	external	not conserved: changes by about +0.43 N·s
3 ball + Earth	internal	conserved: Earth receives about -0.43 N·s

Illustrative: 57 g ball, 4.43 m/s down, 3.10 m/s up: $I = 0.057(3.10 + 4.43) \approx 0.43$ N·s.

Earth: $\Delta v \approx 0.43 / (6.0 \times 10^{24}) \approx 7 \times 10^{-26}$ m/s.

Gravity over a ≈ 5 ms contact: ≈ 0.003 N·s, neglected.

Observe

Idealize

Model

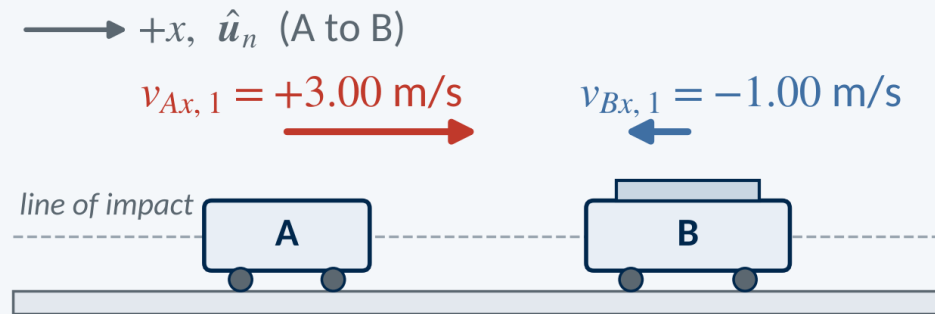
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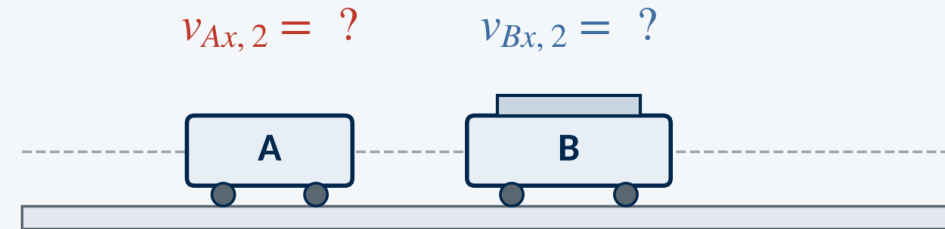
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Line of Impact, Approach and Separation



state 1: just before contact (approaching)



state 2: just after contact (separating)

DEFINITIONS (ALONG THE LINE OF IMPACT)

approach velocity: $v_{An,1} - v_{Bn,1} > 0$

separation velocity: $v_{Bn,2} - v_{An,2}$

$\hat{u}_n$ points from A toward B. Here it is $+x$, so $v_{An} = v_{Ax}$.

- **Central impact:** both velocities lie along the line of impact through the contact.
- Velocity components are signed (+3.00, -1.00 m/s); speeds are never negative.
- Approach here: $3.00 - (-1.00) = 4.00 \text{ m/s}$, larger than either speed.



Coefficient of Restitution

MOMENTUM: ONE EQUATION, TWO UNKNOWNNS

$$m_A v_{An,1} + m_B v_{Bn,1} = m_A v_{An,2} + m_B v_{Bn,2}$$

Valid when the external impulse along $\hat{u}_n$ is negligible.

RESTITUTION: THE SECOND RELATIONSHIP

$$e = \frac{v_{Bn,2} - v_{An,2}}{v_{An,1} - v_{Bn,1}} = \frac{\text{separation velocity}}{\text{approach velocity}}$$

- Relative **normal** velocities, just after over just before.
- Passive impacts: $0 \leq e \leq 1$. Empirical: a property of the contacting **pair** and the impact conditions (speed, temperature).

WHAT e IS NOT

- not the ratio of the two bodies' individual speeds;
- not a ratio of kinetic energies, and not the fraction of kinetic energy kept;
- not a measure of the impact force;
- not a law for every collision: active or energy-releasing impacts (explosive, spring-launched) can fall outside 0 to 1.



Restitution Is Not an Energy Ratio

YOUR PREDICTION

A ball rebounds from a stationary floor with $e = 0.70$.
What fraction of its pre-impact kinetic energy remains just after impact?

- A** 70%
- B** 49%
- C** 30%
- D** it depends on the ball's mass

ANSWER: B, 49%

$$\frac{T_2}{T_1} = \left(\frac{v_{sp,2}}{v_{sp,1}} \right)^2 = e^2 = 0.49$$

- Only because the floor stays still, so the ball's speed ratio equals e .
- For two moving bodies, e^2 is not the ratio of total kinetic energies.
- Example A, next: $e = 0.60$, yet the pair keeps **41.5%** of its kinetic energy, not 60% and not 36%.



Two Carts, Head-On

GIVEN (IDEALIZED)

- A: 0.50 kg, $v_{Ax,1} = +3.00$ m/s
- B: 0.75 kg, $v_{Bx,1} = -1.00$ m/s
- $e = 0.60$, **stipulated**
- external impulse along x negligible

PREDICT

Which direction will cart A move after the impact? Commit before the algebra.

1 MOMENTUM (BOTH CARTS)

$$0.50(3.00) + 0.75(-1.00) = 0.50v_{Ax,2} + 0.75v_{Bx,2} \\ = +0.75$$

2 RESTITUTION

$$v_{Bx,2} - v_{Ax,2} = 0.60[3.00 - (-1.00)] = 2.40$$

$$v_{Ax,2} = -0.84 \text{ m/s}, \quad v_{Bx,2} = +1.56 \text{ m/s}$$

A reverses. Check: $0.50(-0.84) + 0.75(1.56) = +0.75 \checkmark$; $1.56 - (-0.84) = 2.40 \checkmark$.

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Idealize ›

Model ›

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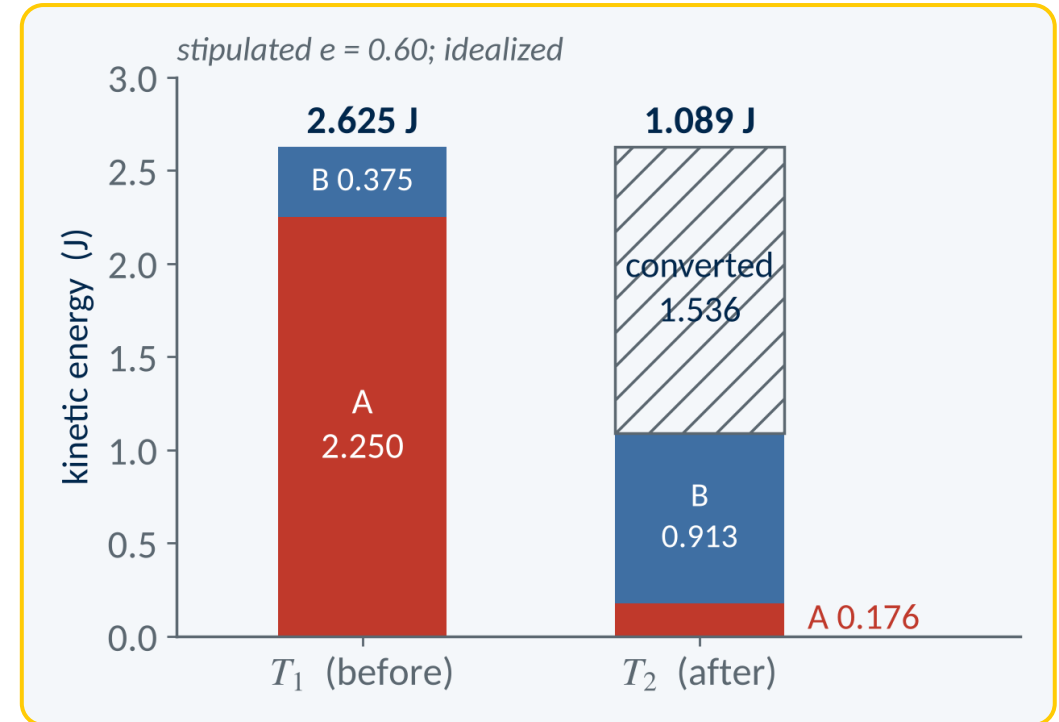


Momentum Is Conserved; Kinetic Energy Is Not

MOMENTUM LEDGER

Cart	$L_{x,1}$ (kg·m/s)	$L_{x,2}$ (kg·m/s)	change (N·s)
A	+1.50	-0.42	-1.92
B	-0.75	+1.17	+1.92
both	+0.75	+0.75	0

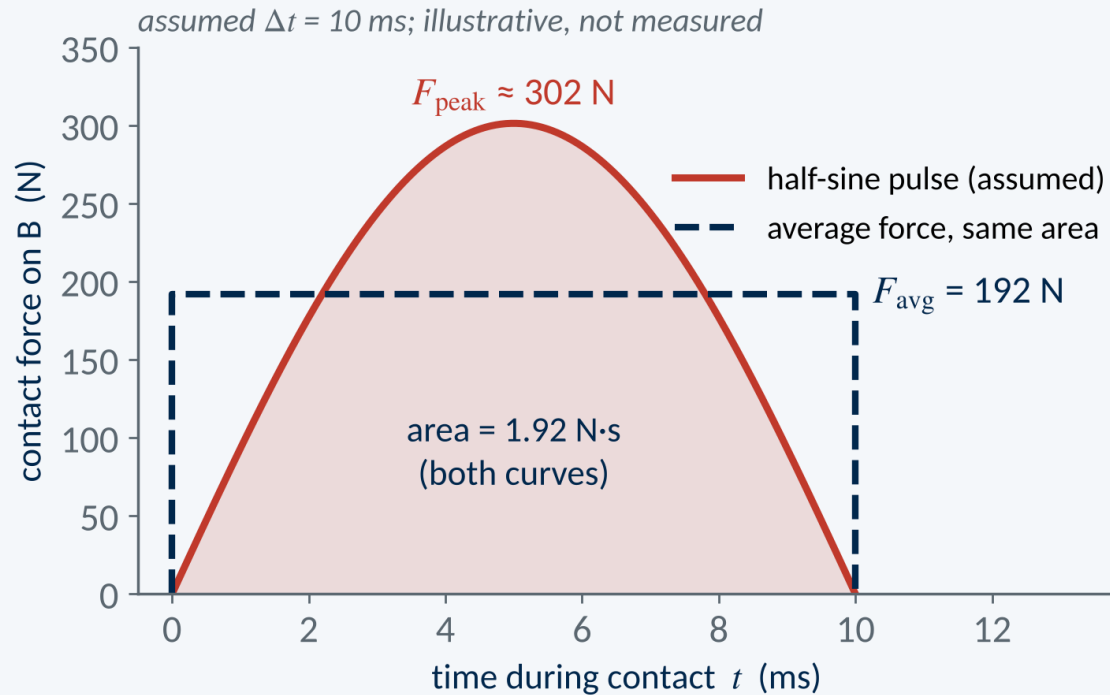
- The contact impulses are equal and opposite:
- $I_{Ax,1 \rightarrow 2}^{(B)} = -1.92 \text{ N}\cdot\text{s}$ on A; $I_{Bx,1 \rightarrow 2}^{(A)} = +1.92 \text{ N}\cdot\text{s}$ on B.
- Each cart's momentum changes; the pair's does not.



$T_1 - T_2 = 1.536 \text{ J}$ (58.5%) **converted** to sound, vibration and heat. 41.5% kept.



Impulse, Average Force, Peak Force



$$I_{B_x, 1 \rightarrow 2}^{(A)} = \int_0^{\Delta} F_{B_x}^{(A)} dt = +1.92 \text{ N} \cdot \text{s}$$

$$F_{\text{avg}} = \frac{I}{\Delta t} = \frac{1.92}{0.010} = 192 \text{ N}$$

$$F_{\text{peak}} = \frac{\pi}{2} F_{\text{avg}} \approx 302 \text{ N}$$

Peak value valid only for the assumed half-sine history; $\Delta t = 10$ ms is also assumed.

Impulse comes from the momentum change. **Average force** also needs Δt . **Peak force** needs the force history. e sets none of them: a softer bumper with the same e lasts longer and pushes less hard.

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From $e = 0$ to $e = 1$, and What Is Physically Allowed

Example A with	$v_{Ax,2}$ (m/s)	$v_{Bx,2}$ (m/s)	T_2 (J)	T_2/T_1
$e = 0$	+0.60	+0.60	0.225	8.6%
$e = 0.60$	-0.84	+1.56	1.089	41.5%
$e = 1$	-1.80	+2.20	2.625	100%

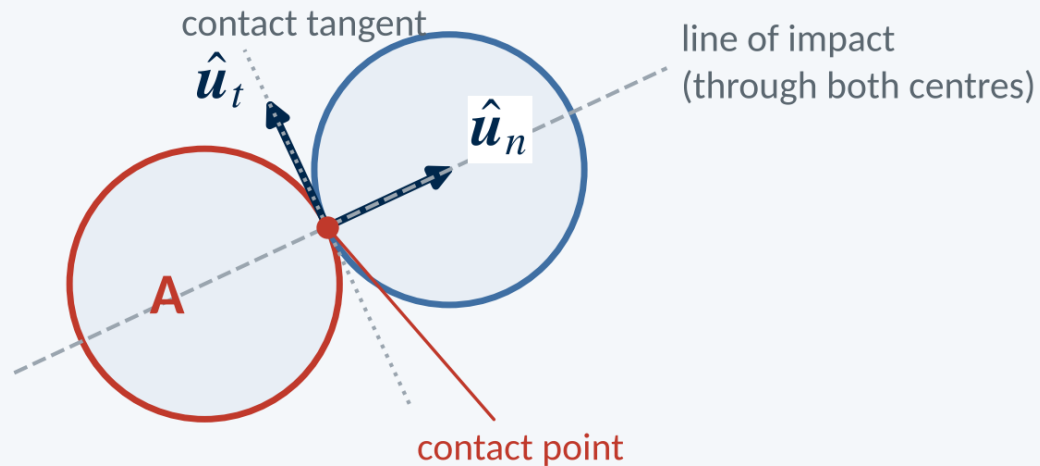
- $e = 0$: zero relative normal separation speed at the end of contact. The bodies need not stay attached; other forces or tangential motion can separate them later.
- $e = 1$: kinetic energy conserved in the ideal smooth particle model. Real impacts also excite rotation, vibration and heating.

THREE PHYSICAL CHECKS

- **Approaching before:** $v_{An,1} - v_{Bn,1} > 0$, or there is no impact.
- **No interpenetration after:** $v_{Bn,2} \geq v_{An,2}$.
- **No energy gain:** $T_2 \leq T_1$ for a passive impact (no stored energy released). In the smooth particle model any $0 \leq e \leq 1$ satisfies this.



Which Velocity Components Can the Contact Change?



THINK

Two pucks on an air table touch at a slant. Which of the four velocity components can the contact force change?

IDEAL SMOOTH CONTACT

- Contact impulse acts along $\hat{u}_n$ only: no tangential impulse.
- So each body's $\hat{u}_t$ component is unchanged.
- Along $\hat{u}_n$: momentum and restitution, exactly as in central impact.
- The force passes through both centres: no torque, spin unchanged.

Here $\hat{u}_n$ lies along the line of impact. It is not the path-coordinate $\hat{u}_n$ of Chapter 2, which points toward a centre of curvature.

Observe

Idealize

Model

Solve / Analyze

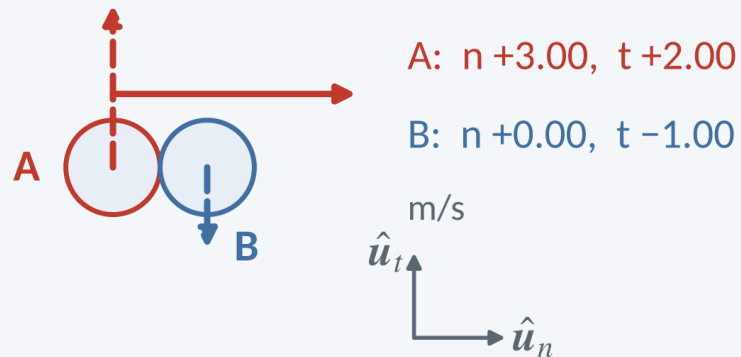
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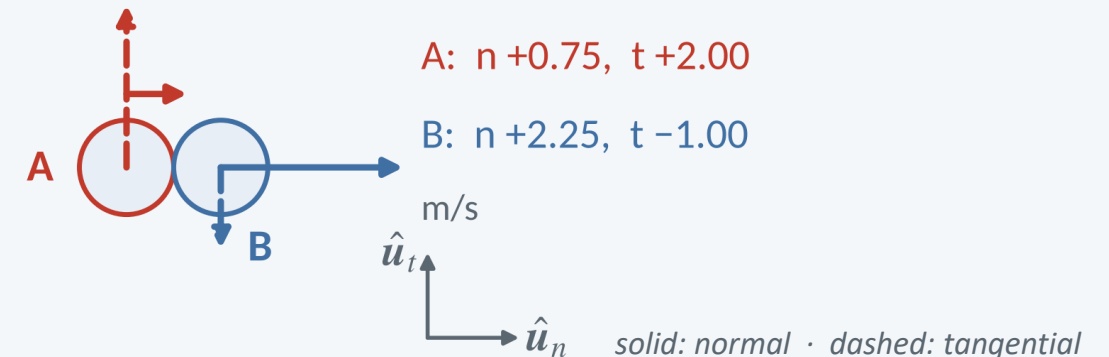


Two 1.00 kg Pucks, $e = 0.50$, Smooth Contact

state 1: just before



state 2: just after



$$n: 3.00 + 0 = v_{An,2} + v_{Bn,2},$$

$$v_{Bn,2} - v_{An,2} = 0.50(3.00) = 1.50$$

$$v_{An,2} = +0.75 \text{ m/s}, \quad v_{Bn,2} = +2.25 \text{ m/s}$$

$$t: v_{At,2} = +2.00, \quad v_{Bt,2} = -1.00 \text{ m/s (unchanged)}$$

If $e = 0$: both normal components 1.50 m/s, yet the t -components (+2.00, -1.00) still carry the pucks apart.

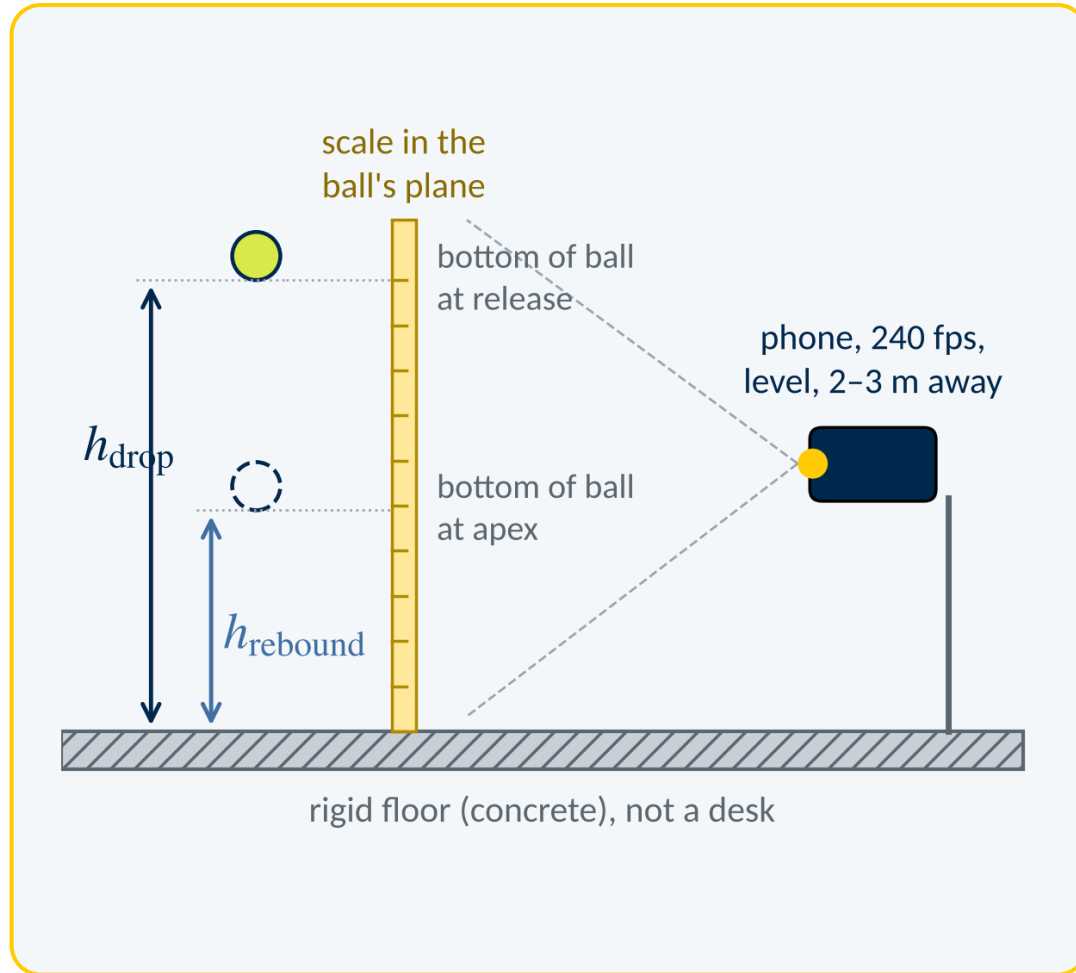
$$\vec{v}_{A,2} = 0.75 \hat{u}_n + 2.00 \hat{u}_t \text{ m/s}$$

$$\vec{v}_{B,2} = 2.25 \hat{u}_n - 1.00 \hat{u}_t \text{ m/s}$$

$T_1 = 7.00 \text{ J}$, $T_2 \approx 5.31 \text{ J}$. The 1.69 J converted comes entirely from the **normal** relative motion.



Rebound Height Gives Restitution



$$v_{sp,1} = \sqrt{2gh_{drop}}, \quad v_{sp,2} = \sqrt{2gh_{rebound}}$$

$$e = \frac{v_{sp,2}}{v_{sp,1}} = \left(\frac{h_{rebound}}{h_{drop}} \right)^{1/2}$$

Illustrative: 1.00 m and 0.49 m $\rightarrow$ 4.43 m/s down, 3.10 m/s up, $e \approx 0.70$.

ASSUMPTIONS AND PROTOCOL

- Floor stays still, so e reduces to a speed ratio; drag, spin negligible.
- Tennis ball (not ping-pong); concrete floor. Bottom of the ball for both heights.
- Camera level and perpendicular; scale in the ball's plane; ≥ 5 drops.
- Result belongs to this ball-floor pair at ≈ 4.4 m/s.



Measurement Uncertainty and Model Limits

Source	Effect on e	Control
Reading the heights	$\approx \pm 0.004$ (± 3 mm on drop, ± 5 mm on rebound)	zoom, fit the apex
Drop-to-drop scatter	expect ± 0.01 – 0.02	≥ 5 drops; mean and spread
Air drag	biases e low, ≈ 1 – 2% (tennis ball)	dense ball; no ping-pong
Ball moves toward or away from camera	scale error that does not cancel	camera 2–3 m away, check vertical drop
Spin, off-vertical bounce	rebound not purely normal	discard those drops
Soft surface	lower e	e belongs to the pair

INDEPENDENT CHECK

- Also fit the speeds just before and after contact from the video.
- e from speeds $\approx e$ from heights: model supported.
- Disagreement points to drag or spin, not automatically to a bad measurement. Measurement uncertainty, parallax, tracking errors, and extrapolation errors are also possible explanations.

Planning estimates, not results. A measured e must be reported with its spread and impact speed.

Observe ›

Idealize ›

Model ›

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Three Plausible but Wrong Explanations



AI CLAIM 1

“Momentum is conserved for the ball alone during impact.”



AI CLAIM 2

“A restitution coefficient of 0.70 means the collision retains 70% of its kinetic energy.”



AI CLAIM 3

“The tangential component also changes according to the coefficient of restitution.”

FAULTY ASSUMPTION

Wrong system. The floor’s push is external to the ball alone; its momentum reverses ($\approx +0.43 \text{ N}\cdot\text{s}$). Conservation holds for ball + Earth.

FAULTY ASSUMPTION

e relates relative normal velocities. A ball on a fixed floor keeps $e^2 = 49\%$; two moving bodies depend on masses too (Example A: 41.5% at $e = 0.60$).

FAULTY ASSUMPTION

Restitution applies along $\hat{u}_n$ only. Smooth contact gives no tangential impulse, so $\hat{u}_t$ components are unchanged.

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Four Tools for Any Impact

1

Momentum conservation

Decided by the system you choose and the external impulse on it.

2

Restitution

Relates the normal relative velocity after impact to that before. Normal direction only.

3

Energy accounting

Compute T from the velocities: kept, converted, never created by a passive impact.

4

Impact force

Needs impulse and contact time; the peak also needs the force history.

Workflow: system → line of impact → momentum along $\hat{u}_n$ → restitution → unchanged $\hat{u}_t$ → energy and force checks.

Next — Part 5: If an impact force acts away from a reference point, can it change something besides linear momentum?

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

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CHAPTER - 5 · PART 4

APPENDIX

Appendix — Supplemental Material

A1 closed-form central impact · A2 kinetic-energy loss · A3 successive rebounds · A4 rotated frame · A5 rough contact (preview) · A6 rebound experiment and uncertainty.

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Solving the Two Equations Once

FROM MOMENTUM + RESTITUTION

$$v_{Ax,2} = \frac{m_A v_{Ax,1} + m_B v_{Bx,1} - m_B e(v_{Ax,1} - v_{Bx,1})}{m_A + m_B}$$

$$v_{Bx,2} = \frac{m_A v_{Ax,1} + m_B v_{Bx,1} + m_A e(v_{Ax,1} - v_{Bx,1})}{m_A + m_B}$$

Check with Example A: $(0.75 - 1.80)/1.25 = -0.84$ m/s and $(0.75 + 1.20)/1.25 = +1.56$ m/s.

CENTRE-OF-MASS FORM

- $v_{Gx} = (m_A v_{Ax,1} + m_B v_{Bx,1}) / m_{\text{sys}}$ is unchanged.
- Relative to G , each velocity reverses and is scaled by e .
- Example A: $v_{Gx} = 0.60$ m/s; A: $0.60 - 1.44 = -0.84$ m/s.

Use the formulas to check work, not to replace it: they hide the sign convention (+ x from A toward B) and the assumption of negligible external impulse. Writing the two equations is faster to verify and harder to misuse.

Observe ›

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How Much Kinetic Energy Is Converted?

SMOOTH IMPACT OF TWO PARTICLES, NORMAL DIRECTION

$$T_1 - T_2 = \frac{1}{2} \frac{m_A m_B}{m_A + m_B} (1 - e^2) (v_{An,1} - v_{Bn,1})^2$$

- Example A: $\frac{1}{2}(0.30)(1 - 0.36)(4.00)^2 = 1.536 \text{ J} \checkmark$
- Example B (normal part): $\frac{1}{2}(0.50)(1 - 0.25)(3.00)^2 = 1.6875 \text{ J} \checkmark$

WHAT IT SHOWS

- Only the relative normal motion loses energy; the centre-of-mass motion keeps its share.
- Fixed floor (m_B very large): the ball keeps e^2 of its kinetic energy.

- The mass factor $m_A m_B / (m_A + m_B)$ is written out in full; we do not give it a Greek symbol, because μ already means a friction coefficient in this course.
- Assumptions: particles, smooth contact, negligible external impulse during contact. In the core, compute T_1 and T_2 directly from the velocities instead.

Observe ›

Idealize ›

Model ›

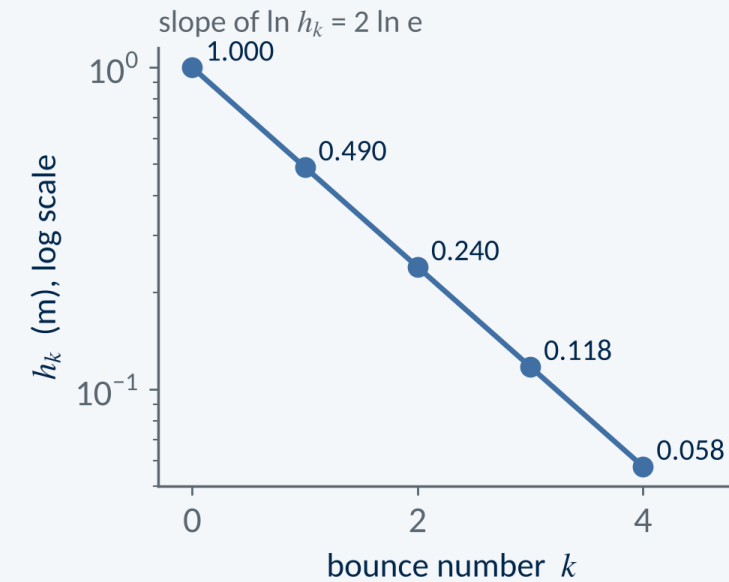
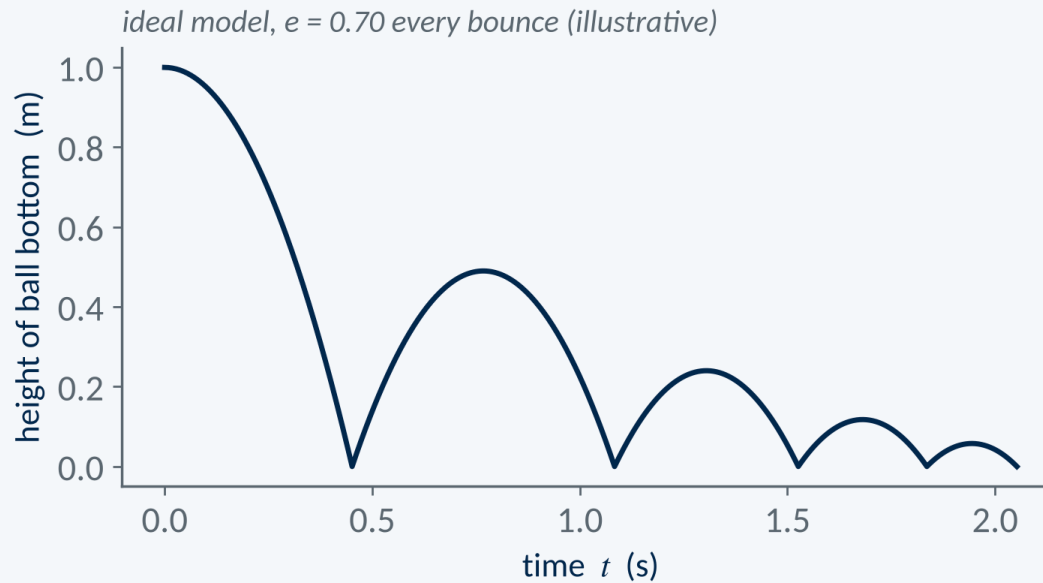
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One Drop, Many Bounces



$$h_{k+1} = e^2 h_k, \quad \ln h_k = \ln h_0 + 2k \ln e$$

Ideal model with constant e .

Homework idea: track five bounces, plot $\ln h_k$ against k , and fit the slope. Curvature in the plot means e changes with impact speed, which real balls often show.

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When the Data Are Given in x and y

- 1 Find the contact normal** $\hat{\mathbf{u}}_n$ along the line of centres, from A toward B; $\hat{\mathbf{u}}_t$ perpendicular to it.
- 2 Project** $v_n = \mathbf{v} \cdot \hat{\mathbf{u}}_n$ and $v_t = \mathbf{v} \cdot \hat{\mathbf{u}}_t$ for each body.
- 3 Solve in n, keep t** Momentum and restitution along $\hat{\mathbf{u}}_n$; tangential components unchanged.
- 4 Transform back** $\mathbf{v}_2 = v_{n,2}\hat{\mathbf{u}}_n + v_{t,2}\hat{\mathbf{u}}_t$, then to x and y.

EXAMPLE B, LINE OF IMPACT AT 30° TO x

- $\hat{\mathbf{u}}_n = (0.866, 0.500)$, $\hat{\mathbf{u}}_t = (-0.500, 0.866)$
- Before: $\mathbf{v}_{A,1} = (1.60, 3.23)$, $\mathbf{v}_{B,1} = (0.50, -0.87)$ m/s
- After: $\mathbf{v}_{A,2} = (-0.35, 2.11)$, $\mathbf{v}_{B,2} = (2.45, 0.26)$ m/s
- Total momentum (2.10, 2.37) kg·m/s before and after ✓

The physics is unchanged by the frame. Choosing $\hat{\mathbf{u}}_n$ and $\hat{\mathbf{u}}_t$ first turns a 2-D problem into a 1-D impact plus two unchanged components.



What Changes When the Contact Has Friction



Tangential impulse

Friction at the contact adds an impulse along $\hat{u}_t$, so tangential components can change.



Spin changes

A tangential force at the surface exerts a torque about each centre: spin enters the problem.



More unknowns

Need a friction model (slip or stick during contact) and rotational dynamics.



Where it is taught

Rigid-body impact belongs with rigid-body kinetics later in the course; not part of this lecture.



Tracker Workflow and Uncertainty

1 Set up

Concrete floor; tennis ball; metre stick in the drop plane; phone on a tripod, level, 2–3 m away.

2 Record

240 fps slow motion; at least five drops from the same marked height. Set the true capture rate in Tracker.

3 Heights

Track the ball's bottom edge; fit a parabola near the apex for h_{rebound} .

4 Speeds

Fit parabolas before and after contact; extrapolate to the contact instant; compare e from speeds.

DATA TABLE (TO BE FILLED IN; NO DATA YET)

Drop	h_{drop} (m)	h_{rebound} (m)	e
1			
2			
3			
4			
5			
mean $\pm$ spread			

$$\frac{\delta e}{e} = \frac{1}{2} \left(\left(\frac{\delta h_{\text{rebound}}}{h_{\text{rebound}}} \right)^2 + \left(\frac{\delta h_{\text{drop}}}{h_{\text{drop}}} \right)^2 \right)^{1/2}$$

e.g. ± 5 mm on 0.49 m and ± 3 mm on 1.00 m give $\delta e \approx 0.004$

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