

Formula Summary for CHAPTERS 1–6

Engineering models and computational logic through numerical
evidence, root finding, interpolation, and engineering judgment



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CHAPTER 1 · Engineering Model & Validation

Ideal free fall: $t = \sqrt{2h/g}$

Inverse forms: $g = 2h/t^2$ and $h = \frac{1}{2}gt^2$

Standard gravity: $g = 9.81 \text{ m/s}^2$

Scaling: $t \propto \sqrt{h}$ $9\times \text{height} \rightarrow 3\times \text{predicted time}$

Validation: model prediction $\leftrightarrow$ measured result

CHAPTER 2 · Algorithms, Decisions & Repetition

Reaction distance: $d_r = v t_r$

Braking distance: $d_b = v^2/(2a)$ $a = \text{positive deceleration magnitude}$

Total stopping distance: $d_s = v t_r + v^2/(2a)$

Decision test: SAFE if $d_s \leq d_{\text{available}}$

WHILE loop: repeat while condition is true *condition + body + mover*

Boundary logic: use $<, \leq, >, \geq$ as required *avoid equality tests on changing numerical values*

CHAPTER 3 · Numerical Accuracy, Resolution & Convergence

Absolute error: $E_{abs} = |x_{approx} - x_{ref}|$ *same units as x*

Relative error: $E_{rel} = \frac{|x_{approx} - x_{ref}|}{|x_{ref}|}$ *dimensionless*

Percent error: $E_{\%} = 100 (E_{rel}) \%$

Successive change: $\Delta_n = |x_n - x_{n-1}|$

Tolerance: required stopping threshold *report only justified precision*

Convergence evidence: changes shrink as resolution is refined *converged $\neq$ correct*

CHAPTER 4 · Root Finding by Bisection

Root form: $f(x^*) = 0$

Certified bracket: $f(a) \cdot f(b) \leq 0$ *continuous f on $[a,b]$*

Midpoint: $m = (a + b)/2$

Bracket update: if $f(a) \cdot f(m) \leq 0$, set $b = m$; otherwise set $a = m$

Width after n halvings: $W_n = W_0/2^n$

Midpoint error bound: $|x^* - m| \leq W_0/2^{n+1}$

Stopping requirement: choose n so $W_0/2^{n+1} \leq \text{tol}$

CHAPTER 5 · Newton–Raphson

Newton update:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Correction:

$$\Delta x_n = -\frac{f(x_n)}{f'(x_n)}$$

Stopping rule: $|\Delta x_n| \leq \text{tol}$

Two-point slope check:

$$f'(x) \approx \frac{[f(x + \Delta x) - f(x)]}{\Delta x}$$

Guard: verify every iterate remains in the physical domain

After convergence: check residual $f(x^*)$, physics, units, and reasonableness

CHAPTER 6 · Missing Data & Linear Interpolation

Local slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Linear interpolation:

$$y(x) = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) = y_1 + m(x - x_1)$$

Interpolation: $x_1 \leq x \leq x_2$ *outside this interval is extrapolation*

Circular-motion physics check: $x^2 + y^2 \approx R^2$ *when applicable to the geometry/model*

Data label: gap values are ESTIMATED / INTERPOLATED *an estimate does not become a measurement*

EXAM HABIT Write units. State assumptions. Predict before computing. Report only justified precision. A program that runs is not evidence that the answer is correct.
