

Self-Assessment 4

Brackets · Sign Change · Bisection · Guaranteed Bounds · The Contract

Ungraded · for your own learning · work at your own pace



AI First Engineering

Figure: AI First Engineering (AIFE) course logo

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AI TUTOR MODE — read this first

You do not need to complete every question in one sitting. Start with each group. If you can confidently solve the first questions independently, sample the remaining questions. Spend more time in groups where you need AI or human help. In Chapter 4 your AI is an engineering analyst that may PROPOSE a method — which is exactly why every recommendation it makes must arrive with its contract: the bracket signs, where the bound comes from, who owns the tolerance. If you become stuck, instead of asking for the answer, try:

“I am working on this problem. Do not solve it or compute the answer for me. Ask me one question or give me one small hint at a time. Help me reason through the sign of f , the bracket, the continuity condition, the halving schedule, the bound, or what the guarantee does not cover, until I can explain it myself.”

The purpose of this self-assessment is to determine what YOU understand — not what your AI can answer. This is the working method in action: WRITE IT › TRAP IT › HUNT IT › TRUST IT — the judgment stays yours.

After each group, rate yourself with the four checkboxes. Your pattern of ratings — not a score — is the output: it tells you exactly what to review, practice, or bring to office hours — before Lab-4 asks you to certify a root against a real cup and Homework 4 asks you to certify one for a channel. A calculator is all you need — no computer, and no clock: work at your own pace. Every question here is one question in disguise: how do we find the crossing without walking the whole road — and when does the guarantee deserve to be trusted?

GROUP 1 · Write It — $f(x) = 0$ and the Sign

One scenario, one equation. Answer in your own words — a sentence or a short calculation each. Keep every unit.

A spherical navigation buoy of radius $R = 0.50$ m and mass $m = 300$ kg is lowered into fresh water ($\rho = 1000$ kg/m³). It settles at the draft d — the depth of its lowest point below the waterline — at which the mass of displaced water equals its own mass. The displaced volume of a sphere submerged to depth d is provided: $V(d) = \pi \cdot d^2 \cdot (3R - d) / 3$, valid for $0 \leq d \leq 2R$. Three evaluations are given: $\rho \cdot V(0.25) = 81.8$ kg, $\rho \cdot V(0.50) = 261.8$ kg, $\rho \cdot V(0.75) = 441.8$ kg.

1. Write the floating condition in the chapter’s shape, $f(d) = 0$, with units. Then give each sign its physical meaning: what is happening to a buoy for which $f(d) < 0$ at the trial draft d , and for which $f(d) > 0$? Finally, what is the widest interval of d that makes physical sense — and why can an engineer defend it without computing anything?

2. Using the three given evaluations: (a) write $f(0.25)$, $f(0.50)$, $f(0.75)$ with their signs; (b) which pair of consecutive drafts certifies a bracket, and which pair certifies nothing — say precisely what “nothing” means (no root, or possibly two); (c) state the theorem’s hinge for THIS f : why is f continuous on the bracket, and why can there be only one crossing there?

RATE YOURSELF on this group before moving on:

- I can do this independently.
- I can do this with a small AI hint.
- I need substantial AI help.
- I still need human/instructor help.

GROUP 2 · Trap It — Brackets and Their Fine Print

Commit each answer before reading past it. A bracket is a promise — read its fine print.

A rural radio link’s signal margin $M(d)$, in dB, is computed at a few distances d from the tower; a positive margin means the link works. A relay amplifier is programmed to switch ON at exactly 22 km. The computed values: $M(5) = +9$, $M(10) = +4$, $M(15) = -1$, $M(20) = -3$, $M(25) = +2$, $M(30) = -2$ dB. The engineering question: what is the maximum distance at which the link works WITHOUT the relay?

3. (a) List every consecutive pair of distances across which M changes sign. (b) Which of those brackets contains the crossing the engineering question actually needs — and why are the others the wrong root for this decision? (c) The sign change between 20 and 25 km: does the guarantee promise a root inside it? Name the condition that may fail there and say what evidence you would demand before trusting that bracket.

A bisection routine a teammate wrote — read it like an engineer; do not run it:

```
def bisection(f, a, b, tol):
    while (b - a) > tol:
        m = (a + b) / 2
        if f(a) * f(m) <= 0:
            b = m
        else:
            a = m
    return (a + b) / 2, tol

root, bound = bisection(f, 0.0, 0.50, 0.01)
print("root =", root, "+/-", bound)
```

4. Read the program like an engineer; do not run it. (a) What is the largest possible error of the returned root when the loop exits — and is the reported bound honest, generous, or overclaimed? (b) Name the one line of Chapter 4's pseudocode that is missing, and say what it would have done. (c) Suppose the buoy's f from Group 1 is passed with the interval $[0.0, 0.50]$. Walk the first two halvings with signs only and state what number the program prints — and what that number is not.

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GROUP 3 · Hunt It — Halvings, the Schedule & the Price

Hand work and the schedule — the exam rep. The arithmetic is Group 1's; the judgment is which half survives.

Reminder: $f(d) = \rho \cdot V(d) - m$ with $V(d) = \pi \cdot d^2 \cdot (3R - d)/3$, $R = 0.50$ m, $\rho = 1000$ kg/m³, $m = 300$ kg. From Group 1: $f(0.50) = -38.2$ kg, $f(0.75) = +141.8$ kg.

n	midpoint m (m)	$\rho \cdot V(m)$ (kg)	sign of $f(m)$ · bracket kept · half-width (m)
1	0.625		
2			
3			

5. Starting from the certified bracket $[0.50, 0.75]$ of Group 1, run THREE halvings by hand for the buoy. For each row record the midpoint m , $\rho \cdot V(m)$ in kg, the sign of $f(m)$, the bracket kept, and the half-width. Then report the draft as midpoint $\pm$ half-width — and state whether the reference draft (given afterward as 0.5488 m) sits inside your promise.

6. The buoy’s specification sheet must state the draft to the nearest centimetre, so the requirement fixes $\text{tol} = \pm 0.005$ m. (a) Using $W_0 / 2^{n+1} \leq \text{tol}$, how many halvings are needed from $[0.50, 0.75]$? From the physical cage $[0, 1.00]$? Show the inequality. (b) The cage was four times wider — how many extra halvings did that cost, and why does bisection “forgive a generous bracket”? (c) Tightening the requirement tenfold to ± 0.0005 m costs how many MORE halvings? Compare with what a tenfold refinement costs an incremental search, and name the asymmetry.

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GROUP 4 · Trust It — Debug the Log, Audit the Bound & BREAK THE AI

Deliver each judgment with evidence. A guarantee that survives your audit is one you may use — and you state how you verified it.

A teammate's hand log of bisection on the buoy, starting from [0.500, 0.750] (values in m and kg):

n	a	b	m	f(a)	f(m)	bracket kept
1	0.5000	0.7500	0.6250	-38.2	+57.9	[0.5000, 0.6250]
2	0.5000	0.6250	0.5625	-38.2	+10.6	[0.5000, 0.5625]
3	0.5000	0.5625	0.5313	-38.2	-13.7	[0.5000, 0.5313]
4	0.5000	0.5313	0.5156	-38.2	-25.9	[0.5156, 0.5313]
5	0.5156	0.5313	0.5234	-25.9	-19.8	[0.5234, 0.5313]
...						"converged: d = 0.531 ± 0.004 m"

7. DEBUG the log. (a) Find the first row in which the contract was broken and say exactly what went wrong in that row's decision. (b) Explain why every later row looks perfectly "normal" — the half-width still halves — even though the search is now lost. (c) The log ends with a converged value. Using one evaluation from the log, show that 0.531 m cannot be the root, and name the cheapest test that would have caught this.

8. Two claims, two audits. (a) "I bisected on [0, 10] for 10 iterations, so the root is known to ± 0.001 ." Check the arithmetic; if the claim fails, state the bound actually earned and the iteration count that WOULD earn ± 0.001 . (b) A different program stops when $|f(m)| < 0.001$ and reports "the root is known to ± 0.001 ." What does a small value of $f(m)$ guarantee about the location of the root — and what is the quantity that does carry the guarantee?

9. BREAK THE AI. Asked to certify the buoy, an AI analyst reports: "Bisection on [0, 1.00 m] with a certified sign change ($f(0) = -300$, $f(1.00) = +223.6$) converges in 7 iterations to $d = 0.551 \text{ m} \pm 0.004 \text{ m}$, guaranteed by the intermediate value theorem. The buoy design is therefore validated." Deliver your verdict — ACCEPT · MODIFY · REJECT — then critique in three to five sentences: which parts of the claim survive your audit (check the halving count against the schedule, and the bound against the cage), and which word in the last sentence does the theorem not support? What would a correctly scoped conclusion say?

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GROUP 5 · Decide It — The Method, the Edge & the Contract

The chapter's last principle earns its keep: a guarantee holds only where its conditions hold — and choosing the method is engineering judgment.

10. Three jobs, one buoy model. Choose the method for each and defend it in one sentence: (a) certify the draft of THIS buoy to ± 5 mm for its specification sheet; (b) produce a catalog curve of draft versus mass for buoys from 100 to 500 kg; (c) a new, oddly shaped hull whose $V(d)$ comes from a CAD table nobody on the team has looked at yet. Your choices: bisection · a coarse sweep · reconnaissance first, then bisection.

11. The buoy carries a light that must stay at least 0.40 m above the waterline: $\text{freeboard} = 2R - d \geq 0.40$ m. Your bisection ends with the bracket [0.545, 0.555] m. (a) What draft do you print on the specification sheet, and with what $\pm$? (b) Which number do you use to check the 0.40 m requirement, and does the buoy pass? (c) In one sentence: why are the midpoint and the safe edge both legitimate engineering — and what question does each answer?

12. A teammate re-runs the buoy calculation with R typed as 50 (centimetres) while p stays 1000 kg/m^3 and m stays 300 kg, and brackets on [0, 100]. The sign change is certified ($f(0) = -300$; $f(100)$ is enormous and positive), $V(d)$ is still a polynomial, and bisection converges in 14 halvings to $d = 0.046 \pm 0.003$. (a) Did the guarantee hold? Say exactly what it guaranteed. (b) What is wrong, and how does the number itself scream it — give the physical-reasonableness argument. (c) Name TWO checks

outside the theorem that would have caught this, and say what each one checks that the contract never can.

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Reading your ratings. Mostly “independently” → you own Chapter 4: you are ready to certify a root in Lab-4 and in Homework 4. “Small AI hint” clusters → redo just those questions tomorrow without the hint. “Substantial AI help” → rework the matching Chapter 4 lecture part (Write it · Trap it · Hunt it · Trust it), then retry. Any “human/instructor help” → bring that exact question to your instructor or ask an expert — that is what they are for. If you can answer these small scenarios, you can answer the chapter’s question: how do we find the crossing without walking the whole road — and when does its guarantee deserve to be trusted? AI assists. Humans verify. Engineers decide.