

**AI-AUGMENTED ENGINEERING DYNAMICS:
A CONVERSATIONAL APPROACH**

Chapter 5 — Momentum Methods for Particles

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

CHAPTER - 5

PART 6 · ≈ 60 MIN

How Do Moments Change Angular Momentum, and When Is It Conserved?

A moment about O changes angular momentum about O. Added up over time, it predicts the change; when it is zero throughout, angular momentum about O stays constant, even while speed and energy change.

Observe

Idealize

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Validate

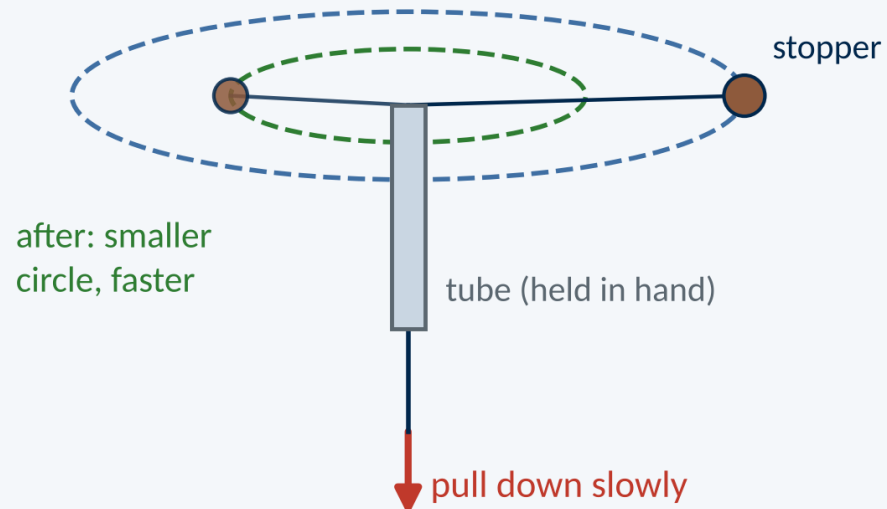
Report



Pull the String In: What Happens to the Motion?

SCHEMATIC · QUALITATIVE

before: larger circle, slower



LIVE DEMO, IF AVAILABLE

- Whirl a rubber stopper on a string threaded through a short tube.
- Slowly pull the string down through the tube.
- Watch: does the stopper go around faster or slower? Does your hand do any work?

The demo is 3-D and qualitative (gravity tilts the string). **Today's model:** an idealized puck on a smooth horizontal table, string through a hole at O.

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Pull the String In. Then What?

PREDICTION 1

Angular momentum about the hole
O ...

- A** increases
- B** stays the same
- C** decreases

Class vote: A: B: C:

Test: slide 12 (we will come back)

PREDICTION 2

The puck's speed ...

- A** increases
- B** stays the same
- C** decreases

Class vote: A: B: C:

Test: slides 13–14 (we will come back)

PREDICTION 3

The puck's kinetic energy ...

- A** increases
- B** stays the same
- C** decreases

Class vote: A: B: C:

Test: slide 14 (we will come back)

Observe

Idealize

Model

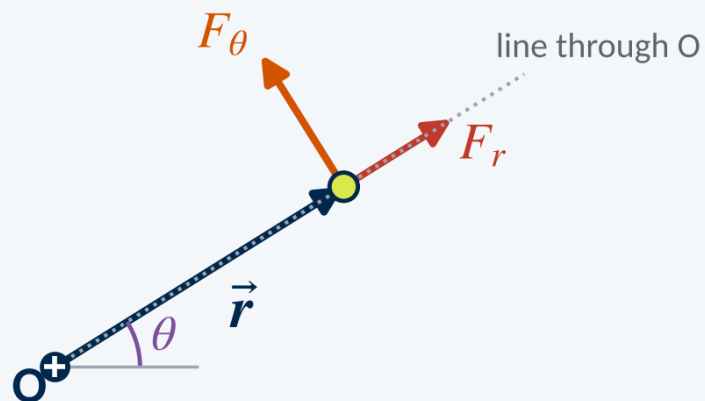
Solve / Analyze

Validate

Report



Moment About O Sets the Rate of Change



F_r : zero moment about O

F_θ : moment rF_θ

⊙ CCW positive, $\hat{u}_z$ out of the screen

PARTICLE, O FIXED IN AN INERTIAL FRAME, CONSTANT m

$$\Sigma M_{Oz} = \frac{dH_{Oz}}{dt}$$

$$H_{Oz} = mrv_\theta = mr^2\dot{\theta}$$

$$M_{Oz} = rF_\theta \quad (\text{only the transverse part of a force})$$

- Counterclockwise about O is positive ($\hat{u}_z$ out of the screen).
- A force whose line of action passes through O has $M_{Oz} = 0$ at that instant.

Part 5 answered **how fast** H_{Oz} changes at one instant. Today: **how much** it changes over an interval.

Observe

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Angular Impulse–Momentum About O

FROM RATE TO CHANGE, t_1 TO t_2

$$\Sigma M_{Oz} dt = dH_{Oz}$$

$$\int_{t_1}^{t_2} \Sigma M_{Oz} dt = H_{Oz,2} - H_{Oz,1}$$

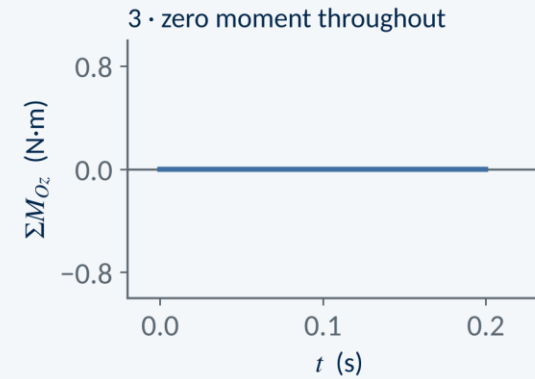
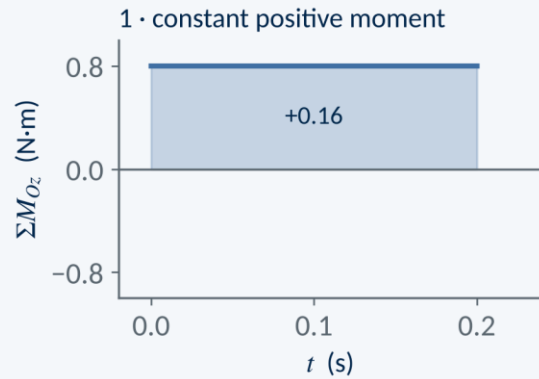
$$H_{Oz,1} + \int_{t_1}^{t_2} \Sigma M_{Oz} dt = H_{Oz,2}$$

	Linear (Part 2)	Angular, about fixed O
Momentum	$\vec{L} = m\vec{v}$	$\vec{H}_O = \vec{r} \times m\vec{v}$
Accumulates	impulse $\vec{I}_{1 \rightarrow 2} = \int \Sigma \vec{F} dt$	angular impulse $\int \Sigma \vec{M}_O dt$
Balance	$\vec{L}_1 + \vec{I}_{1 \rightarrow 2} = \vec{L}_2$	$\vec{H}_{O,1} + \int \Sigma \vec{M}_O dt = \vec{H}_{O,2}$
Units	N·s = kg·m/s	N·m·s = kg·m ² /s

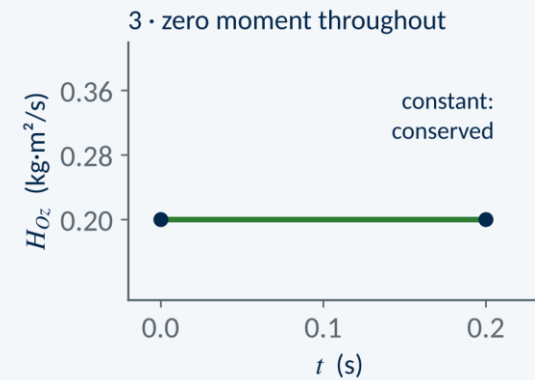
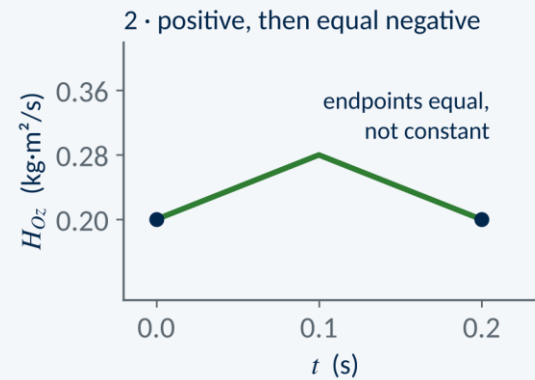
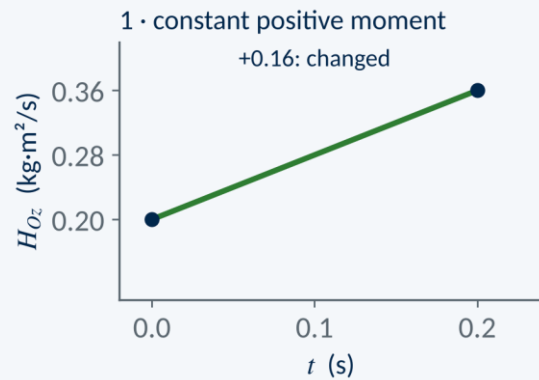
- **Angular impulse about O** is the moment integral itself; it gets no new symbol. Planar motion: only the z-component.
- It is the signed area under the ΣM_{Oz} – t graph: counterclockwise moments add, clockwise moments subtract.
- In general it is not $\vec{r} \times \vec{I}$: $\vec{r}$ changes while the force acts (a brief impact is the exception).



Equal Endpoints or Conserved Throughout?



- Each starts at $H_{O_z} = 0.20 \text{ kg}\cdot\text{m}^2/\text{s}$.
- **Sketch** $H_{O_z}(t)$ for each.
- Which are conserved throughout?



ANSWER

- 1: +0.16, changed.
- 2: endpoints equal, **not** conserved.
- 3: conserved.

Observe

Idealize

Model

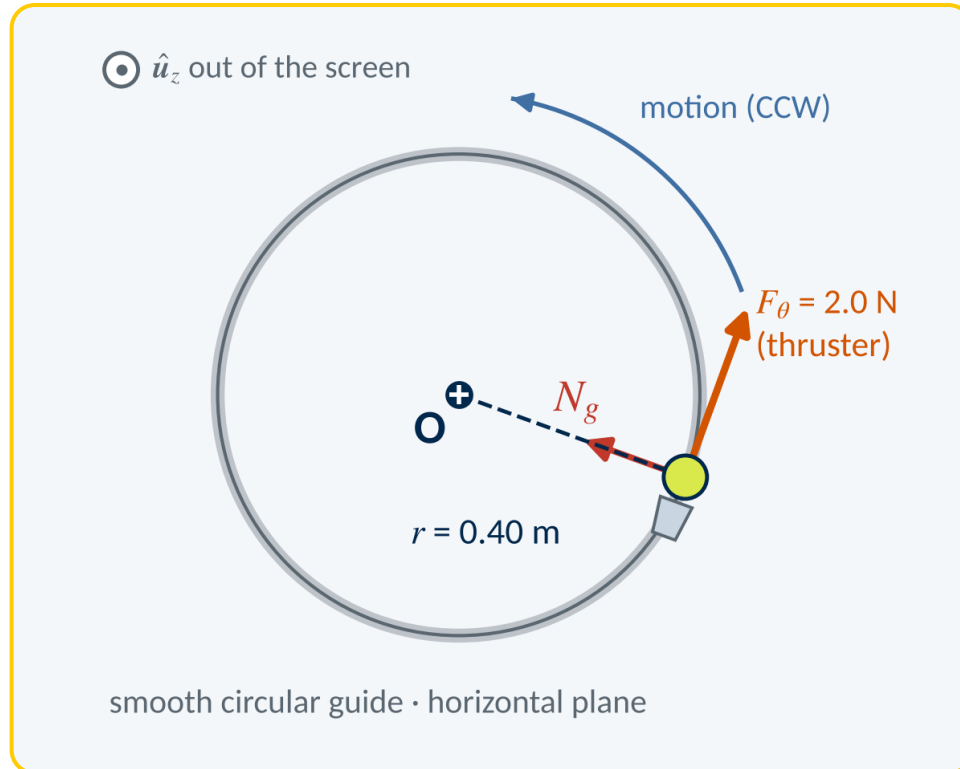
Solve / Analyze

Validate

Report



A Thruster on a Circular Guide



- Idealized data: $m = 0.50$ kg, $r = 0.40$ m, $v_{\theta,1} = +1.0$ m/s.
- Thruster: constant $F_{\theta} = +2.0$ N for $\Delta t = 0.20$ s. Find $H_{Oz,2}$ and $v_{\theta,2}$.

$$\Sigma M_{Oz} = rF_{\theta} = (0.40)(2.0) = +0.80 \text{ N} \cdot \text{m}$$

$$\int_{t_1}^{t_2} \Sigma M_{Oz} dt = (0.80)(0.20) = +0.16 \text{ N} \cdot \text{m} \cdot \text{s}$$

$$H_{Oz,1} = mr v_{\theta,1} = (0.50)(0.40)(1.0) = +0.20 \text{ kg} \cdot \text{m}^2/\text{s}$$

$$H_{Oz,2} = 0.20 + 0.16 = +0.36 \text{ kg} \cdot \text{m}^2/\text{s}$$

$$v_{\theta,2} = \frac{H_{Oz,2}}{mr} = 0.36/0.20 = +1.8 \text{ m/s}$$

Idealized: horizontal plane; smooth guide, so its reaction N_g is radial (zero moment); weight and vertical support have no z-moment; propellant mass neglected (constant m).



Work–Energy Agrees; Linear Impulse Would Struggle

INDEPENDENT CHECK · WORK–ENERGY

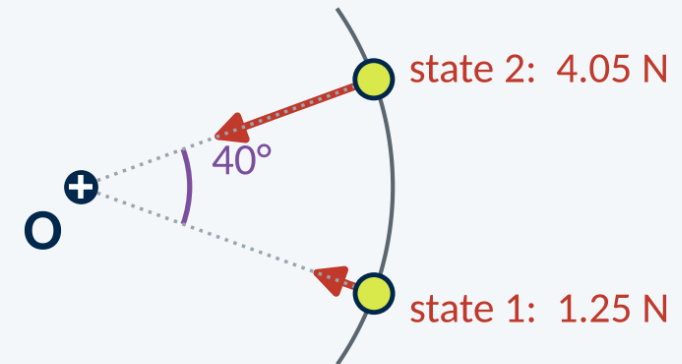
$$a_{\theta} = F_{\theta}/m = 4.0 \text{ m/s}^2 \text{ (along the path; } r \text{ constant)}$$

$$s = (1.0)(0.20) + \frac{1}{2}(4.0)(0.20)^2 = 0.28 \text{ m}$$

$$W = F_{\theta}s = (2.0)(0.28) = 0.56 \text{ J}$$

$$\Delta T = \frac{1}{2}m(v_{\theta,2}^2 - v_{\theta,1}^2) = 0.56 \text{ J } \checkmark$$

- The guide reaction is perpendicular to the velocity: it does no work.
- $\Delta v_{\theta} = a_{\theta}\Delta t = 0.80 \text{ m/s}$ is the same equation $\div r$: a consistency check, not independent.



$N_g = mv_{\theta}^2/r$: size and direction change
its moment about O is zero throughout

Why use angular impulse? Linear impulse needs $\int N_g^{\rightarrow} dt$ of a force that grows $1.25 \rightarrow 4.05 \text{ N}$ and turns through 40° . About O its moment is zero at every instant: it drops out.



Conditions for Conservation About O

$$H_{Oz,1} + \int_{t_1}^{t_2} \Sigma M_{Oz} dt = H_{Oz,2}$$

Your turn: list every way the moment integral could be zero. Then decide: conserved throughout, or only equal at the endpoints?

CONSERVED THROUGHOUT

$\Sigma M_{Oz} = 0$ at every instant:

- No force at all (Part 5, Example A).
- Every force's line of action passes through O (central forces).
- Forces parallel to $\hat{u}_z$: weight and table support on a horizontal table.
- Nonzero moments that cancel at every instant.

EQUAL ENDPOINTS ONLY: NET AREA ZERO

- Moments act, but their signed areas cancel over $[t_1, t_2]$ (graph 2).
- Then $H_{Oz,2} = H_{Oz,1}$, but H_{Oz} changed in between.
- Not conservation: do not use it to predict intermediate states.

Fine print: O fixed in an inertial frame, the same O throughout, constant m , and every force counted.

Observe ›

Idealize ›

Model ›

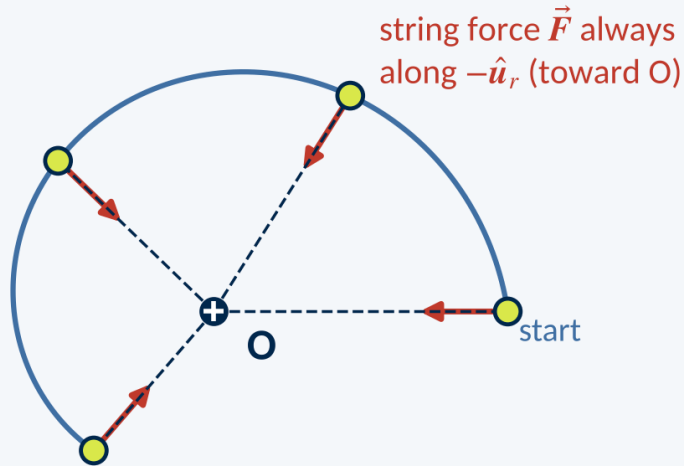
Solve / Analyze ›

Validate ›

Report



A Force Through O Has No Moment About O



$$\vec{r} \times \vec{F} = \vec{0} \text{ at every instant}$$

CENTRAL FORCE ABOUT O

Line of action always passes through the fixed point O (toward it or away from it).

$$\vec{F} \parallel \vec{r} \Rightarrow \vec{M}_O = \vec{r} \times \vec{F} = \vec{0}$$

CAN CHANGE

- direction of $\vec{v}$
- $\vec{L}$ (it is a net force)
- v_r , speed, T

CANNOT CHANGE

- $\vec{H}_O$ about that same O
- (other points: slide 15)

Prediction 1, resolved: in the ideal model, angular momentum about the hole **stays the same** while the string is pulled.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

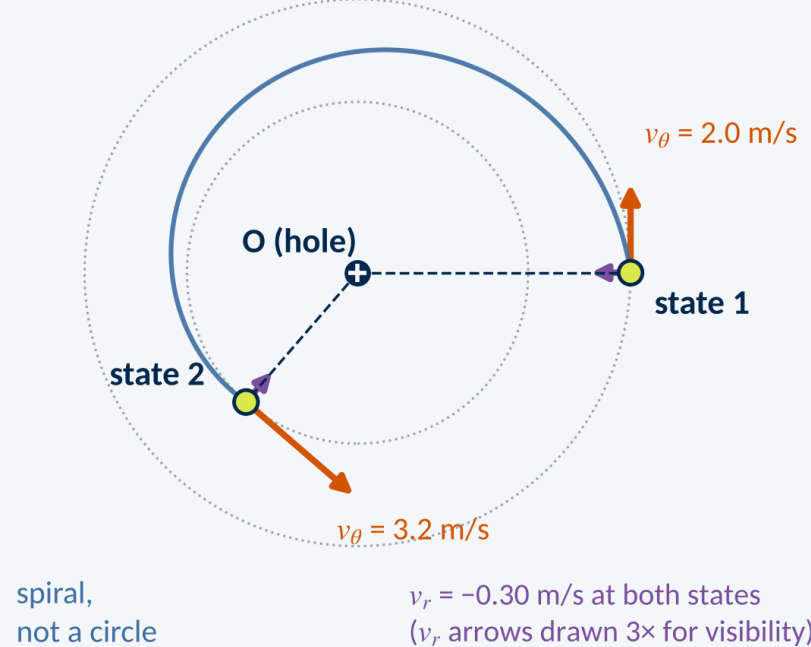
Validate ›

Report



Pulling the Puck Inward

top view · r : 0.80 m → 0.50 m · swept 4.0 rad
string force: along each dashed line, toward O



IDEALIZE

- Puck as a particle, $m = 0.60$ kg; smooth horizontal table.
- Massless, inextensible string through a hole at fixed O.
- State 1: $r_1 = 0.80$ m, $v_{\theta,1} = +2.0$ m/s. State 2: $r_2 = 0.50$ m.

$$H_{Oz} = mr_1 v_{\theta,1} = (0.60)(0.80)(2.0) = +0.96 \text{ kg} \cdot \text{m}^2/\text{s}$$

$$v_{\theta,2} = \frac{H_{Oz}}{mr_2} = \frac{0.96}{(0.60)(0.50)} = +3.2 \text{ m/s}$$

$$\dot{\theta}_1 = v_{\theta,1}/r_1 = 2.5 \text{ rad/s}, \quad \dot{\theta}_2 = v_{\theta,2}/r_2 = 6.4 \text{ rad/s}$$

$$\text{Check: } (0.60)(0.50)(3.2) = 0.96 \text{ kg} \cdot \text{m}^2/\text{s}.$$

H_{Oz} fixes v_{θ} only. The radial component v_r is set by how the string is pulled: here steadily, $v_r = -0.30$ m/s throughout.

Observe

Idealize

Model

Solve / Analyze

Validate

Report



Angular Momentum Constant, Kinetic Energy Up 1.87 J

Quantity	State 1	State 2	Constant?
r (m)	0.80	0.50	no
v_r (m/s)	-0.30	-0.30	yes (prescribed)
v_θ (m/s)	+2.0	+3.2	no
v_{sp} (m/s)	2.02	3.21	no
H_{Oz} (kg·m ² /s)	+0.96	+0.96	yes
T (J)	1.23	3.10	no: +1.87 J

Idealized model values; $v_{sp} = \sqrt{v_r^2 + v_\theta^2}$.

KINETIC ENERGY

$$T_1 = \frac{1}{2}(0.60)(0.30^2 + 2.0^2) = 1.23 \text{ J}$$

$$T_2 = \frac{1}{2}(0.60)(0.30^2 + 3.2^2) = 3.10 \text{ J}$$

$$\Delta T = +1.87 \text{ J}$$

Supplied by the hand, through the string.

- String force inward, puck moves inward: positive work.
- Computed: $\int (\text{string force})(\text{pull speed}) dt = 1.87 \text{ J} \checkmark$

Central is not conservative. Zero moment about O says nothing about energy. **Predictions 2 and 3, resolved:** speed and kinetic energy both increase.

Observe ›

Idealize ›

Model ›

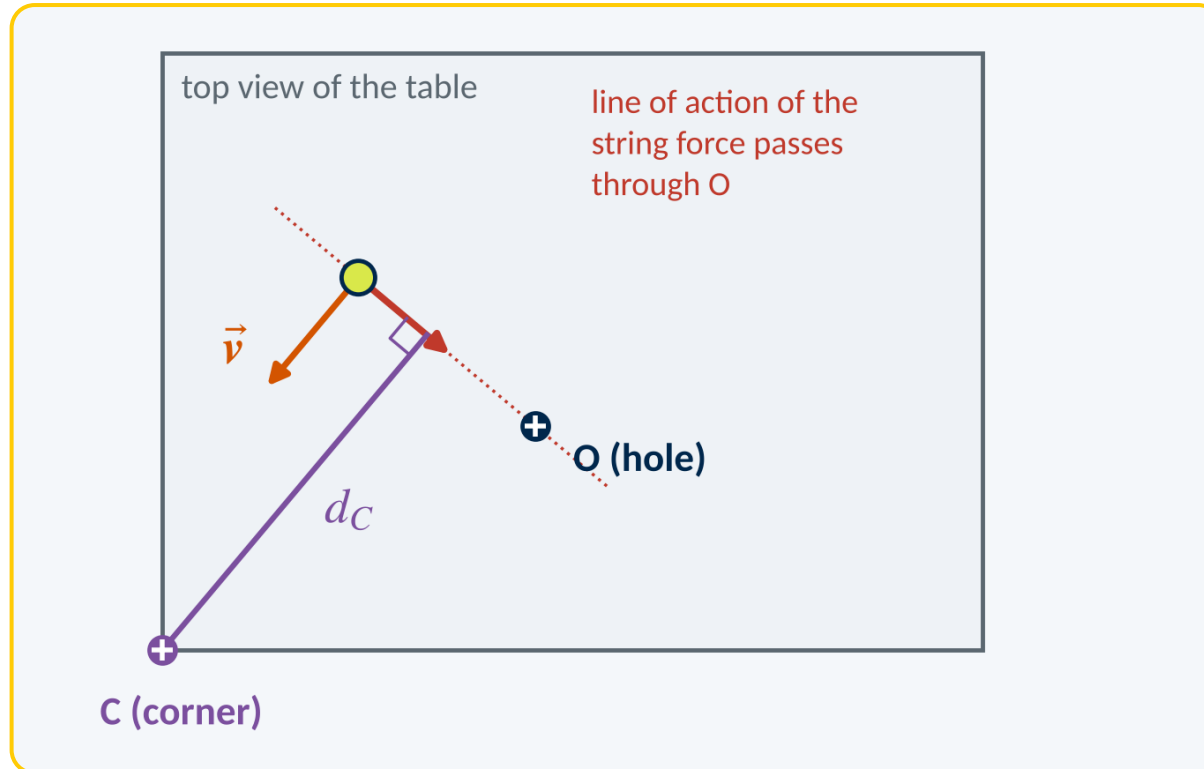
Solve / Analyze ›

Validate ›

Report



Same Puck, Same Motion, Two Reference Points



- Is the puck's angular momentum conserved about the hole O?
- About the fixed table corner C?

ANSWER

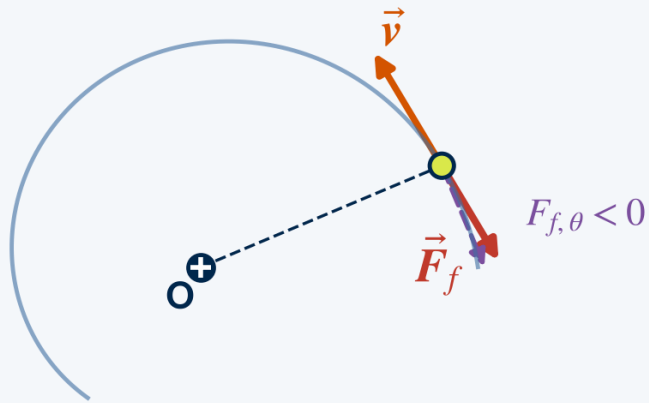
- **About O: yes.** The string force passes through O, so $M_{Oz} = 0$.
- **About C: no.** Its moment arm $d_C \neq 0$, so $M_{Cz} \neq 0$. Along the steady-pull path, H_{Cz} goes from +2.27 to $-1.10 \text{ kg}\cdot\text{m}^2/\text{s}$.

Conservation is a statement **about a point**. Choose O first, then audit every moment about that O.



Table Friction Has a Moment About O

⊙ motion counterclockwise



kinetic friction opposes $\vec{v}$, so its transverse part opposes v_θ : $M_{Oz} = rF_{f,\theta} < 0$

ASSUMPTIONS

- Kinetic friction of size $\mu_k N$, directed opposite $\vec{v}$.
- Counterclockwise motion: its transverse part opposes v_θ .

$$M_{Oz} = rF_{f,\theta} < 0 \Rightarrow H_{Oz} \text{ decreases}$$

- Illustrative $\mu_k = 0.05$ during the 1.0 s pull: H_{Oz} falls from 0.96 to about 0.77 kg·m²/s (computed).
- So $v_{\theta,2} \approx 2.6$ m/s, not 3.2 m/s.

Conservation is a **model**: good when the friction moment's angular impulse is small compared with H_{Oz} .

Observe

Idealize

Model

Solve / Analyze

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Three Confident Claims About Conservation



AI CLAIM 1

"A nonzero force always changes a particle's angular momentum."

WHAT IS WRONG

Wrong. Only a moment about O changes H_O . The string force is nonzero and grows as the puck moves in, yet H_{Oz} stays 0.96 kg·m²/s: its line of action passes through O.



AI CLAIM 2

"If angular momentum is conserved about the hole, it must also be conserved about the table corner."

WHAT IS WRONG

Wrong. Conservation holds about a point. About C the string force has a moment arm d_C : H_{Cz} goes from +2.27 to -1.10 kg·m²/s.



AI CLAIM 3

"If angular momentum is conserved, kinetic energy must also be conserved."

WHAT IS WRONG

Wrong. In Example B, H_{Oz} is constant while T rises 1.87 J: the hand does positive work through the string. Central is not conservative.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



Two Impulse–Momentum Balances, One Choice

LINEAR IMPULSE–MOMENTUM

$$\vec{L}_1 + \vec{I}_{1 \rightarrow 2}^{(\text{ext})} = \vec{L}_2$$

ANGULAR IMPULSE–MOMENTUM ABOUT FIXED O

$$\vec{H}_{O,1} + \int_{t_1}^{t_2} \Sigma \vec{M}_O dt = \vec{H}_{O,2}$$

CHOOSING A METHOD

- What quantity do we need?
- Which forces or moments are unknown?
- Can a reference point eliminate an unknown force?
- What assumptions does the method need?
- How can we validate the result independently?
- Remember: conserved throughout $\neq$ equal endpoints.

Next — Part 7: For the puck on the string, angular momentum gives the final transverse velocity in one line. Which method would we need to find the string tension at state 2, or the work done by the hand?

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report

CHAPTER - 5 · PART 6

APPENDIX

Appendix — Supplemental Material

A1 vector form and components · A2 variable moment histories · A3 friction and decay · A4 numerical check · A5 reference points and measurement · A6 the taut-string condition. Not part of the 56-minute core.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Components Can Be Conserved Separately

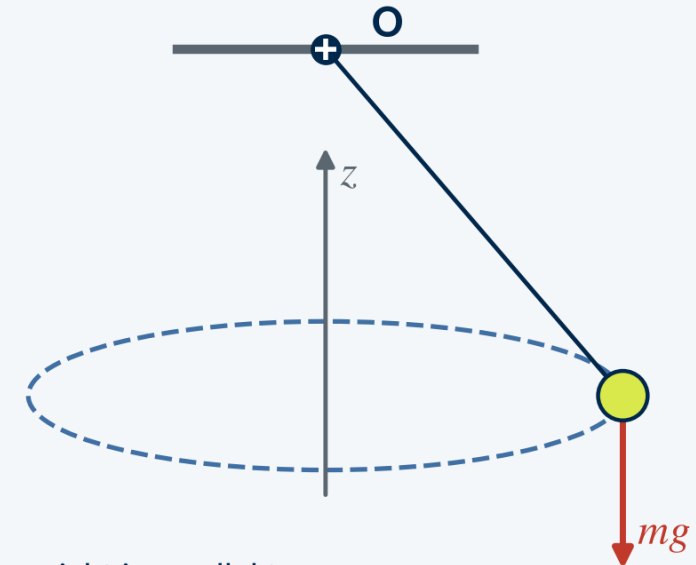
THREE-DIMENSIONAL FORM, O FIXED

$$\vec{H}_{O,1} + \int_{t_1}^{t_2} \Sigma \vec{M}_O dt = \vec{H}_{O,2}$$

- One balance per component: x , y and z separately.
- A component is conserved when the matching component of $\Sigma \vec{M}_O$ is zero at every instant, even if the others are not.

EXAMPLE: CONICAL PENDULUM, O AT THE SUPPORT

- String force passes through O: no moment about O.
- Weight is parallel to z : no z -moment, so H_{Oz} is constant.
- Weight does have x - and y -moments about O: the horizontal part of $\vec{H}_O$ turns with the bob.



weight is parallel to z :
no moment about the z -axis

This is also why the tilted stopper demo still has a conserved vertical component (ideally).

Observe

Idealize

Model

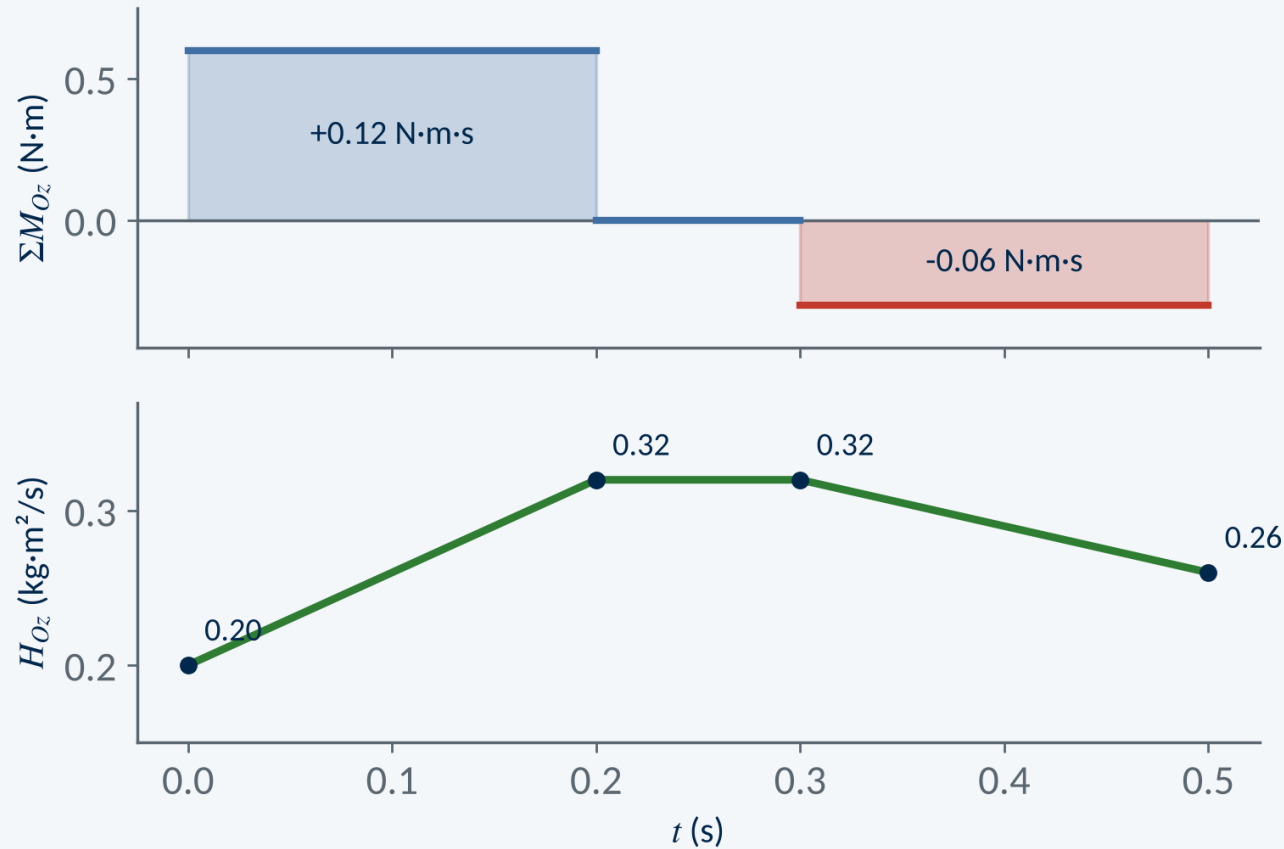
Solve / Analyze

Validate

Report



Angular Impulse as a Signed Running Area



WORKED (IDEALIZED)

- $H_{O_z} = 0.20 \text{ kg}\cdot\text{m}^2/\text{s}$ at $t = 0$.
- $0-0.2 \text{ s}$: $+0.60 \text{ N}\cdot\text{m} \rightarrow +0.12$; $H_{O_z} = 0.32$.
- $0.2-0.3 \text{ s}$: no moment $\rightarrow$ stays 0.32 .
- $0.3-0.5 \text{ s}$: $-0.30 \text{ N}\cdot\text{m} \rightarrow -0.06$; $H_{O_z} = 0.26$.

- Net angular impulse $+0.06 \text{ N}\cdot\text{m}\cdot\text{s}$ over 0.5 s .
- Conserved only on $0.2-0.3 \text{ s}$, where the moment is zero.
- Try: what constant moment on $0.3-0.5 \text{ s}$ would bring H_{O_z} back to 0.20 ?

Observe

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How Fast Does a Rough Table Remove H?

CONSTANT RADIUS (STRING HELD), ILLUSTRATIVE

$m = 0.60 \text{ kg}$, $r = 0.50 \text{ m}$, $v_\theta = 3.2 \text{ m/s}$, illustrative $\mu_k = 0.05$, $N = mg$.

$$M_{Oz} = -r\mu_k mg = -(0.50)(0.05)(0.60)(9.81) = -0.147 \text{ N} \cdot \text{m}$$

after 1.0 s: $H_{Oz} = 0.96 - 0.147 = 0.81 \text{ kg} \cdot \text{m}^2/\text{s}$ (−15%)

Velocity is purely transverse here, so the whole friction force has a moment about O.

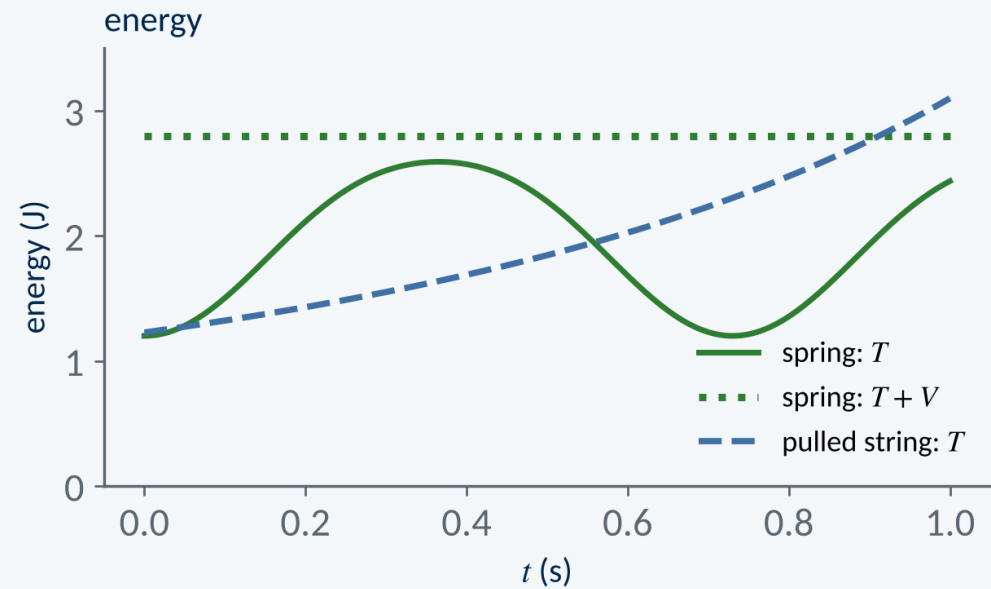
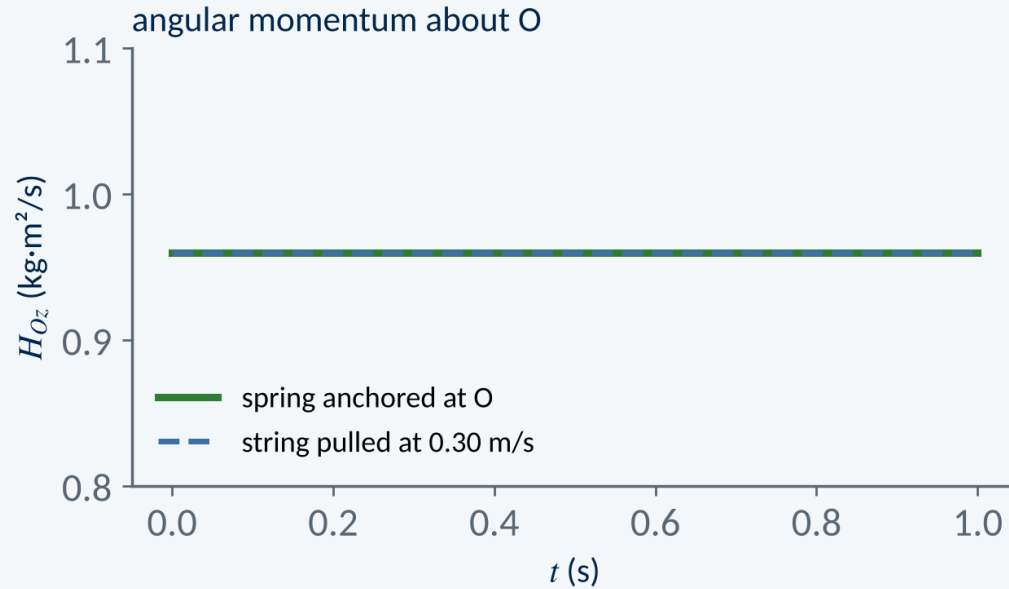
DURING THE STEADY PULL (COMPUTED)

- Same μ_k , r from 0.80 to 0.50 m in 1.0 s.
- $H_{Oz,2} \approx 0.77 \text{ kg} \cdot \text{m}^2/\text{s}$ instead of 0.96.
- $v_{\theta,2} \approx 2.6 \text{ m/s}$ instead of 3.2 m/s.
- Numerical integration of the idealized model.

- **When is conservation a good model?** For under 1% loss in 1.0 s at constant radius, need $\mu_k g(1.0 \text{ s})/v_\theta < 0.01$, so μ_k below about 0.003.
- Question: why does the fractional loss grow when the puck moves faster? Or does it?



Spring Anchored at O Versus Hand-Pulled String



- **Spring at O** ($k = 20$ N/m, natural length 0.40 m; start 0.80 m, $v_\theta = 2.0$ m/s, $v_r = 0$): central **and** conservative.
- $H_{O_z} = 0.96$ constant; T varies; $T + V = 2.80$ J constant.

- **Hand-pulled string** ($v_r = -0.30$ m/s): central, **not** conservative.
- $H_{O_z} = 0.96$ constant; T rises 1.23 $\rightarrow$ 3.10 J; no potential energy accounts for it.
- Computed from idealized models, not measured.

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Angular Momentum About O Versus About C

TRANSFORMATION (PART 5, APPENDIX A5)

$$\vec{H}_C = \vec{H}_O - \vec{r}_C \times \vec{L}$$

C fixed at $(-1.0, -0.6)$ m from O. Steady-pull model, states 1 and 2 of Example B.

	$H_{Oz} \text{ (kg}\cdot\text{m}^2/\text{s)}$	$H_{Cz} \text{ (kg}\cdot\text{m}^2/\text{s)}$
State 1	+0.96	+2.27
State 2	+0.96	-1.10

About C the string force has a moment, so H_{Cz} changes, even sign.

IF YOU MEASURE IT (VIDEO TRACKING, ILLUSTRATIVE UNCERTAINTIES)

- Uncertainties assumed: 5 mm in r , 0.05 m/s in v_θ . Then $H_{Oz} = mrv_\theta$ is uncertain by about 2.6% at state 1 and 1.9% at state 2.
- A measured change smaller than about 3% cannot distinguish 'conserved' from 'slowly decaying'.
- Locate the hole carefully: tracking about the wrong point mixes in a moment about that point.

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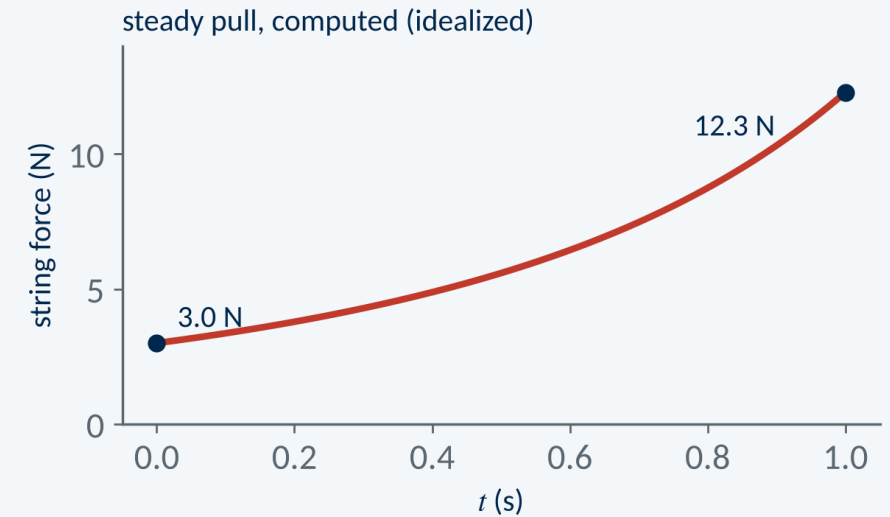
A String Can Pull but Cannot Push

RADIAL EQUATION, STRING FORCE TOWARD O

$$-F_{\text{str}} = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_{\text{str}} = m(r\dot{\theta}^2 - \ddot{r}) \geq 0$$

Taut only while $d^2r/dt^2 \leq r(d\theta/dt)^2$. F_{str} is the string force (not T , which is kinetic energy).



- **Steady pull** ($d^2r/dt^2 = 0$): $F_{\text{str}} = mv_\theta^2/r$ rises from 3.0 N to 12.3 N, always positive: the string stays taut.
- **Let go** ($F_{\text{str}} = 0$): no horizontal force, so the puck moves in a straight line, and H_{Oz} is still $0.96 \text{ kg}\cdot\text{m}^2/\text{s}$.
- **Limit**: the string can slow an inward rush only while the outward d^2r/dt^2 it needs stays below $r(d\theta/dt)^2$. Beyond that it would have to push, which a string cannot do.