

CHAPTERS 1–6 COMPREHENSIVE ASSESSMENT

Engineering models and computational logic through numerical
evidence, root finding, interpolation, and engineering judgment



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Purpose. This comprehensive assessment is a learning checkpoint for Chapters 1–6. It asks you to demonstrate how an engineer turns an engineering problem into a computational solution that can be understood, tested, validated, and trusted. Across the assessment, you will work with engineering models, Excel and Python logic, numerical accuracy and convergence, root-finding methods, interpolation, and engineering judgment. The recurring workflow is

PROBLEM → MODEL → COMPUTE → ANALYZE → VALIDATE → IMPROVE.

The goal is not to reproduce code from memory, but to understand the model and algorithm, follow the computation, evaluate the evidence, and decide what the result does—and does not—justify.

How this comprehensive assessment works

- Seven parts, 100 points, 90 minutes. The parts move from engineering models and computational logic through numerical evidence, root finding, interpolation, and engineering judgment.
- Complete the assessment independently. NO calculator, NO electronic devices, NO internet, and NO AI tools. Use only pen/pencil, this assessment booklet, and the provided Chapters 1-6 Formula Summary.
- Numbers are intentionally selected for hand calculation. Show the requested work in the spaces provided; the purpose is to assess computational reasoning, not calculator arithmetic.
- Read Excel formulas and Python code as engineering computational procedures. Focus on variables, decisions, repetition, functions, stopping criteria, and what the computation means.
- For numerical methods, distinguish numerical evidence from engineering validation. A calculation that runs or converges does not, by itself, prove that the model or physical result is correct.
- For multiple-choice and checkbox questions, mark your selection clearly. For numerical answers, include units when requested.

COMPREHENSIVE ASSESSMENT

Time: 90 minutes	Total: 100 points
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Assessment Conditions

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Numbers are selected for hand calculation. Show requested work in the spaces provided.

For multiple-choice and checkbox questions, mark your answer clearly, such as [X] .

PART 1 • ENGINEERING MODEL + EXCEL

13 points · Target time: 12 minutes

Free-Fall Model

A ball is released from rest from a height h . Neglect air resistance. Use the free-fall model provided on your Formula Summary.

For this problem, use $g = 10 \text{ m/s}^2$, a hand-calculation approximation to the standard value 9.81 m/s^2 .

Row	A: Height h (m)	B: g (m/s^2)	C: Predicted Time t (s)
2	5	10	=SQRT(2*A2/\$B\$2)
3	20		
4	45		

1A • Predict — 2 pts

Before calculating, compare Row 2 and Row 3. When height increases from 5 m to 20 m, the predicted fall time will:

- ☐ A. stay the same ☐ B. double ☐ C. increase by a factor of 4 ☐ D. decrease by half

1B • Trace the Excel Formula — 3 pts

The formula in C2 is copied down to C3. Write the formula that appears in C3.

C3 = _____

Classify each reference:

In standard Excel terminology, a **RELATIVE** reference changes when copied, while an **ABSOLUTE** reference stays fixed when copied.

- ☐ A3: RELATIVE ☐ A3: ABSOLUTE
☐ \$B\$2: RELATIVE ☐ \$B\$2: ABSOLUTE

1C • Execute the Spreadsheet — 3 pts

For Row 3, use $h = 20 \text{ m}$ and $g = 10 \text{ m/s}^2$. Calculate the value that appears in C3.

$$t = \sqrt{(2h/g)}$$

C3 = _____

Units: _____

1D • Read the Pattern — 2 pts

Complete the statements.

Height changes from 5 m to 20 m: factor = _____

Predicted fall time changes from 1 s to _____ s: factor = _____

1E • Validation — 3 pts

Excel predicts 2.0 s. A Python program using the same model also predicts 2.0 s. Which conclusion is supported?

- ☐ A. The real falling ball has been physically validated.
- ☐ B. The agreement provides evidence that the Excel and Python implementations are consistent, but it does not physically validate the model.
- ☐ C. The agreement proves that $g = 10 \text{ m/s}^2$ is the correct value for the actual experiment.
- ☐ D. Two programs giving the same answer proves that the answer is correct.

PART 2 • ALGORITHM + PYTHON

17 points • Target time: 15 minutes

Stopping-Distance Search

Use the stopping-distance model from your Formula Summary:

$$d_s = v t_r + v^2 / (2a)$$

For this problem: $t_r = 1 \text{ s}$, $a = 8 \text{ m/s}^2$, and $d_{\text{available}} = 24 \text{ m}$.

```
def stopping_distance(v, tr, a):  
    return v * tr + v**2 / (2 * a)
```

```
tr = 1  
a = 8  
d_available = 24
```

```
v = 4  
step = 4
```

```
while stopping_distance(v + step, tr, a) <= d_available:  
    v = v + step
```

```
print(v)
```

2A • Read the Function Call — 2 pts

On the first evaluation of the WHILE condition, $v = 4$ and $\text{step} = 4$. Python evaluates the function call: `stopping_distance(v + step, tr, a)`

What numerical value is passed as the first argument to `stopping_distance()` function?

Value passed (first argument) _____ m/s

2B • Execute the Function — 3 pts

For that first call, complete:

$$d_s = (\text{____})(1) + (\text{____})^2 / [2(8)]$$

$d_s =$ _____
m

2C · Trace the WHILE Loop — 6 pts

Complete the table as Python executes the program.

Check	v before	v + step tested	Returned d_s	$d_s \leq 24$	v after
1	4	8	12	TRUE	8
2				TRUE / FALSE	
3				TRUE / FALSE	

2D · Program Output — 2 pts

What does the program print?

2E · Assignment — 2 pts

During Check 2, the condition is TRUE and Python executes: $v = v + \text{step}$

Immediately before this line: $v = 8$ and $\text{step} = 4$.

Immediately after this line, $v =$ _____

What does $=$ mean here?

- ☐ A. Test whether the two sides are mathematically equal.
- ☐ B. Calculate the right-hand side and assign that value to the variable on the left.
- ☐ C. Permanently define v as $v + 4$.
- ☐ D. Compare v with step .

2F · Engineering Meaning of step — 2 pts

The program reports 12 m/s. Which statement is correct?

- ☐ A. The program has mathematically proven that 12 m/s is the exact maximum safe speed.
- ☐ B. 12 m/s is the largest speed tested on this 4 m/s search grid that satisfies the requirement; a smaller step could refine the search.
- ☐ C. The program stopped because 12 m/s is unsafe.
- ☐ D. Changing step changes the physical stopping-distance model.

PART 3 · NUMERICAL EVIDENCE

15 points · Target time: 13 minutes

An engineering quantity x is computed using progressively smaller step sizes.

Step size Δ	Computed x (mm)
0.40	13.0
0.20	12.5
0.10	12.3
0.05	12.2

An analytical solution using the same engineering model gives $x_{\text{ref}} = 12.0$ mm.

3A · Resolution — 2 pts

From the first row to the last row, the numerical resolution is becoming:

☐ A. coarser ☐ B. finer ☐ C. unchanged

3B • Successive Change — 3 pts

Calculate the successive change between the final two computed values.

$$\Delta x = |x_{\text{new}} - x_{\text{previous}}|$$

$$\Delta x = | \underline{\hspace{1cm}} - \underline{\hspace{1cm}} | = \underline{\hspace{1cm}} \text{ mm}$$

3C • Error Relative to the Reference — 3 pts

Calculate the absolute error of the final result relative to the analytical reference.

$$\text{Absolute Error} = | \underline{\hspace{1cm}} - \underline{\hspace{1cm}} | = \underline{\hspace{1cm}} \text{ mm}$$

3D • Convergence Evidence — 3 pts

Which observation provides the strongest evidence that the numerical result is converging?

- ☐ A. The computed values change less as the step size becomes smaller.
- ☐ B. The final value contains three significant digits.
- ☐ C. The analytical result is exactly 12.0 mm.
- ☐ D. The calculation was performed using a computer.

3E • Convergence vs. Validation — 4 pts

Select ALL statements supported by the information given.

- ☐ A. The numerical results show evidence of convergence as resolution is refined.
- ☐ B. The final numerical result is close to the analytical result from the same model.
- ☐ C. The agreement proves that the engineering model represents the real physical system correctly.
- ☐ D. Additional physical or experimental evidence would be needed to validate the model against the real system.

PART 4 • BISECTION — HAND TRACE

17 points · Target time: 15 minutes

An engineering equation has been written in root form $f(x) = 0$. The function is continuous over the interval being considered.

Required root-location tolerance: ± 0.20

Initial information: $a = 2$, $b = 4$, $f(2) = -6$, $f(4) = +10$.

4A • Certified Starting Bracket — 2 pts

Does $[2, 4]$ provide a certified starting bracket containing at least one root?

☐ YES ☐ NO

Select the reason:

- ☐ A. $f(2)$ and $f(4)$ have opposite signs and $f(x)$ is continuous.
- ☐ B. The midpoint of 2 and 4 is 3.
- ☐ C. Both endpoint values are nonzero.

4B • Trace the Bisection Iterations — 10 pts

Complete the table as the bisection method proceeds. The function values at the three midpoints are provided; use their signs to update the bracket.

Given: $f(m_1) = -1$, $f(m_2) = +3.75$, $f(m_3) = +1.19$.

Stage	a	b	Midpoint m	Sign of f(m)	New bracket
Start	2	4	—	—	[2, 4]
Iteration 1	2	4	3.0	NEGATIVE	[_____ , _____]
Iteration 2	_____	_____	_____	POSITIVE	[_____ , _____]
Iteration 3	_____	_____	_____	POSITIVE	[_____ , _____]

4C • Final Estimate + Tolerance Check — 5 pts

Using your final bracket from Iteration 3:

Bracket width $W =$ _____

Midpoint estimate $x_{\text{estimate}} =$ _____

Guaranteed maximum root-location error $= \pm$ _____

Has the required tolerance of ± 0.20 been satisfied?

☐ YES ☐ NO

PART 5 • NEWTON-RAPHSON — HAND TRACE

20 points • Target time: 17 minutes

An engineering equation has been written in root form $f(x) = 0$. Use the Newton-Raphson relationship from your Formula Summary.

$$\Delta x_n = -f(x_n) / f'(x_n) \quad \text{and} \quad x_{n+1} = x_n + \Delta x_n$$

Required correction tolerance: $|\Delta x| \leq 0.10$. Start with $x_0 = 2.0$.

5A • First Newton Step — 5 pts

At $x_0 = 2.0$, you are given: $f(x_0) = -2.0$ and $f'(x_0) = 4.0$.

$$\Delta x_0 = -(\text{_____})/(\text{_____}) = \text{_____}$$

$$x_1 = x_0 + \Delta x_0 = \text{_____}$$

5B • Second Newton Step — 5 pts

At x_1 , you are given: $f(x_1) = +0.5$ and $f'(x_1) = 5.0$.

$$\Delta x_1 = -(\text{_____})/(\text{_____}) = \text{_____}$$

$$x_2 = x_1 + \Delta x_1 = \text{_____}$$

5C • Stopping Decision — 3 pts

Based on the second correction, $|\Delta x_1| =$ _____

Has the required correction tolerance $|\Delta x| \leq 0.10$ been satisfied?

☐ YES ☐ NO

5D • What the Correction Means — 3 pts

Select the best statement.

- ☐ A. A small Newton correction proves that the physical engineering model is correct.
- ☐ B. A small Newton correction is evidence that the numerical iterations are settling, but it does not validate the physical model.
- ☐ C. A small Newton correction guarantees the same root-location bound as bisection.
- ☐ D. A small Newton correction proves all input values and units are correct.

5E • Newton vs. Bisection — 2 pts

Which method uses a sign-changing bracket to provide a root-location guarantee?

- ☐ A. Newton-Raphson
- ☐ B. Bisection
- ☐ C. Both
- ☐ D. Neither

5F • Physical Domain Check — 2 pts

Suppose x represents a physical length and the Newton iteration returns $x = -2.4$ m. What should the engineer do?

- ☐ A. Accept it because Newton converged.
- ☐ B. Reject or investigate it because the result violates the physical domain for a length.
- ☐ C. Round it to $+2.4$ m.
- ☐ D. Accept it if the residual is small.

PART 6 • MISSING DATA + LINEAR INTERPOLATION

13 points • Target time: 13 minutes

A position measurement is missing at $t = 0.25$ s. Assume linear interpolation is reasonable between the two measured points shown.

Time t (s)	Position x (m)	Status
0.20	1.40	MEASURED
0.25	?	MISSING
0.30	2.20	MEASURED

6A • Local Slope — 3 pts

Calculate the local slope between the measured points.

$$m = (x_2 - x_1) / (t_2 - t_1)$$

$m = (\underline{\hspace{2cm}} - \underline{\hspace{2cm}}) / (\underline{\hspace{2cm}} - \underline{\hspace{2cm}}) = \underline{\hspace{2cm}} \text{ m/s}$

6B • Interpolate the Missing Value — 3 pts

The missing time $t = 0.25$ s is halfway between 0.20 s and 0.30 s.

Interpolated position $x(0.25) = \underline{\hspace{2cm}} \text{ m}$

6C • Label the Value — 2 pts

How should $x(0.25)$ be labeled?

- ☐ A. MEASURED
- ☐ B. EXACT
- ☐ C. ESTIMATED / INTERPOLATED
- ☐ D. VALIDATED

6D • Interpolation or Extrapolation? — 2 pts

Using the same two measured points to estimate x at $t = 0.35$ s would be:

- ☐ A. INTERPOLATION ☐ B. EXTRAPOLATION

6E • Quick Reasonableness Check — 2 pts

A Python program reports $x(0.25) = 2.60$ m. For linear interpolation between 1.40 m and 2.20 m, this result is:

- ☐ A. REASONABLE ☐ B. SUSPICIOUS

Select the reason:

- ☐ A. An interior linear interpolation should lie between the two bounding measured values.
☐ B. Python always adds numerical error.
☐ C. Interpolated values must be larger than both measurements.

6F • Precision vs. Confidence — 1 pt

Python prints 1.800000000 m. Does the number of printed decimal places, by itself, establish high confidence in the estimate?

- ☐ YES ☐ NO

PART 7 • ENGINEER DECIDES

5 points · Target time: 5 minutes

An AI assistant gives the following conclusion:

“The Excel and Python results agree. The numerical solution converges as resolution is refined. Bisection provides a root-location guarantee, and Newton-Raphson converges to a small correction. Therefore, the engineering model has been physically validated and the final engineering result must be correct.”

7A • Review the AI Conclusion — 5 pts

The AI conclusion should be:

- ☐ A. ACCEPTED AS WRITTEN ☐ B. MODIFIED ☐ C. REJECTED IN FULL

Select ALL statements that identify what should be preserved or corrected:

- ☐ A. Agreement between Excel and Python can support implementation consistency.
☐ B. Numerical convergence can provide evidence that a computational result is settling.
☐ C. A bisection bracket can provide a numerical root-location guarantee when its conditions are satisfied.
☐ D. These numerical checks alone physically validate the engineering model and real system.

EXPECTED ANSWERS — SAMPLE EXAM

Use this section after attempting the sample exam. The answers are intentionally concise so that students can check the required result, decision, or interpretation.

Part 1

- 1A: B — double.
- 1B: $=\text{SQRT}(2*A3/\$B\$2)$; A3 = RELATIVE; $\$B\$2$ = ABSOLUTE.
- 1C: 2 s.
- 1D: height factor = 4; predicted time = 2 s; time factor = 2.
- 1E: B.

Part 2

- 2A: 8 m/s.
- 2B: 8, 8, and $d_s = 12$ m.
- 2C Check 2: v before 8; tested 12; $d_s = 21$; TRUE; v after 12.
- 2C Check 3: v before 12; tested 16; $d_s = 32$; FALSE; v after 12.
- 2D: 12.
- 2E: $v = 12$; B.
- 2F: B.

Part 3

- 3A: B — finer.
- 3B: 0.1 mm.
- 3C: 0.2 mm.
- 3D: A.
- 3E: A, B, D.

Part 4

- 4A: YES; A.
- 4B Iteration 1: new bracket [3, 4].
- 4B Iteration 2: $a = 3$, $b = 4$, $m_2 = 3.5$, POSITIVE, new bracket [3, 3.5].
- 4B Iteration 3: $a = 3$, $b = 3.5$, $m_3 = 3.25$, POSITIVE, new bracket [3, 3.25].
- 4C: $W = 0.25$; midpoint estimate = 3.125; guaranteed maximum root-location error = ± 0.125 ; YES.

Part 5

- 5A: $\Delta x_0 = -(-2.0)/4.0 = +0.5$; $x_1 = 2.5$.
- 5B: $\Delta x_1 = -(+0.5)/5.0 = -0.1$; $x_2 = 2.4$.
- 5C: $|\Delta x_1| = 0.10$; YES.
- 5D: B.
- 5E: B — Bisection.
- 5F: B.

Part 6

- 6A: $(2.20 - 1.40)/(0.30 - 0.20) = 0.80/0.10 = 8$ m/s.
- 6B: 1.80 m.
- 6C: C — ESTIMATED / INTERPOLATED.
- 6D: B — EXTRAPOLATION.
- 6E: B — SUSPICIOUS; reason A.

- 6F: NO.

Part 7

- 7A: B — MODIFIED. Select A, B, and C. Do not select D. The numerical evidence can support implementation consistency, convergence, and a numerical guarantee, but does not by itself physically validate the engineering model or real system.

INSTRUCTOR / TA REVIEW NOTES

Please attempt the sample exam as a student would: no calculator, no electronics, and using only the provided Formula Summary. Record total completion time and flag any wording that is ambiguous, any arithmetic that feels too time-consuming by hand, any answer that could reasonably have more than one interpretation, and any question where the expected answer is not obvious from the prompt.

Design constraints: 90 minutes; 100 points; hand-solvable arithmetic; familiar course terminology; Excel/Python assessed through tracing; numerical methods assessed through hand execution; responses structured for reliable AI-assisted grading.