

FUNDAMENTALS OF AI-AUGMENTED ENGINEERING: A CONVERSATIONAL APPROACH

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The engineering workflow you will use throughout this book

CHAPTER - 8

PART 2 · ≈ 35 MIN

From Physical Laws to a Matrix

Part 1 showed that V_1 and V_2 must be found together. Part 2 writes the equations — Ohm's law for every resistor, conservation at every node — and organizes them into one matrix equation in which every number has a physical address.

2

Problem

›

Model

›

Compute

›

Analyze

›

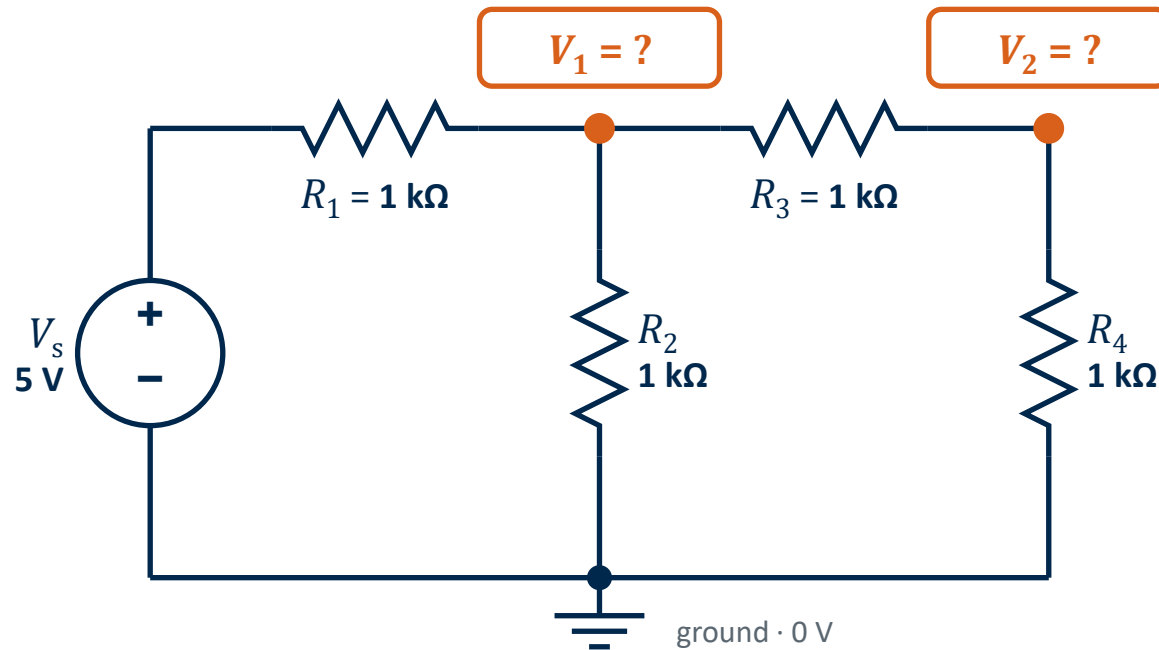
Validate

›

Improve



Two unknowns, two balances — now write them as equations



The same circuit as Part 1. 5 V source, four 1 kΩ resistors, ground = 0 V.

FROM PART 1

Balance at node V1: $f_1(V_1, V_2) = 0$

Balance at node V2: $f_2(V_1, V_2) = 0$

Both must hold at once.

ASK THE ROOM

What must we know to turn each balance into an equation we can actually write down?

UNITS FOR THE CHAPTER

voltage V · resistance kΩ · current mA · conductance mS

Every current must be written in terms of V1 and V2. Ohm's law does that — one resistor at a time.

Problem



Model



Compute



Analyze



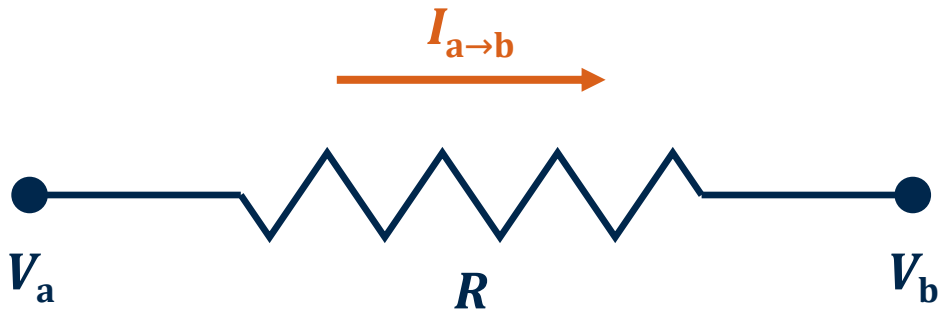
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One resistor links the voltages at its two ends



$$I_{a \rightarrow b} = (V_a - V_b) / R$$

Voltage at the arrow's tail minus voltage at its head, divided by the resistance.

Units: volts ÷ kilohms = milliamps.

ASK THE ROOM

Reverse the arrow so it points from V_b to V_a . Write the new expression.

How is it related to the old one?

REVIEW NOTE Assumed direction and sign

The arrow is a bookkeeping choice made **before** solving. Always write (tail – head) / R.

If the result is negative, the real current flows against the arrow. The size of the current is the same either way — only the sign records the direction.

One resistor = one relationship. Its current depends only on the voltages at its own two ends.

Problem



Model



Compute



Analyze



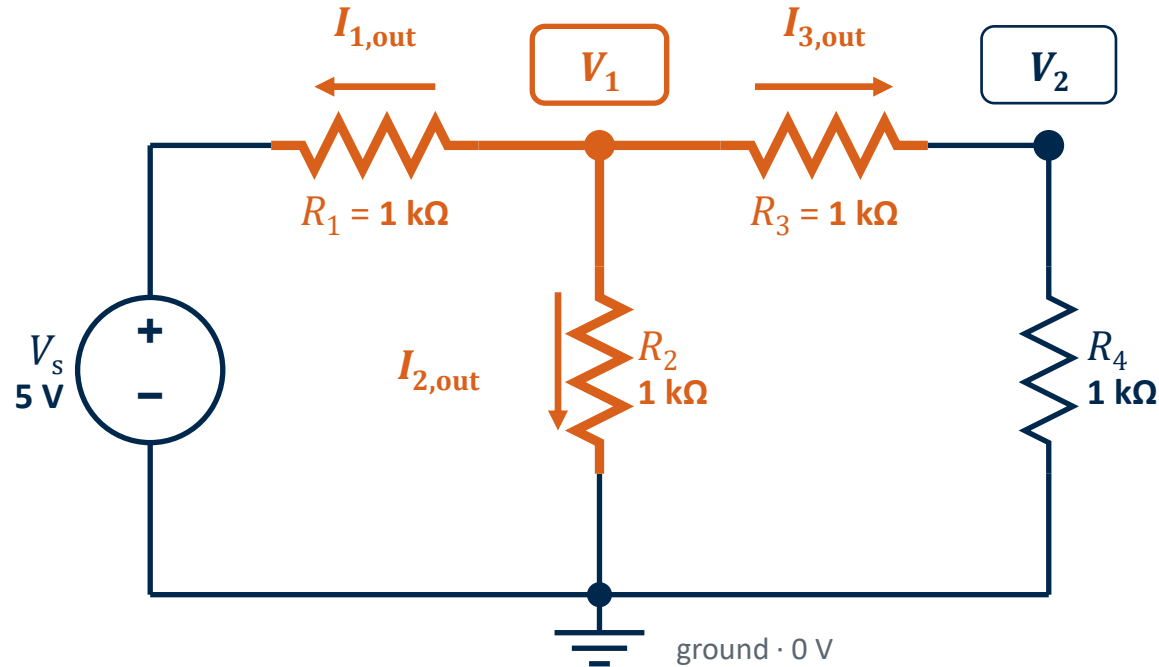
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Three currents meet at V1 — define each one as leaving



CURRENTS LEAVING V1

$$I_{1,\text{out}} = (V_1 - V_s) / R_1$$

$$I_{2,\text{out}} = (V_1 - 0) / R_2 = V_1 / R_2$$

$$I_{3,\text{out}} = (V_1 - V_2) / R_3$$

A NEGATIVE CURRENT IS FINE

Current in R1 really flows **into** V1 from the 5 V side, so $I_{1,\text{out}}$ will be negative. The definition is still correct.

ASK THE ROOM

Which current depends on BOTH unknowns?

Every arrow points **AWAY** from V1. One rule for every node: define currents as leaving.

Same rule, every resistor: (tail – head) / R. The tail is always V1, because every arrow leaves V1.

Problem



Model



Compute



Analyze



Validate



Improve



Conservation turns three currents into one equation

KIRCHHOFF'S CURRENT LAW
AT V1

$$\Sigma I_{\text{leaving V1}} = 0 \rightarrow I_{1,\text{out}} + I_{2,\text{out}} + I_{3,\text{out}} = 0$$

$$\frac{(V_1 - V_s)}{R_1} + \frac{V_1}{R_2} + \frac{(V_1 - V_2)}{R_3} = 0$$

branch ①

branch ②

branch ③

ASK THE ROOM

Name each branch: which one leads to the source, which one to ground, and which one couples V_1 to V_2 ?

REVIEW NOTE An algebraic current balance

Kirchhoff's Current Law: the currents leaving a node add up to zero. Because every current is **defined** as leaving, some come out negative — those really enter. "Sum leaving = 0" and Part 1's "in = out" say the same thing.

The first node equation — straight from physics. One unknown too many to solve alone: V_2 is still in it.

Problem



Model



Compute



Analyze



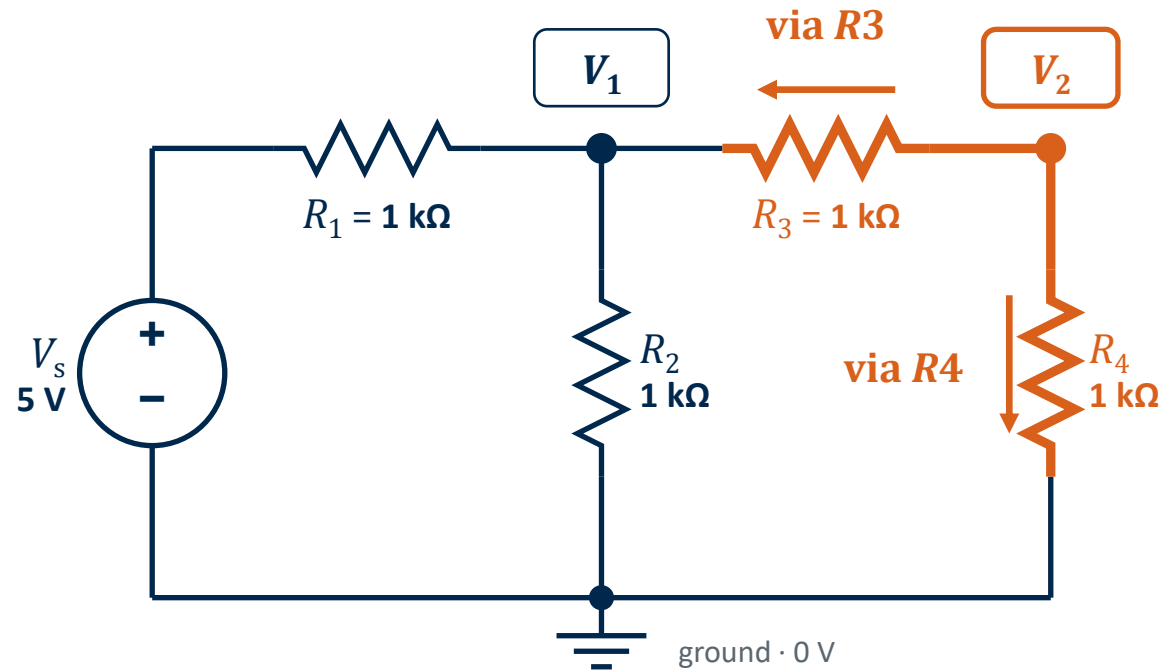
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YOUR TURN



Both arrows point **AWAY** from **V2** — the same rule as at V1.

Write the current balance at node V_2 yourself.

YOUR WORKSHEET

current leaving through R3 = _____

current leaving through R4 = _____

sum of currents leaving V2 = **0**

Then compare with a neighbor. Where do your R3 terms differ from the V1 equation?

2 minutes. Use (tail – head) / R. The tail is V2 for both arrows.

Try it before the next slide. Same rule, same convention — a new node.

Problem



Model



Compute



Analyze



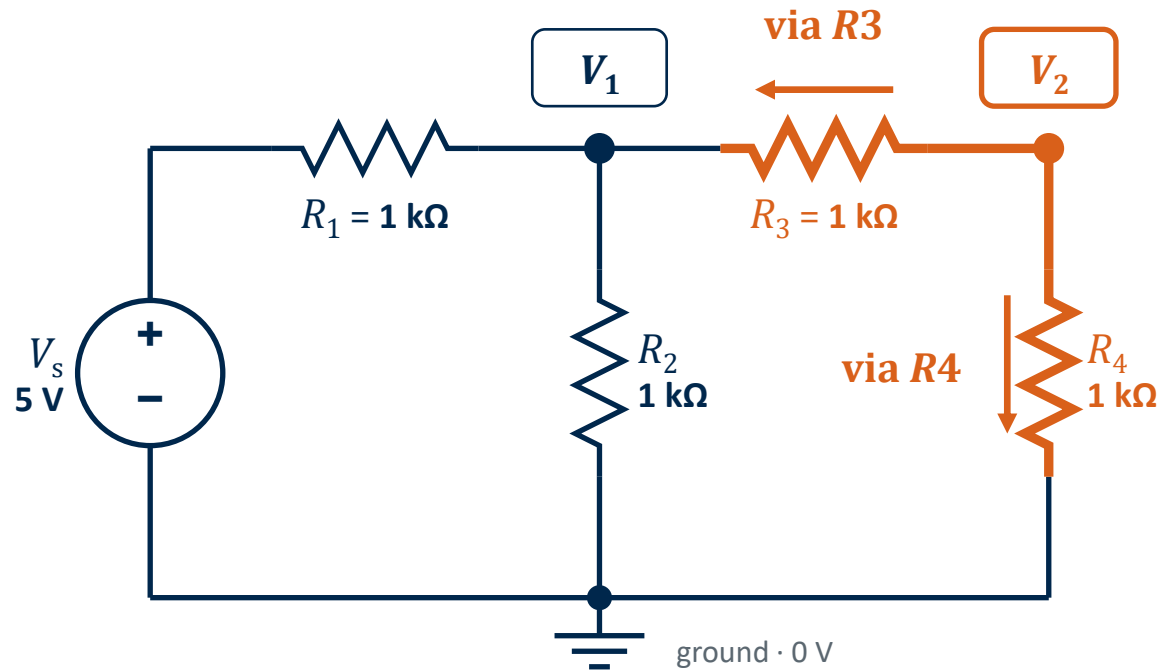
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The second node supplies the second equation



No source touches V2. Only R3 and R4 carry current to or from it.

CURRENTS LEAVING V2

via R3: $(V_2 - V_1) / R_3$

via R4: $(V_2 - 0) / R_4 = V_2 / R_4$

$$(V_2 - V_1) / R_3 + V_2 / R_4 = 0$$

WHY THE R3 TERM FLIPPED

At V1: $(V_1 - V_2) / R_3$ leaving V1

At V2: $(V_2 - V_1) / R_3$ leaving V2

Same resistor, same physical current, opposite reference direction — so opposite sign. Both are correct.

Two nodes, two equations. Both written with one rule — so they agree with each other.

Problem



Model



Compute



Analyze



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Improve



Collect the V1 terms and the V2 terms

NODE V1

$$(V_1 - V_s)/R_1 + V_1/R_2 + (V_1 - V_2)/R_3 = 0$$

$$\rightarrow (1/R_1 + 1/R_2 + 1/R_3) V_1 - (1/R_3) V_2 = V_s/R_1$$

NODE V2

$$(V_2 - V_1)/R_3 + V_2/R_4 = 0$$

$$\rightarrow -(1/R_3) V_1 + (1/R_3 + 1/R_4) V_2 = 0$$

ASK THE ROOM

Both coupling coefficients are **negative**. Which resistor puts them there — and why the minus sign?

THE GENERAL FORM

$$a_{11} V_1 + a_{12} V_2 = b_1$$

$$a_{21} V_1 + a_{22} V_2 = b_2$$

$$a_{11} = 1/R_1 + 1/R_2 + 1/R_3 \quad a_{12} = -1/R_3$$

$$a_{21} = -1/R_3 \quad a_{22} = 1/R_3 + 1/R_4$$

$$b_1 = V_s/R_1 \quad b_2 = 0$$

REVIEW NOTE

What makes it linear?

Each unknown appears to the first power, times a constant. No $V_1 V_2$, no V_1^2 , no $\sin(V)$.

Every coefficient is built from resistances. The known source term moves to the right-hand side.

Problem



Model



Compute



Analyze



Validate



Improve



Two equations, written once: $A x = b$

THE TWO EQUATIONS

$$a_{11} V_1 + a_{12} V_2 = b_1$$

$$a_{21} V_1 + a_{22} V_2 = b_2$$



MATRIX FORM

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$A \quad x \quad b$

A

coefficient matrix

how the circuit is connected · mS

x

unknown vector

the node voltages V_1, V_2 · V

b

right-hand side

what the source drives in · mA

REVIEW NOTE

Matrix × vector = the original equations

Row 1 of A times x : $a_{11} \cdot V_1 + a_{12} \cdot V_2$ — exactly the left side of equation 1. Row 2 times x gives equation 2. **$A x = b$ is the two equations, written at once.**

Nothing new has been added. $A x = b$ is the same two node balances, organized for a computer.

Problem



Model



Compute



Analyze



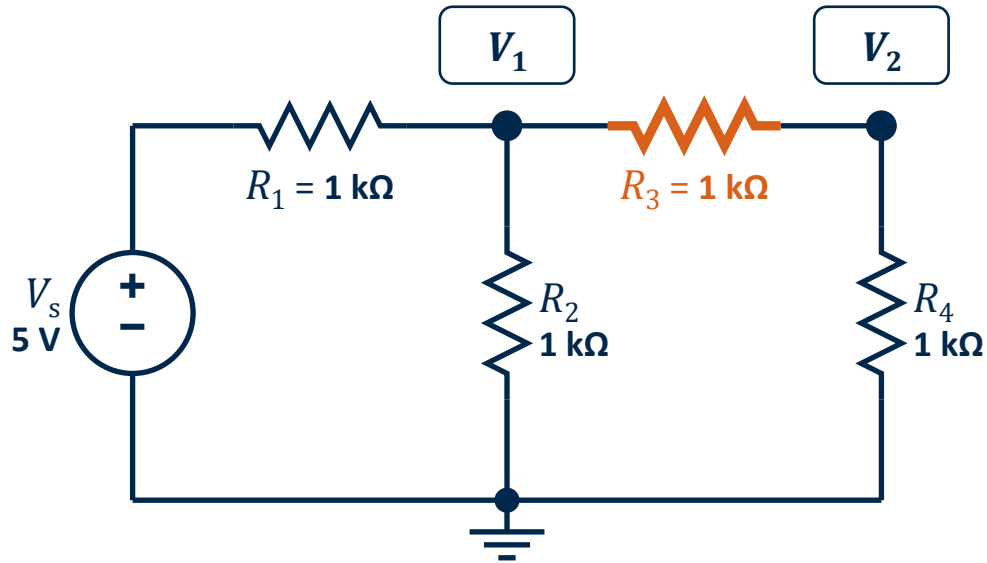
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Every entry of the matrix has a physical address



Rows are node equations (row 1 = node V1, row 2 = node V2); **columns** collect the coefficients of one unknown.

$$\begin{array}{c} \text{column: } \times V_1 \qquad \text{column: } \times V_2 \\ \left[\begin{array}{cc} 1/R_1 + 1/R_2 + 1/R_3 & -1/R_3 \\ -1/R_3 & 1/R_3 + 1/R_4 \end{array} \right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_s/R_1 \\ 0 \end{bmatrix}
 \end{array}$$

☐ **DIAGONAL** all conductances touching that node: V1 has R1, R2, R3; V2 has R3, R4

☐ **OFF-DIAGONAL** minus the conductance directly linking V1 and V2 — R3, highlighted in both places

☐ **RIGHT-HAND SIDE** what the source pushes into each node: through R1 into V1; nothing into V2

Highlight R3 in the circuit — it appears twice in A. Coupling lives off the diagonal.

Problem

**Model**

Compute



Analyze



Validate



Improve



Numbers with physical meaning

WHERE EACH NUMBER COMES FROM

$$a_{11} = 1/1 + 1/1 + 1/1 = 3 \text{ mS}$$

R1, R2, R3 touch V1

$$a_{22} = 1/1 + 1/1 = 2 \text{ mS}$$

R3, R4 touch V2

$$a_{12} = a_{21} = -1/1 = -1 \text{ mS}$$

R3 links V1 and V2

$$b_1 = 5 \text{ V} / 1 \text{ k}\Omega = 5 \text{ mA}$$

source, through R1

$$b_2 = 0 \text{ mA}$$

no source at V2

REVIEW NOTE Conductance and units

Conductance $G = 1/R$, unit siemens (S): how easily current flows. $1 / (1 \text{ k}\Omega) = 1 \text{ mS}$. The entries of A are conductances — not pure numbers.

$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

mS V mA

Unit check: mS × V = mA — every row is a current balance.

ASK THE ROOM

Point to the part of the circuit that produces each number:
3, 2, -1, -1, 5, 0.

Which numbers would change if R4 changed?

3, 2, -1, -1, 5, 0 — each is a piece of the circuit. Change a resistor and you change a number.

Problem



Model



Compute



Analyze



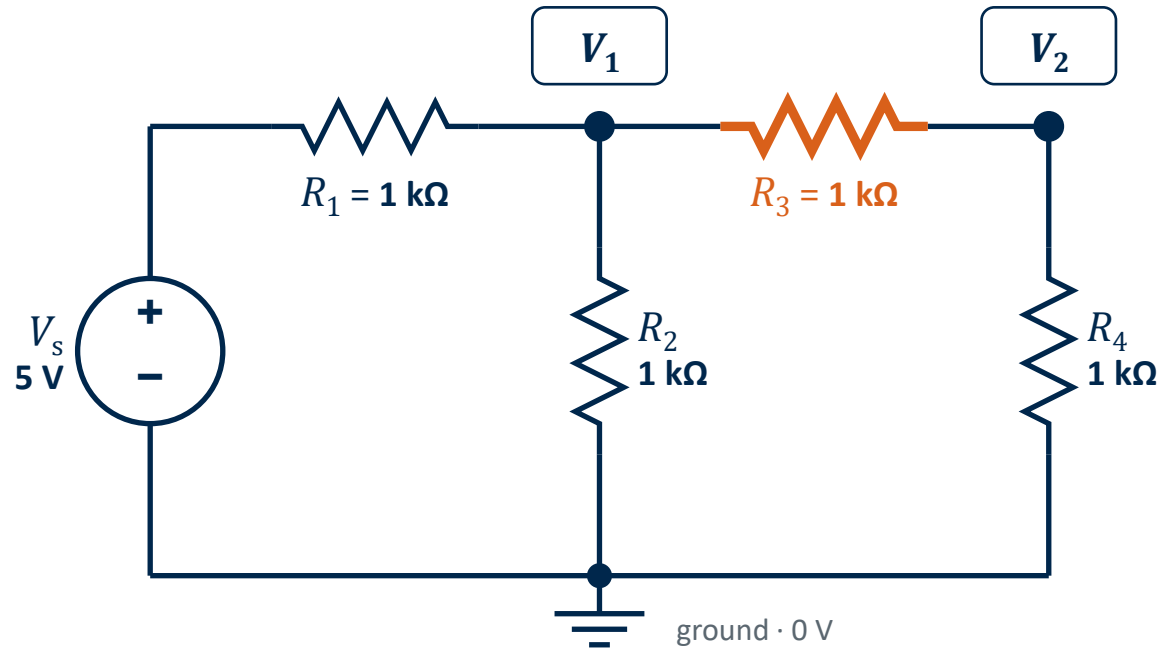
Validate



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Can we read the circuit from the matrix?



$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

- 1 Why are the two off-diagonal entries equal?
- 2 Remove R_3 (let $R_3 \rightarrow \infty$). What happens to those entries — and to the diagonal?
- 3 Why is the second entry of b zero?
- 4 Does $b_2 = 0$ mean no current flows through the resistors connected to V_2 — or $V_2 = 0$?

Think physically, not algebraically. Every answer should point at a wire or a resistor.

The matrix is a map of the circuit. If you cannot name a physical reason for an entry, check the model.

Problem



Model



Compute



Analyze



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Improve



The model is ready. Is the answer?

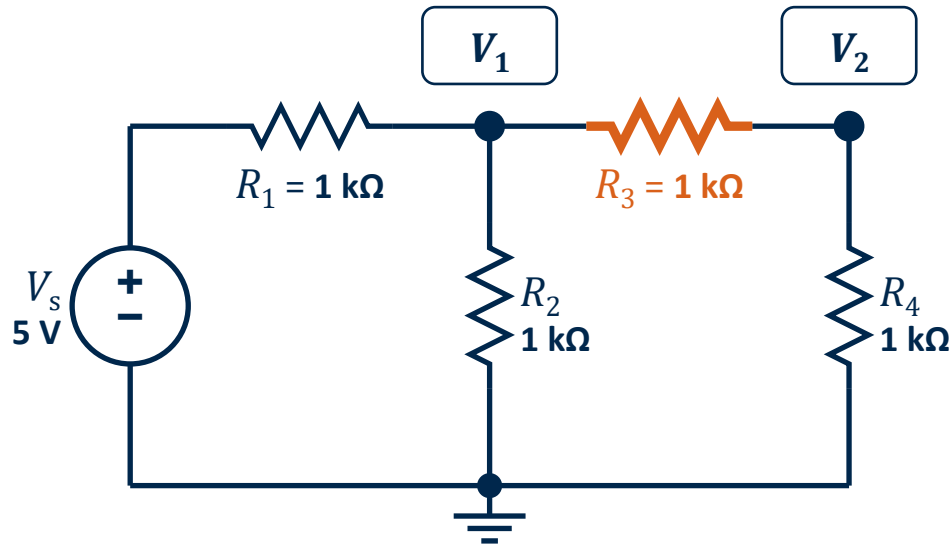
PHYSICAL SYSTEM

OHM'S LAW

CONSERVATION

LINEAR EQUATIONS

MATRIX



$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

A in mS
x in V
b in mA

NEXT — PART 3

What operations can we perform on these equations without changing their solution?

Part 3 · Solve it by hand, then let the computer solve it

The model is ready. The answer is not — yet. Part 3 solves it: first by hand, then by computer.

Problem

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