

# FUNDAMENTALS OF AI-AUGMENTED ENGINEERING: A CONVERSATIONAL APPROACH

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The engineering workflow you will use throughout this book

CHAPTER - 8

PART 3 · ≈ 40 MIN

# Solve It by Hand, Then Let the Computer Solve It

The matrix from Part 2 holds the whole model, but not yet a single number we could measure. Solve it by hand to see WHY elimination works, solve it in MATLAB to see how engineers do it — then check what the answer does and does not prove.

3

Problem

›

Model

›

**Compute**

›

Analyze

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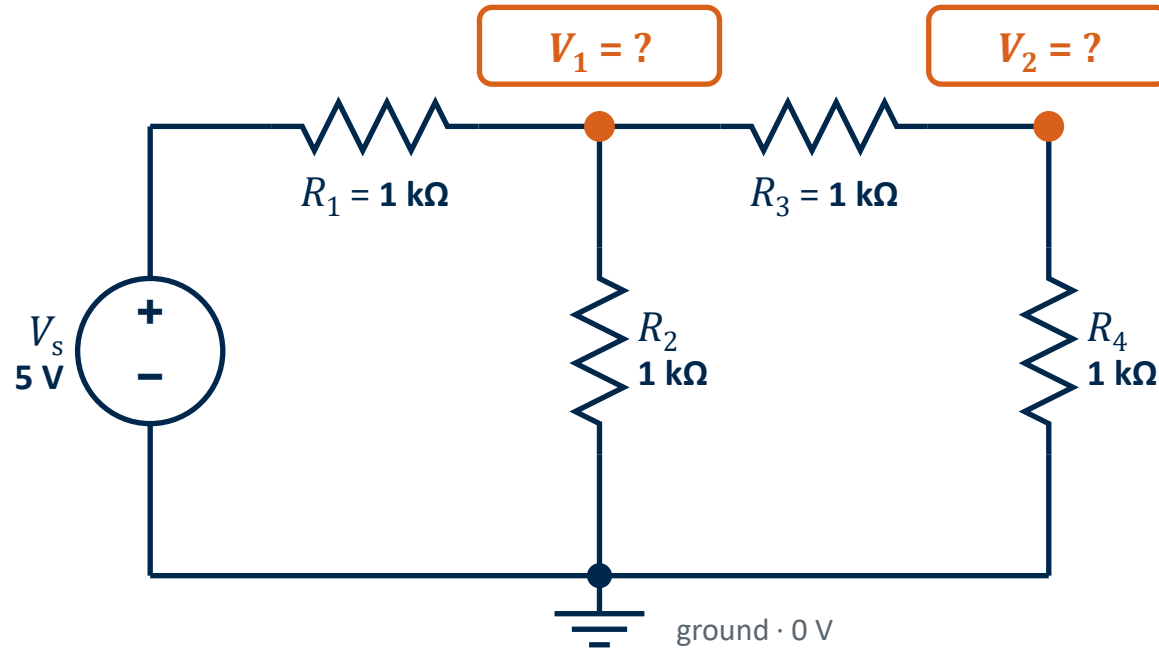
Validate

›

Improve



# The model is ready. Where are the voltages?



Same circuit, same matrix as Part 2. A in mS, x in V, b in mA.

## THE MODEL FROM PART 2

$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad \begin{matrix} \text{mS} \cdot \text{V} \\ = \text{mA} \end{matrix}$$

## MODEL ≠ PREDICTION

We know the equations  $V_1$  and  $V_2$  must satisfy — but not yet one number a meter could check.

## ASK THE ROOM

Which values of  $V_1$  and  $V_2$  satisfy BOTH equations at once?

**A model becomes a prediction only when it is solved.** Today: by hand, then by computer — then we check.

Problem



Model



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# What can we change without changing the answer?

Our two equations:  $3V_1 - V_2 = 5$  (row 1)     $-V_1 + 2V_2 = 0$  (row 2)

## 1 INTERCHANGE

$$R_1 \leftrightarrow R_2$$

Swap two rows — list the same equations in a different order.

## 2 SCALE

$$R_2 \leftarrow c R_2, \quad c \neq 0$$

Multiply both sides of one equation by a nonzero number. Example:  $\times 2$  gives  $-2V_1 + 4V_2 = 0$ .

## 3 REPLACE

$$R_2 \leftarrow R_2 + k R_1$$

Add a multiple of one equation to another: adding two true balances gives a true balance.

### REVIEW NOTE    Equivalent equations

Two systems are equivalent if they have exactly the same solutions. Each operation can be undone — swap back, divide by  $c$ , subtract  $k R_1$  — so no solution is gained or lost.

### ASK THE ROOM

Why is multiplying an equation by **zero** not allowed?

**Row operations change how the equations look,** never which voltages satisfy them.

Problem



Model



Compute



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# The augmented matrix: a compact workspace

## THE EQUATIONS

$$3V_1 - 1V_2 = 5$$

$$-1V_1 + 2V_2 = 0$$

units:  $mS \cdot V = mA$



## THE AUGMENTED MATRIX [ A | b ]

$$\left[ \begin{array}{cc|c} 3 & -1 & 5 \\ -1 & 2 & 0 \end{array} \right]$$

V1 column   V2 column   right side

left of the bar: A (mS)  
right of the bar: b (mA)

## THE WHOLE ROW MOVES TOGETHER

A row operation acts on **every** entry of the row — including the number to the right of the bar. Forgetting the right side is the most common hand-elimination error.

### REVIEW NOTE

### Reading an augmented matrix

A and b side by side, separated by a bar that stands for "=". Each row is one equation. The unknowns are not written — their **positions** are: column 1 always multiplies V1, column 2 always multiplies V2.

**Same equations, less writing.** Every row operation now acts on one row of three numbers.

Problem



Model



Compute



Analyze



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# Eliminate one unknown: remove V1 from row 2

$$\left[ \begin{array}{cc|c} 3 & -1 & 5 \\ -1 & 2 & 0 \end{array} \right] \xrightarrow{R_2 \leftarrow R_2 + (1/3) R_1} \left[ \begin{array}{cc|c} 3 & -1 & 5 \\ 0 & 5/3 & 5/3 \end{array} \right]$$

## ROW 2, ENTRY BY ENTRY

$$-1 + (1/3)(3) = 0 \quad \text{V1 is gone}$$

$$2 + (1/3)(-1) = 5/3$$

$$0 + (1/3)(5) = 5/3 \quad \text{right side too}$$

Why 1/3? It is the multiple of row 1 that cancels the -1 under the pivot 3.

## ASK THE ROOM

After this step, what has changed:

**the physical circuit? the solution? or only how the equations are written?**

Row 2 now reads  **$(5/3) V_2 = 5/3$** .

**Row 2 no longer contains V1.** One equation, one unknown — the coupling has been worked out.

Problem

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Model

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# Turn the second pivot into one

$$\left[ \begin{array}{cc|c} 3 & -1 & 5 \\ 0 & 5/3 & 5/3 \end{array} \right] \xrightarrow{R_2 \leftarrow (3/5) R_2} \left[ \begin{array}{cc|c} 3 & -1 & 5 \\ 0 & 1 & 1 \end{array} \right]$$

## REVIEW NOTE Pivots

A pivot is the diagonal entry used to clear its column. **Normalizing** scales its row so the pivot becomes 1 — every entry, right side too:  $5/3 \times 3/5 = 1$ .

## ASK THE ROOM

Read row 2 as an equation. What does it say?

## FIRST RESULT

$$0 \cdot V_1 + 1 \cdot V_2 = 1$$

$$V_2 = 1.00 \text{ V}$$

$$1 \text{ mA} \div 1 \text{ mS} = 1 \text{ V}$$

**A pivot of 1 lets us read an unknown directly.**  $V_2 = 1.00 \text{ V}$  — the first number this chapter has computed.

Problem

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# Finish the elimination: the identity matrix appears

$$\left[ \begin{array}{cc|c} 3 & -1 & 5 \\ 0 & 1 & 1 \end{array} \right] \xrightarrow{R_1 \leftarrow R_1 + R_2} \left[ \begin{array}{cc|c} 3 & 0 & 6 \\ 0 & 1 & 1 \end{array} \right] \xrightarrow{R_1 \leftarrow (1/3) R_1} \left[ \begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array} \right]$$

identity | solution

$$\begin{aligned} V_1 &= 2.00 \text{ V} \\ V_2 &= 1.00 \text{ V} \end{aligned}$$

## THE ARITHMETIC

Row 1 + Row 2:

$$[3 \ -1 \ | \ 5] + [0 \ 1 \ | \ 1] = [3 \ 0 \ | \ 6]$$

Row 1 ÷ 3:

$$[3 \ 0 \ | \ 6] \div 3 = [1 \ 0 \ | \ 2]$$

## REVIEW NOTE

The identity matrix

1s on the diagonal, 0s elsewhere. When the left side is the identity, each row reads **one unknown = one number**:  $1 \cdot V_1 + 0 \cdot V_2 = 2$  and  $0 \cdot V_1 + 1 \cdot V_2 = 1$ .

## ASK THE ROOM

**Is this plausible for a 5 V supply?**

Between 0 and 5 V? Which node should be higher, and why?

**Gauss-Jordan:  $V_1 = 2.00 \text{ V}$ ,  $V_2 = 1.00 \text{ V}$ .** Solved by hand, using only operations that keep the answer.

Problem



Model



Compute



Analyze



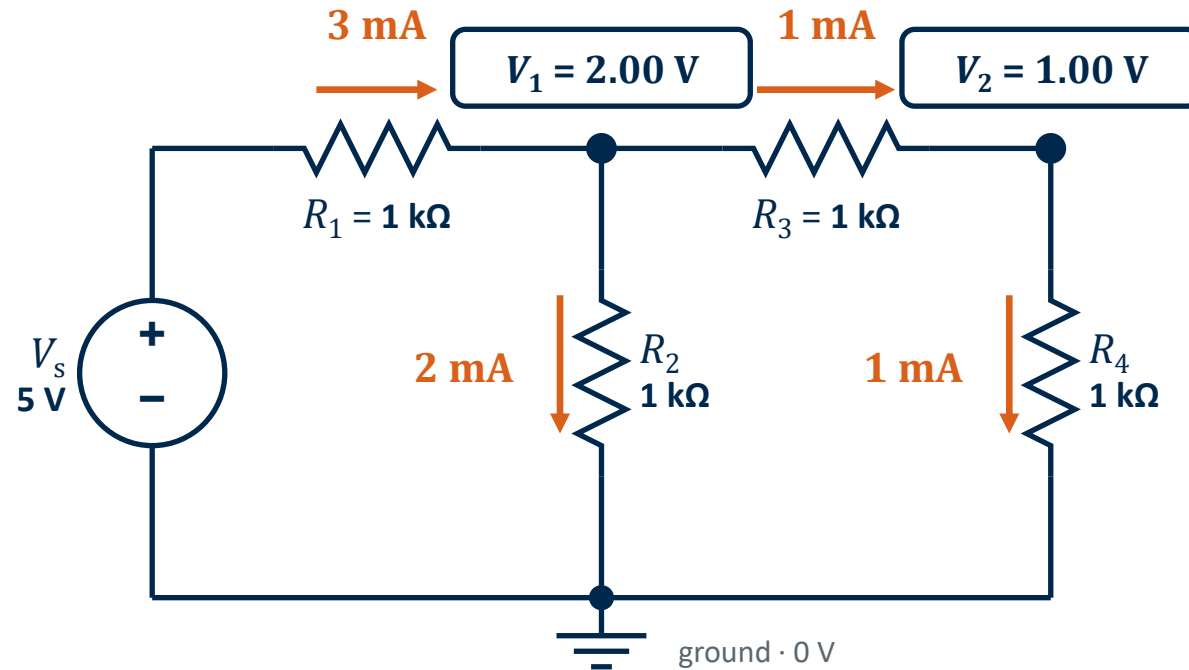
Validate



Improve



# Do the voltages make physical sense?



Arrows show the actual directions. Every current flows from the higher voltage to the lower one.

## BRANCH CURRENTS

$$R1: (5 - 2) / 1 = \mathbf{3\text{ mA}} \text{ source} \rightarrow V1$$

$$R2: (2 - 0) / 1 = \mathbf{2\text{ mA}} \text{ } V1 \rightarrow \text{ground}$$

$$R3: (2 - 1) / 1 = \mathbf{1\text{ mA}} \text{ } V1 \rightarrow V2$$

$$R4: (1 - 0) / 1 = \mathbf{1\text{ mA}} \text{ } V2 \rightarrow \text{ground}$$

## CURRENT BALANCE

$$\text{At } V1: 3\text{ mA in} = 2\text{ mA} + 1\text{ mA out} \checkmark$$

$$\text{At } V2: 1\text{ mA in} = 1\text{ mA out} \checkmark$$

$$\text{Part 2's } I_{1,\text{out}} = (2 - 5)/1 = -3\text{ mA: negative, as expected.}$$

**Consistent with our model — not yet validated.** These checks reuse the assumed circuit. Only a measurement tests it.

Problem



Model



Compute



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# Now let MATLAB solve it

```
% Part 2 model: A in mS, b in mA, x in V
A = [3 -1; -1 2];
b = [5; 0];

x = A\b
```

## COMMAND WINDOW

```
x =
    2.0000
    1.0000
```

**Same as the hand solution: 2 V and 1 V.** Round-off decides whether MATLAB prints 2 or 2.0000.

## REVIEW NOTE MATLAB matrix syntax

[ ] builds a matrix; a space separates columns; ; inside [ ] starts a new row.

[5; 0] is a column vector. A ; at the end of a line hides the output.

**A\b** means: solve  $Ax = b$  for  $x$ .

## MATLAB SEES ONLY NUMBERS

A holds mS, b holds mA, x comes out in V — but only because **we** kept the units consistent. Type 1000 for a 1 k $\Omega$  resistor and MATLAB will solve a different circuit without complaint.

**Run the original system and confirm the hand solution. We will investigate changes to the physical system in Part 4.**

Problem



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**Compute**

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# Why $A \setminus b$ instead of $\text{inv}(A) * b$ ?

$$x = A \setminus b$$

## SOLVE

Asks MATLAB to solve  $A x = b$  directly.

MATLAB inspects  $A$  (symmetric? triangular? ...) and picks a suitable numerical method.

**The preferred general way to solve a linear system.**

$$x = \text{inv}(A) * b$$

## INVERT, THEN MULTIPLY

First builds the whole inverse matrix, then multiplies it by  $b$ .

More work than needed, and usually no more accurate — often slightly less.

**Rarely necessary: we want  $x$ , not  $\text{inv}(A)$ .**

## WHAT BACKSLASH IS NOT

It is not our classroom Gauss-Jordan procedure run by a computer. Gauss-Jordan shows the **logic** of elimination; MATLAB uses numerical algorithms suited to the matrix it is given.

## ASK THE ROOM

Both give 2 V and 1 V here. So why should the choice matter?

**Solve, don't invert.** Ask the computer for what you need:  $x$ .

Problem



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**Compute**



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# The residual: does the answer satisfy the equations?

$$r = A x - b$$

$$r = A * x - b$$

One number per equation: how far that node balance is from zero.

## NUMERICAL VERIFICATION

Does the computed  $x$  satisfy the equations **we supplied**?

**Tool: the residual. Today: yes.**

## FOR OUR SOLUTION

Row 1:  $3(2) - 1(1) - 5 = 0 \text{ mA}$

Row 2:  $-1(2) + 2(1) - 0 = 0 \text{ mA}$

## PHYSICAL VALIDATION

Do those equations — and their predictions — agree with the **real** circuit?

**Tool: measurement (Lab 8). Today: not yet tested.**

## REVIEW NOTE Numerical residuals

Units: mA of current imbalance at each node. Tiny values such as  $1e-16$  are round-off, not mistakes.

**Same word, new job.** Chapter 7's residual compared a model with measured data. This residual compares an answer with its own equations.

**A small residual confirms that the computed solution closely satisfies the supplied equations.  
It does not establish that the model represents the real system.**

Problem



Model



Compute



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# A solved system is not yet a validated system

PHYSICAL MODEL ›

MATRIX ›

HAND SOLUTION ›

MATLAB SOLUTION ›

NUMERICAL  
VERIFICATION

## WHAT WE NOW KNOW

$$V_1 = 2.00 \text{ V} \quad V_2 = 1.00 \text{ V}$$

They satisfy our equations — by hand, in MATLAB, and by the residual.

## ASK THE ROOM

Suppose R2 is really 1.2 kΩ, but our model says 1 kΩ. Could MATLAB still return a residual near zero?

## NEXT — PART 4 · WHEN THE SYSTEM CHANGES AND GROWS

### What happens when the physical system changes?

*Your R4 prediction from Part 1 returns — still unanswered.*

**Yes — MATLAB solves the model we give it, right or wrong.** Only measurement can tell which.

Problem ›

Model ›

Compute ›

Analyze ›

**Validate** ›

Improve