

Self-Assessment 5

The Slope · The Newton Step · Convergence Evidence · When Newton Goes Wrong · Bracket First

Ungraded · for your own learning · work at your own pace



AI First Engineering

Figure: AI First Engineering (AIFE) course logo

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AI TUTOR MODE — read this first

You do not need to complete every question in one sitting. Start with each group. If you can confidently solve the first questions independently, sample the remaining questions. Spend more time in groups where you need AI or human help. In Chapter 5 your AI is an engineering analyst turning CRITIC — it may propose a start, a slope and a stop, which is exactly why every proposal it makes must arrive with its reasons: where the derivative came from, why the start is defensible, and what its convergence does NOT certify. If you become stuck, instead of asking for the answer, try:

“I am working on this problem. Do not solve it or compute the answer for me. Ask me one question or give me one small hint at a time. Help me reason through the sign of f , the meaning of f' , the tangent step, the stopping rule, the physical domain, or what a converged iteration does not certify, until I can explain it myself.”

The purpose of this self-assessment is to determine what YOU understand — not what your AI can answer. This is the working method in action: USE IT › RUN IT › BREAK IT › CHOOSE IT — the judgment stays yours.

After each group, rate yourself with the four checkboxes. Your pattern of ratings — not a score — is the output: it tells you exactly what to review, practice, or bring to office hours — before Lab-5 asks you to march Newton along a real beam and Homework 5 asks you to solve a circuit no algebra can. A calculator is all you need — no computer, and no clock: work at your own pace. Every question here is one question in disguise: can the hunt read the map while it runs — and when does “converged” deserve to be trusted?

GROUP 1 · Use It — $f(h) = 0$ and the Slope

One scenario, one equation, one slope. Answer in your own words — a sentence or a short calculation each. Keep every unit.

A horizontal circular drain pipe of radius $R = 0.30$ m runs partly full. A level sensor must be mounted at the water depth h (measured from the bottom of the pipe) at which the flow cross-section — the wetted area — equals $A_{\text{target}} = 0.10$ m². The wetted area of a circle filled to depth h is provided: $A(h) = R^2 \cdot \arccos(1 - h/R) - (R - h) \cdot \sqrt{2Rh - h^2}$, valid for $0 \leq h \leq 2R$, with the arccos in radians. Its derivative is also provided — and it has a name you can see: $A'(h) = 2 \cdot \sqrt{2Rh - h^2}$, the WIDTH of the water surface at depth h . Three evaluations are given: $A(0.20) = 0.0825$ m², $A(0.25) = 0.1115$ m², $A(0.30) = 0.1414$ m² (half full); and the surface widths $A'(0.20) = 0.5657$ m, $A'(0.25) = 0.5916$ m, $A'(0.30) = 0.6000$ m.

1. Write the sensor condition in the chapter’s shape, $f(h) = 0$, with units. Give each sign its physical meaning: what is true of the water for which $f(h) < 0$ at the trial depth h , and

for which $f(h) > 0$? Then explain, in one sentence and with units, why the derivative of the wetted area is the surface width — what does “m² per m” mean physically here — and state the widest interval of h that makes physical sense.

2. Using the given values at $h = 0.20$ m: (a) compute the Newton correction $-f/f'$ and the next depth h_1 , keeping units at every step; (b) say in words what the correction means — shortfall divided by what? — and why it is NOT the midpoint of any bracket; (c) a classmate proposes starting at the pipe’s bottom, $h_0 = 0$, “because it is the simplest number.” What is $A'(0)$, and what happens to the first correction?

RATE YOURSELF on this group before moving on:

- I can do this independently.
- I can do this with a small AI hint.
- I need substantial AI help.
- I still need human/instructor help.

GROUP 2 · Run It — Newton Steps by Hand and in Code

Hand work and the stopping rule — the exam rep. The arithmetic is Group 1’s; the judgment is whether the correction is small enough to stop, and what to check afterward.

Reminder: $f(h) = A(h) - 0.10$ with $A'(h) = 2 \cdot \sqrt{(2Rh - h^2)}$, $R = 0.30$ m. From Group 1: $f(0.20) = -0.0175$ m², $A'(0.20) = 0.5657$ m. Run the steps in the table below; the second row needs one new evaluation, which is given here so the arithmetic stays short: $A(0.2309) = 0.1003$ m², $A'(0.2309) = 0.5839$ m.

n	h_n (m)	$f(h_n) = A(h_n) - 0.10$ (m ²)	$f'(h_n) = \text{surface width}$ (m)	correction $-f/f'$ (m) · h_{n+1} (m)
0	0.2000	-0.0175	0.5657	
1				

3. Starting from $h_0 = 0.20$ m, run TWO Newton steps by hand in the table (row 0 uses your Group 1 numbers; row 1 uses $A(0.2309) = 0.1003$ m² and $A'(0.2309) = 0.5839$ m). For each row record $f(h_n)$, $f'(h_n)$, the correction, and h_{n+1} . Then: the sensor mount is specified to the nearest millimetre, so the requirement fixes $\text{tol} = 0.0005$ m. After which

row does the correction satisfy $|\Delta h| \leq \text{tol}$ — and what TWO things must you still check before reporting h^* ? Report h^* to the digits the requirement justifies.

A Newton routine a teammate wrote — read it like an engineer; do not run it:

```
def newton(f, fp, h0):
    h = h0
    for i in range(3):          # three is plenty
        h = h - f(h) / fp(h)
    return h

h_star = newton(f, fp, 0.01)
print("sensor depth =", h_star, "m")
```

4. Read the program like an engineer; do not run it. (a) The loop stops on a fixed count of three. Name the stopping rule Chapter 5 uses instead, and say what a fixed count would have reported from a start that needs five steps. (b) The program never checks $0 < h < 2R$. Trace what happens from $h_0 = 0.01$ m: $f(0.01) = -0.0990$ m², $A'(0.01) = 0.1536$ m — compute h_1 , say where it lies relative to the top of the pipe ($2R = 0.60$ m), and what the NEXT evaluation of A' would try to do. (c) Write the one-line guard that would have stopped the run honestly, and say what it should print.

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GROUP 3 · Break It — Debug the Log, Bad Starts & the Domain

Deliver each judgment with evidence. Newton converges to a root — you decide whether it is the answer.

A teammate's Newton log for the pipe, starting from $h_0 = 0.20$ m (values in m and m^2). The teammate typed the derivative as $\sqrt{(2Rh - h^2)}$ — half the surface width — and did not notice:

n	h_n	$f(h_n)$	f' used	correction	h_{n+1}
0	0.2000	-0.0175	0.2828	+0.0619	0.2619
1	0.2619	+0.0186	0.2976	-0.0624	0.1995
2	0.1995	-0.0178	0.2827	+0.0629	0.2624
3	0.2624	+0.0189	0.2976	-0.0634	0.1990
4	0.1990	-0.0181	0.2825	+0.0639	0.2629
...					"still running — 50 steps, no convergence"

5. DEBUG the log. (a) The corrections never shrink: the iterate bounces between ≈ 0.20 and ≈ 0.26 for ever. Using row 0, show that the correction is exactly twice what the true surface width would give, and explain why doubling every step makes Newton overshoot to the mirror image of where it started. (b) Name the cheapest check that would have caught the derivative error before any loop ran (Chapter 5, Part 1). (c) A guard that stops after 50 steps would end this run — with what message, and would that message be honest?

6. Bad starts. (a) From $h_0 = 0.59$ m — one centimetre below the top of the pipe — $f(0.59) = +0.1817 \text{ m}^2$ and $A'(0.59) = 0.1536 \text{ m}$. Compute h_1 and say what a negative depth means. (b) Why are the slopes at $h = 0.01$ and $h = 0.59$ identical, and why is the tangent so misleading near both the bottom and the top of the pipe? (Think about the shape of the water surface there.) (c) Sketch, in words, the region of starts from which you would EXPECT Newton to behave — and what a start outside it needs before the loop is allowed to run.

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GROUP 4 · Trust It — Audit the Claims & BREAK THE AI

Convergence evidence is not a certificate. Audit each claim against what the evidence can actually support.

An AI analyst's Newton log for the pipe from $h_0 = 0.30$ m (values in m and m^2), as printed by its program:

n	h_n	$f(h_n)$	$f'(h_n)$	correction	h_{n+1}
0	0.3000	+0.04137	0.6000	−0.06895	0.23105
1	0.23105	+0.00037	0.5839	−0.00063	0.23042
2	0.23042	+0.0000000	0.5836	−0.0000000	0.23042
					“converged in 3 iterations: $h = 0.2304$ m, residual $3\text{e-}10$ ”

7. Two claims, two audits. (a) “The residual $f(h^*)$ is $3 \times 10^{-10} \text{ m}^2$, so the depth is known to ten decimal places.” What does a tiny residual actually establish, what quantity would carry a bound, and where would that bound have to come from? (b) “Newton is quadratically convergent, so three iterations are always enough.” Using the pipe (from 0.30: corrections −0.0690, −0.0006, ~0) and the log in Group 3, say when digit-doubling holds and when it does not — and what should decide the number of steps instead.

8. A teammate re-runs the pipe calculation with R typed as 300 (millimetres) while A_{target} stays 0.10 (m^2). Newton from $h_0 = 1$ converges in 7 steps to $h^* = 0.0211$ with $f(h^*) = 2 \times 10^{-10}$, every iterate positive. (a) Did Newton converge? Say exactly what it converged to, and to what equation. (b) What is wrong, and how does the number itself scream it — give the physical-reasonableness argument. (c) Name TWO checks outside the loop that would have caught this, and say what each one checks that convergence never can.

9. BREAK THE AI. Asked to place the sensor, an AI analyst reports: “Newton-Raphson from $h_0 = 0.30$ m converged in 3 iterations to $h = 0.2304$ m with residual 3×10^{-10} m². The depth is therefore certified to ± 0.0001 m and the sensor position is validated.” Deliver your verdict — ACCEPT · MODIFY · REJECT — then critique in three to five sentences: which parts of the claim survive your audit (check the log above against the digit-doubling pattern and the domain), which word carries a bound the evidence never produced, and which word the mathematics does not support at all. What would a correctly scoped conclusion say — and what two evaluations would turn the ± 0.0001 m into a real certificate?

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GROUP 5 · Choose It — Which Information Do You Have?

The chapter’s last principle earns its keep: bracket first, move intelligently second — and choosing the method is engineering judgment.

10. Three jobs, one pipe model. Choose the method for each and defend it in one sentence: (a) certify the sensor depth to ± 0.5 mm for a drawing that will be signed; (b) a control program that re-solves $f(h) = 0$ every second as A_{target} changes slightly, starting each time from last second’s answer; (c) a new pipe whose cross-section is an odd shape and whose $A(h)$ exists only as a table of measured values, no formula. Your choices: bisection · Newton · reconnaissance and a bracket first, then Newton.

11. Bracket first, move intelligently second. (a) Certify the cage $[0, 2R]$ for the pipe with physics alone — the signs of $f(0)$ and $f(0.60)$ without a calculator. (b) One halving of the cage gives $h = 0.30$; Newton from there reaches 0.2304 in two steps. Count the model evaluations of the whole hybrid — endpoint checks, the halving, the Newton steps (each f and f') — and compare with bisection alone, which needs 9 halvings from $[0, 0.60]$ for

± 0.001 m. (c) The Newton answer has no bound. Which two evaluations certify it to ± 0.001 m, and what must their signs be?

12. Which information do you have? For each situation say which method you would run FIRST and why, in one sentence: (a) you have $A(h)$ as a formula and its derivative, and a good guess from last year's survey; (b) you have only five measured values of $A(h)$ at five depths, no formula; (c) you have the formula, no idea of the depth, and the pipe may be nearly empty or nearly full — where you know the slope is tiny. Then finish the sentence: "Newton converges to a root — ...".

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Reading your ratings. Mostly "independently" → you own Chapter 5: you are ready to march Newton in Lab-5 and to solve the circuit in Homework 5. "Small AI hint" clusters → redo just those questions tomorrow without the hint. "Substantial AI help" → rework the matching Chapter 5 lecture part (Use it · Run it · Break it · Choose it), then retry. Any "human/instructor help" → bring that exact question to your instructor or ask an expert — that is what they are for. If you can answer these small scenarios, you can answer the chapter's question: can the hunt read the map while it runs — and when does "converged" deserve to be trusted? AI assists. Humans verify. Engineers decide.