

**AI-AUGMENTED ENGINEERING DYNAMICS:
A CONVERSATIONAL APPROACH**

Chapter 5 — Momentum Methods for Particles

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Observe



Idealize



Model



Solve / Analyze



Validate



Report

The engineering workflow you will use throughout this book

CHAPTER - 5

PART 7 · ≈ 60 MIN

Which Dynamics Method Should We Choose?

Newton, work–energy, linear and angular impulse–momentum: each answers a different question. Engineers choose, combine and check them according to the unknown, the information and the assumptions.

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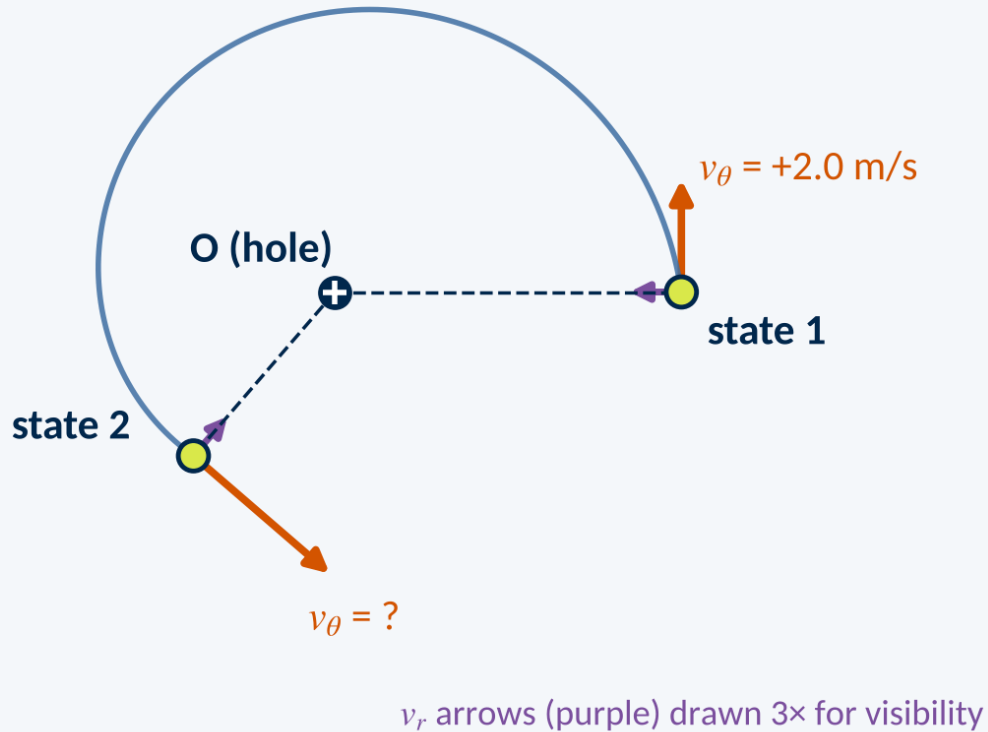
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Back to the Part 6 Puck

top view · smooth horizontal table · $v_r = -0.30$ m/s
string through a frictionless hole, pulled from below



- Idealized: $m = 0.60$ kg; r from 0.80 m to 0.50 m; $v_{\theta,1} = +2.0$ m/s.
- String pulled steadily: $v_r = -0.30$ m/s throughout.

1

What is the final transverse velocity $v_{\theta,2}$?

2

What is the string tension at the instant of state 2?

3

How much work does the hand do during the pull?

4

What if the puck gets a brief transverse impulse of +0.12 N·s at state 2?



Which Method for Each Question, and Why?

QUESTION 1

final $v_{\theta,2}$

- A** Newton's second law
- B** Work–energy
- C** Linear impulse–momentum
- D** Angular impulse–momentum

Votes: A B C D
Unknown it eliminates?
Checked on slide 7

QUESTION 2

tension at state 2

- A** Newton's second law
- B** Work–energy
- C** Linear impulse–momentum
- D** Angular impulse–momentum

Votes: A B C D
Unknown it eliminates?
Checked on slide 7

QUESTION 3

work by the hand

- A** Newton's second law
- B** Work–energy
- C** Linear impulse–momentum
- D** Angular impulse–momentum

Votes: A B C D
Unknown it eliminates?
Checked on slide 7

QUESTION 4

brief impulse

- A** Newton's second law
- B** Work–energy
- C** Linear impulse–momentum
- D** Angular impulse–momentum

Votes: A B C D
Unknown it eliminates?
Checked on slide 8

Observe

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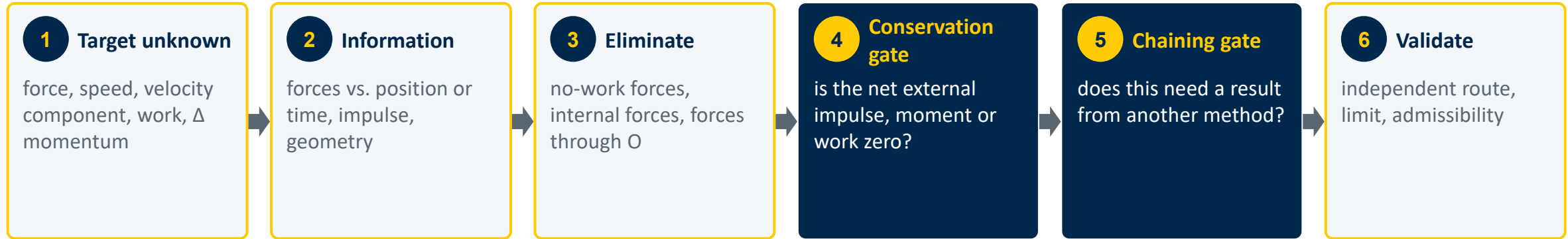
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Choosing and Combining Methods



Method	Typical target	Unknown it can eliminate
Newton's second law	force or acceleration at an instant	forces perpendicular to the chosen component
Work–energy	speed after a displacement, or the work done	forces that do no work (normal forces, smooth guides)
Linear impulse–momentum	velocity after an interval	internal forces, by choosing the system
Angular impulse–momentum about O	transverse velocity; quantities about O	every force whose line of action passes through O

A guide to try, not a rule to always follow: the best method is the one whose eliminated unknown and required information fit the question.





Angular Momentum First, Then Newton and Energy

1 · ANGULAR MOMENTUM

String force passes through O: H_{Oz} constant.

$$v_{\theta,2} = \frac{r_1 v_{\theta,1}}{r_2}$$

$$= (0.80)(2.0)/0.50 = 3.2 \text{ m/s}$$

2a · RADIAL NEWTON AT STATE 2 (uses $v_{\theta,2}$ from step 1)

$$F_{\text{str},2} = \frac{mv_{\theta,2}^2}{r_2} = \frac{(0.60)(3.2)^2}{0.50} = 12.3 \text{ N} \quad (\ddot{r} = 0)$$

2b · WORK-ENERGY, 1 → 2 (uses $v_{\theta,2}$ from step 1)

$$W_{\text{str},1 \rightarrow 2} = T_2 - T_1 = \frac{1}{2}(0.60)[(3.2^2 + 0.30^2) - (2.0^2 + 0.30^2)] = 1.87 \text{ J}$$

- Massless string, frictionless hole: the string's work on the puck is the work supplied by the hand.
- Instantaneous power at state 2: $P_2 = F_{\text{str},2}(0.30 \text{ m/s}) = 3.69 \text{ W}$, a rate; 1.87 J is the total over the pull.

Votes 1–3: angular momentum (Q1); Newton, after Q1 (Q2); work–energy used backwards, after Q1 (Q3).

Observe ›

Idealize ›

Model ›

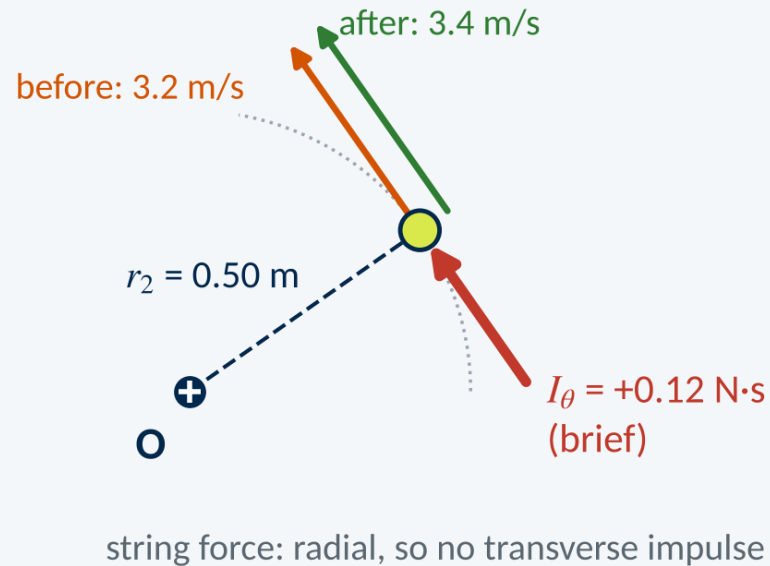
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A Brief Transverse Impulse at State 2



ANGULAR, ABOUT O

$$\Delta H_{Oz} = r_2 I_\theta = +0.060$$

$$H_{Oz}: 0.96 \rightarrow 1.02 \text{ kg} \cdot \text{m}^2/\text{s}$$

$$v_\theta = \frac{1.02}{mr_2}$$

$$= 1.02/0.30 = 3.4 \text{ m/s}$$

LINEAR, TRANSVERSE

$$\Delta v_\theta = \frac{I_\theta}{m} = \frac{0.12}{0.60}$$

$$= 0.20 \text{ m/s}$$

$$v_\theta = 3.2 + 0.20 = 3.4 \text{ m/s}$$

- **A tie:** both are valid because the impulse is brief, so r stays 0.50 m.
- The string is radial: it can supply no transverse impulse.
- Note: The linear transverse impulse equation is appropriate for the stipulated instantaneous impulse. For a finite-duration force during curvilinear motion, the transverse basis rotates, so a scalar component impulse equation generally needs additional care. The current explanation could leave an impression only a changing radius matters, but that may not be the whole physics.

Vote 4: C and D are both right. A tie is a result, not a failure: pick the one you can check fastest.

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What Can Each Method NOT Tell Us?

Newton's second law

Can it give the puck's final v_θ by itself?

Not without integrating the motion over time. It gives forces and accelerations at an instant; accumulated changes need integration.

Work–energy

Can it give the direction of the puck's velocity?

No. Energy is a scalar: it gives speed, not direction, and nothing about time.

Linear impulse–momentum

Can it give velocity without knowing the impulse?

Only if the impulse is known, negligible, or internal to a well-chosen system.

Angular impulse–momentum

Can it give the puck's v_r , or the tension?

No. About θ it gives v_θ only; the string force drops out, so it cannot be found this way.

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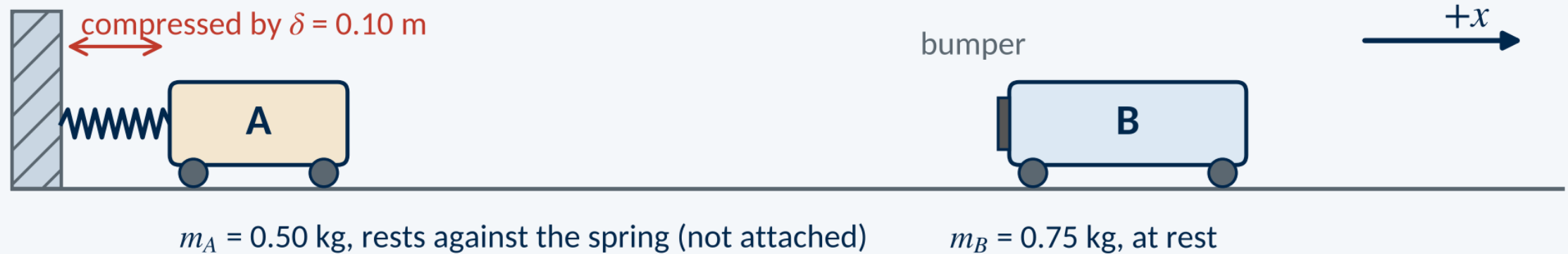
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Where Does One Phase End and the Next Begin?

spring $k = 200 \text{ N/m}$ fixed to the wall · smooth horizontal track · A-B impact $e = 0.50$ · IDEALIZED

**1**

Identify the phases. Where does each begin and end?

2

In each phase: is the linear momentum of A conserved? Of A + B?

3

Which method for each phase, and which unknown does it eliminate?

Observe

Idealize

Model

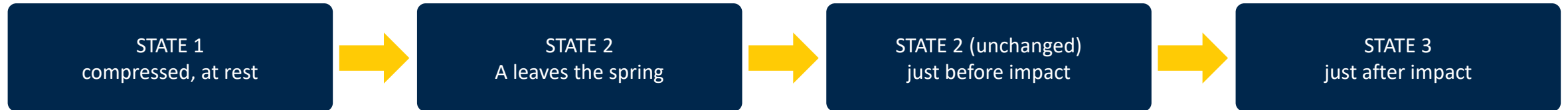
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Launch, Coast, Impact: Three Phases, Three States



LAUNCH · 1 → 2

- **Work–energy.** Normal force and weight do no work.
- **Momentum of A: NOT conserved.** The wall, through the spring, supplies a nonzero external impulse (its size: next slide).

COAST · 2 → just before impact

- **No horizontal force:** v_{Ax} unchanged.
- Nothing to solve, but say so: a smooth track is an assumption.

IMPACT · 2 → 3

- **System A + B:** contact forces are internal; external horizontal impulse negligible during brief contact.
- L_x **conserved, plus restitution** e for the second equation. T is not conserved.

+x to the right in every phase. A and B name carts; 1, 2, 3 name states.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

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Work–Energy From State 1 to State 2

WORK–ENERGY, CART A, 1 → 2

$$V_{\text{spr},1} = \frac{1}{2}k\delta^2 = \frac{1}{2}(200)(0.10)^2 = 1.00 \text{ J}$$

$$\frac{1}{2}m_A v_{Ax,2}^2 = 1.00 \text{ J}$$

$$v_{Ax,2} = +2.0 \text{ m/s}$$

Massless spring; smooth track; normal force and weight do no work.

WHAT ELSE IS TRUE IN THIS PHASE

- Impulse on A: $I_x = m_A v_{Ax,2} - 0 = +1.00 \text{ N}\cdot\text{s}$, delivered by the spring and supported by the wall.
- So momentum of A is **not** conserved here.
- Newton also works, through $v dv/dx$ (A2): same 2.0 m/s, more steps.

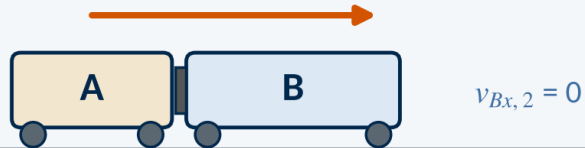
Why work–energy here? The spring force depends on position, the target is a speed, and the forces we do not know (normal, weight) do no work.



Momentum of A + B, Plus Restitution

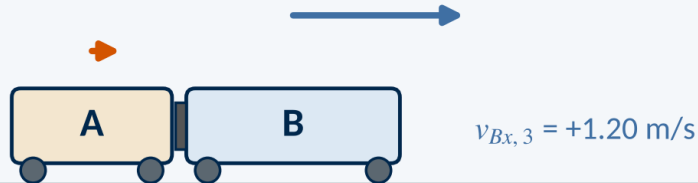
state 2 (just before)

$$v_{Ax,2} = +2.0 \text{ m/s}$$



state 3 (just after), $e = 0.50$

$$v_{Ax,3} = +0.20 \text{ m/s}$$



TWO UNKNOWN, TWO EQUATIONS

$$m_A v_{Ax,2} = m_A v_{Ax,3} + m_B v_{Bx,3}$$

$$0.50(2.0) = 0.50v_{Ax,3} + 0.75v_{Bx,3}$$

$$v_{Bx,3} - v_{Ax,3} = e(v_{Ax,2} - v_{Bx,2}) = 0.50(2.0)$$

$$v_{Ax,3} = +0.20 \text{ m/s}, \quad v_{Bx,3} = +1.20 \text{ m/s}$$

Momentum: contact forces internal. Restitution: the second equation.

Kinetic energy is **not** assumed conserved. The impact model is Part 4's; today's point is **why** these two equations, in this phase.

Observe

Idealize

Model

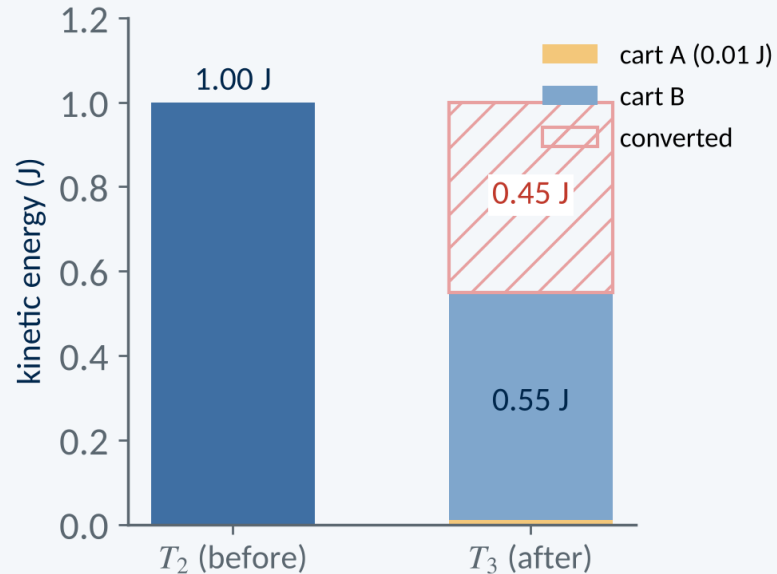
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Is the Impact Result Physically Consistent?



CROSS-CHECKS: MOMENTUM, THEN RESTITUTION

$$v_G \text{ before} = 1.00/1.25 = 0.80 \text{ m/s}$$

$$v_G \text{ after} = [0.50(0.20) + 0.75(1.20)]/1.25 = 0.80 \text{ m/s}$$

$$T_2 - T_3 = \frac{1}{2} \left(\frac{m_A m_B}{m_A + m_B} \right) (1 - e^2) (v_{Ax,2} - v_{Bx,2})^2 = 0.45 \text{ J}$$

- **Admissibility (Part 4):** $T_3 = 0.55 \text{ J} \leq T_2 = 1.00 \text{ J}$; a passive impact cannot add energy.
- $v_{Bx,3} \geq v_{Ax,3}$: B leaves at least as fast as A (no pass-through).

0.45 J of kinetic energy becomes deformation, heat and sound; momentum is unchanged. These are checks **within the model**, not experimental validation.

Observe

Idealize

Model

Solve / Analyze

Validate

Report



What If $e = 0$? What If $e = 1$?

Predict first: for $e = 0$ and $e = 1$, find $v_{Ax,3}$, $v_{Bx,3}$ and T_3 . Which one keeps all the kinetic energy? Does v_G change?

e	$v_{Ax,3}$ (m/s)	$v_{Bx,3}$ (m/s)	T_3 (J)	Converted (J)	v_G (m/s)
0 (zero separation speed)	+0.80	+0.80	0.40	0.60	0.80
0.50 (today)	+0.20	+1.20	0.55	0.45	0.80
1 (elastic)	-0.40	+1.60	1.00	0	0.80

$e = 0$ means zero relative separation speed just after impact; coupling is one way to get it, not the definition. At $e = 1$, A rebounds and no kinetic energy is converted. v_G never changes.

Observe

Idealize

Model

Solve / Analyze

Validate

Report



What Each Method Eliminated, Needed and Could Not Find

Step	Method	Eliminated	Needed first	Could not find
Puck $v_{\theta,2}$	angular momentum about O	string force	—	v_r ; tension
Tension at state 2	radial Newton	—	$v_{\theta,2}$	the work over the pull
Hand's work	work–energy (backwards)	weight, table normal	$v_{\theta,2}$	tension at an instant
Launch	work–energy	weight, track normal	—	launch time, force history
Impact	momentum of A + B, with e	contact force	$v_{Ax,2}$	contact force, duration

- Newton could do the launch (integrate $v dv/dx$) but not the impact without a contact model.
- Energy alone cannot split two unknown velocities when the converted energy is unknown.

- Momentum of A is **not** conserved in the launch: the wall supplies impulse.
- Momentum of A + B **is** conserved during the brief impact: external impulse negligible.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

Validate ›

Report



Three Confident Method Errors



AI CLAIM 1

“Since the carts are moving on a smooth track, kinetic energy must be conserved through their collision.”

WHAT IS WRONG

Invalid assumption: smooth track $\Rightarrow$ elastic impact. The track is not the contact. At $e = 0.50$, 0.45 J is converted.



AI CLAIM 2

“Cart A's linear momentum is conserved from spring release to impact, so its launch velocity follows from momentum conservation.”

WHAT IS WRONG

Invalid assumption: no external impulse during launch. The wall-supported spring delivers +1.00 N·s to A.



AI CLAIM 3

“Work–energy gives the string tension as 1.87 J divided by 0.30 m, or 6.24 N.”

WHAT IS WRONG

Invalid assumption: work $\div$ distance is the tension. 6.24 N is the displacement-average; at state 2 radial Newton gives 12.3 N.

Observe ›

Idealize ›

Model ›

Solve / Analyze ›

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Choose, Justify, Check

1 · After the impact

Cart B ($v_{Bx,3} = 1.20$ m/s) enters a rough patch, $\mu_k = 0.20$. How far does it slide?

2 · A struck ball

A 0.15 kg ball receives an idealized force–time pulse of area 4.5 N·s. Find its change in velocity.

3 · A bead on a wire

A bead starts from rest and drops 1.2 m vertically along a smooth curved wire. Find its speed at the bottom.

For each: **target unknown · method or chain · unknown it eliminates · one check**. At least one needs two methods.



Chapter 5: not one problem, one method, but choose and combine methods by the unknowns, the information and the assumptions, then validate.



CHAPTER - 5 · PART 7

APPENDIX

Appendix — Supplemental Material

A1 comparison matrix · A2 Newton for the spring launch · A3 restitution limits · A4 friction during launch · A5 brief transverse impulse · A6 method-selection worksheet. Not part of the 56-minute core.

Observe



Idealize



Model



Solve / Analyze



Validate



Report



Four Balances Side by Side

Method	Relates	Best when the target is	Eliminates	Misleads when
Newton's second law	$\Sigma \vec{F}$ and $m\vec{a}$ at an instant	a force or acceleration now	forces perpendicular to the chosen component	used for accumulated change without integrating
Work–energy	work and ΔT over a displacement	a speed after a displacement, or the work	forces that do no work	energy is converted (impacts) or direction is needed
Linear impulse–momentum	external impulse and $\Delta \vec{L}$ over an interval	a velocity after an interval	internal forces (system choice)	an external impulse is ignored (the wall in the launch)
Angular impulse–momentum about O	moment integral and $\Delta \vec{H}_O$	v_θ , or quantities about O	forces through O	O is moved, or a force misses O

Use the 'Eliminates' column to choose, the 'Misleads when' column to check.

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Same 2.0 m/s, More Steps

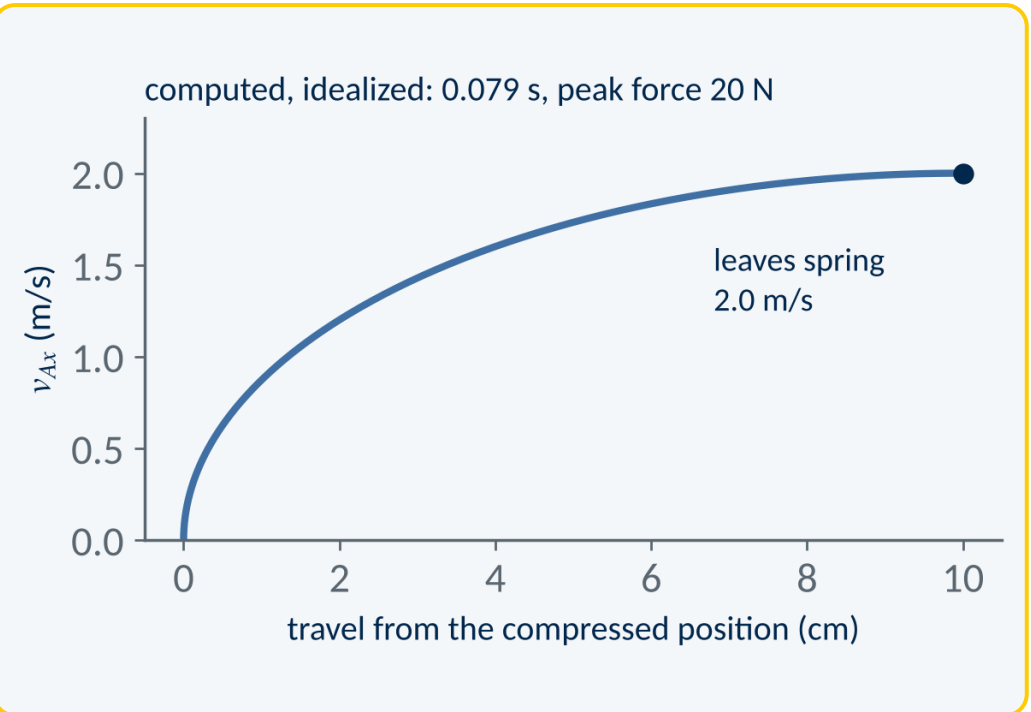
CART A WHILE IN CONTACT, x FROM 0 TO δ

$$m_A v \frac{dv}{dx} = k(\delta - x)$$

$$\frac{1}{2} m_A v_{Ax,2}^2 = k(\delta \cdot \delta - \frac{\delta^2}{2}) = \frac{1}{2} k \delta^2$$

$$v_{Ax,2} = 2.0 \text{ m/s (same as work-energy)}$$

- x : travel from the compressed position.
- The integration is the work-energy theorem, derived again.



- Newton also gives what energy cannot: launch time $(\pi/2)\sqrt{m_A/k} = 0.079 \text{ s}$ and peak force $k\delta = 20 \text{ N}$ (idealized).
- Choose by the target: speed $\rightarrow$ energy; time or force history $\rightarrow$ Newton.

Observe

Idealize

Model

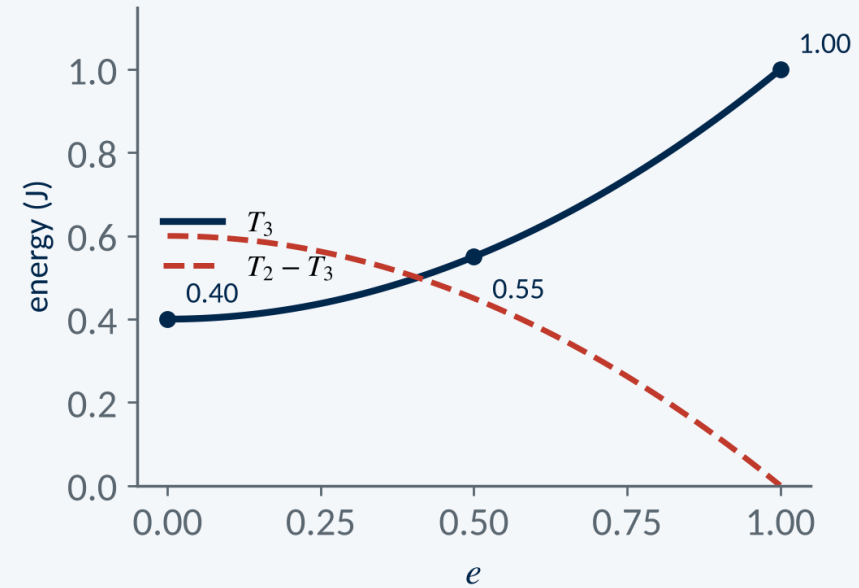
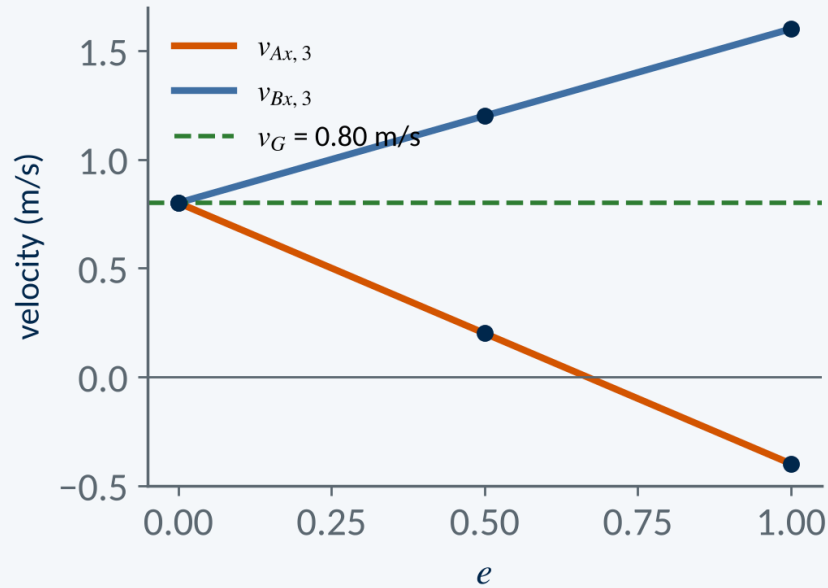
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Velocities and Energy Across $0 \leq e \leq 1$



- $v_G = 0.80$ m/s for every e : momentum of A + B is fixed by the incoming state alone.
- A reverses direction when $m_A - em_B < 0$, i.e. for $e > 0.67$ here.
- Converted energy falls as $1 - e^2$: 0.60 J at $e = 0$, 0.45 J at 0.50, 0 at 1.



What If the Track Is Not Smooth?

Illustrative model (not measured): $\mu_k = 0.10$ on the whole track; coast distance 0.50 m from spring to impact; $g = 9.81 \text{ m/s}^2$. Everything else as in the core.

WORK-ENERGY WITH FRICTION

- Launch: friction work $-\mu_k m_A g \delta = -0.049 \text{ J} \rightarrow T = 0.951 \text{ J} \rightarrow 1.95 \text{ m/s}$.
- Coast: $-\mu_k m_A g (0.50 \text{ m}) = -0.245 \text{ J} \rightarrow T = 0.706 \text{ J} \rightarrow 1.68 \text{ m/s}$.
- Impact ($e = 0.50$): $v_{Ax,3} = +0.17$, $v_{Bx,3} = +1.01 \text{ m/s}$.

WHAT CHANGES IN THE METHOD CHOICE

- Energy still works; it simply gains a friction-work term.
- The coast is no longer trivial: it is a phase with its own balance.
- During the brief impact, friction's impulse is still negligible: momentum of A + B is kept.

Release needs $k\delta = 20 \text{ N}$ to exceed static friction $\mu_s m_A g$: true for any μ_s below about 4.

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Linear and Angular Forms, and When They Differ

VECTOR VIEW AT STATE 2

- $\Delta \vec{L} = I_{\theta} \hat{u}_{\theta}$: magnitude 0.12 N·s, transverse.
- $\Delta H_{Oz} = r_2 I_{\theta} = 0.060 \text{ kg}\cdot\text{m}^2/\text{s}$, because the impulse acts at r_2 .
- $\Delta T = \frac{1}{2}(0.60)(3.4^2 - 3.2^2) = +0.396 \text{ J}$.

WHEN THE TIE BREAKS

- If the push lasts 0.010 s while $v_r = -0.30 \text{ m/s}$, r changes by only 3 mm (0.6%): the tie holds.
- For a long push while r changes, use the moment integral $\int r F_{\theta} dt$; r times $\int F_{\theta} dt$ is no longer exact.
- Note: 10 ms impulse gives the angular result to better than 1% is due to the assumed force history. A small change in radius supports that estimate for a same-direction transverse force impulse but does not establish a universal error bound.

- **After the impulse**, with the string held at $r = 0.50 \text{ m}$: $H_{Oz} = 1.02 \text{ kg}\cdot\text{m}^2/\text{s}$ from then on, and the tension rises to $mv_{\theta}^2/r = 13.9 \text{ N}$.
- If the pull resumes, the new constant is 1.02, not 0.96: every later state follows from it.

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Fill This In Before Writing Any Equation

Item	Your answer
System (what is inside the boundary?)	
Interval or instant; phases and states 1, 2, 3	
Reference point O (for angular momentum)	
Target unknown	
Unknown forces, and which method eliminates each	
Conservation gate: is the external impulse, moment or work zero? Why?	
Chaining gate: what must another method supply first?	
Validation: independent route, limit, admissibility	

AI-RESPONSE RUBRIC

- Names the system and interval?
- States the conservation condition, not just the law?
- Names the reference point?
- Distinguishes instantaneous from total or average?
- Gives a check that does not reuse its own equations?

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