



Self-Assessment 8

Coupled Equations · Building $Ax = b$ · Alb and the Residual · Change and Growth · Questioning the Model

Ungraded · for your own learning · work at your own pace

Taner Onsay, Ph.D. & Sebnem Onsay · © 2026 AI First Engineering

AI TUTOR MODE — read this first

You do not need to complete every question in one sitting. Start with each group. If you can confidently answer the first question independently, sample the rest. Spend more time in groups where you need AI or human help. In Chapter 8 your AI can write the MATLAB in seconds — which is exactly why the formulation is yours: which equation belongs to which unknown, what each coefficient means physically, and what a residual can and cannot prove. If you become stuck, instead of asking for the answer, try:

"I am working on this question. Do not tell me the answer or which choice is correct. Ask me one question or give me one small hint at a time. Help me reason through the physics of each node, the sign of each coefficient, the units, or what the residual actually checks, until I can explain it myself."

The purpose of this self-assessment is to determine what YOU understand — not what your AI can answer. This is the working method in action: THINK › PSEUDOCODE › AI › CODE — the judgment stays yours.

After each group, rate yourself with the four checkboxes. Your pattern of ratings — not a score — is the output: it tells you exactly what to review, practice, or bring to office hours — before Lab-8 asks you to predict a real circuit with a matrix and Homework 8 asks you to model five different systems. Each question has one best answer: choose it, then write one sentence saying WHY. A calculator is all you need — no computer, and no clock. Every question here is one question in disguise: does the matrix describe the system — and how would you know?

GROUP 1 · Couple It — Why One Equation Is Not Enough

Coupled unknowns, coupled equations. Read each choice as an engineering claim.

1. A two-node circuit has two unknown node voltages, V_1 and V_2 . A student writes KCL at node V_1 , then multiplies that same equation by 2 and uses it as the "second equation." Why can this pair NOT determine V_1 and V_2 ?

- A) MATLAB can only solve systems whose equations have different units.
- B) Multiplying by 2 changes the physics, so the second equation is false.
- C) The second equation adds no new information: two unknowns need two INDEPENDENT equations — here, KCL at node V_2 .
- D) The coefficients in the second equation are larger, so the system is ill-conditioned but still solvable.

Your choice: A ☐ B ☐ C ☐ D ☐ **Why (one sentence):**

2. In a nodal matrix A, entry $a_{12} = -G_3$, where R3 joins nodes 1 and 2. What does the NEGATIVE sign mean physically?
- A) Raising V_2 (with V_1 held fixed) REDUCES the current leaving node 1 through R3 — the two nodes are coupled through R3.
 - B) R3 behaves like a negative resistance.
 - C) Current always flows from node 2 to node 1 through R3.
 - D) It is an arbitrary sign convention; only the magnitude G_3 carries meaning.
- Your choice: A ☐ B ☐ C ☐ D ☐ Why (one sentence):

RATE YOURSELF on this group before moving on:

- I can do this independently.
- I can do this with a small AI hint.
- I need substantial AI help.
- I still need human/instructor help.

GROUP 2 · Build It — From the Circuit to $Ax = b$

Construct the system and keep its units honest.

Reference circuit (Questions 3, 7 and 9): a supply $V_s = 10\text{ V}$ feeds node V_1 through $R_1 = 2\text{ k}\Omega$; $R_2 = 5\text{ k}\Omega$ joins V_1 to ground; $R_3 = 1\text{ k}\Omega$ joins V_1 to V_2 ; $R_4 = 4\text{ k}\Omega$ joins V_2 to ground. Units: V, k Ω , mS, mA; KCL with the currents leaving each node summing to zero.

3. Which matrix A and vector b correctly model the reference circuit, with $x = [V_1; V_2]$?
- A) $A = [1.7 \quad 1; 1 \quad 1.25]\text{ mS}$, $b = [5; 0]\text{ mA}$
 - B) $A = [0.7 \quad -1; -1 \quad 1.25]\text{ mS}$, $b = [5; 0]\text{ mA}$
 - C) $A = [1.7 \quad -1; -1 \quad 1.25]\text{ mS}$, $b = [10; 0]\text{ mA}$
 - D) $A = [1.7 \quad -1; -1 \quad 1.25]\text{ mS}$, $b = [5; 0]\text{ mA}$
- Your choice: A ☐ B ☐ C ☐ D ☐ Why (one sentence):

-
4. In a nodal model the entries of A are in mS and the unknowns in V. A supply $V_s = 6\text{ V}$ feeds node 1 through $R_1 = 2\text{ k}\Omega$. What are the units of b, and what is b_1 ?
- A) Volts; $b_1 = 6\text{ V}$
 - B) Milliamperes; $b_1 = G_1 \cdot V_s = 3\text{ mA}$
 - C) Millisiemens; $b_1 = G_1 = 0.5\text{ mS}$
 - D) k $\Omega \cdot \text{mA}$; $b_1 = R_1 \cdot V_s = 12$
- Your choice: A ☐ B ☐ C ☐ D ☐ Why (one sentence):

RATE YOURSELF on this group before moving on:

- I can do this independently.
- I can do this with a small AI hint.
- I need substantial AI help.
- I still need human/instructor help.

GROUP 3 · Solve It — $A \backslash b$ and What the Residual Says

The computer solves. You decide what the solution means.

5. Which MATLAB line is the Chapter 8 way to solve $Ax = b$ for a column vector b ?

- A) $x = b \backslash A$
- B) $x = A * b$
- C) $x = A \backslash b$
- D) $x = b ./ A$

Your choice: A ☐ B ☐ C ☐ D ☐ Why (one sentence):

6. After solving, MATLAB gives $r = A * x - b$ with every entry about 10^{-15} mA. What does this establish?

- A) x satisfies the equations you entered, to round-off — it says nothing about whether A and b describe the real circuit correctly.
- B) The circuit model is validated and the meter will read these voltages.
- C) Measured node voltages will agree with x to within 10^{-15} V.
- D) The matrix A is singular.

Your choice: A ☐ B ☐ C ☐ D ☐ Why (one sentence):

RATE YOURSELF on this group before moving on:

- I can do this independently.
- I can do this with a small AI hint.
- I need substantial AI help.
- I still need human/instructor help.

GROUP 4 · Change It — Parameters and Growth

A model is useful because you can change it. Predict before you compute.

Reference circuit (Questions 3, 7 and 9): a supply $V_s = 10$ V feeds node V_1 through $R_1 = 2$ k Ω ; $R_2 = 5$ k Ω joins V_1 to ground; $R_3 = 1$ k Ω joins V_1 to V_2 ; $R_4 = 4$ k Ω joins V_2 to

ground. Units: V, k Ω , mS, mA; KCL with the currents leaving each node summing to zero.

7. R4 is changed from 4 k Ω to 8 k Ω . Only one entry of A changes (a_{22} : 1.25 $\rightarrow$ 1.125 mS). What happens to the node voltages?

- A) Only V_2 changes, because R4 touches only node 2.
- B) Neither changes, because b is unchanged.
- C) Both node voltages fall.
- D) Both node voltages rise — V_2 by more than V_1 — because the nodes are coupled through R3.

Your choice: A ☐ B ☐ C ☐ D ☐ Why (one sentence):

8. A two-node circuit is extended: R5 joins node V_2 to a new node V_3 , and R6 joins V_3 to ground. Which statement describes the new matrix?

- A) A stays 2×2 ; G5 is simply added to a_{22} .
- B) A becomes 3×3 : a_{22} gains G5, the new row and column for V_3 have $a_{33} = G5 + G6$ and $a_{23} = a_{32} = -G5$, and $a_{13} = a_{31} = 0$.
- C) A becomes 3×3 with $a_{13} = a_{31} = -G5$, because V_3 is connected to the circuit.
- D) A becomes 3×3 and b gains a new entry $G6 \cdot V_s$ in row 3.

Your choice: A ☐ B ☐ C ☐ D ☐ Why (one sentence):

RATE YOURSELF on this group before moving on:

- I can do this independently.
- I can do this with a small AI hint.
- I need substantial AI help.
- I still need human/instructor help.

GROUP 5 · Question It — When the Model or the Matrix Is Wrong

Evidence decides. A solution is not a verdict.

Reference circuit (Questions 3, 7 and 9): a supply $V_s = 10$ V feeds node V_1 through R1 = 2 k Ω ; R2 = 5 k Ω joins V_1 to ground; R3 = 1 k Ω joins V_1 to V_2 ; R4 = 4 k Ω joins V_2 to

ground. Units: V, k Ω , mS, mA; KCL with the currents leaving each node summing to zero.

9. An AI assistant models the reference circuit as $A = \begin{bmatrix} 1.5 & -1 \\ -1 & 1.25 \end{bmatrix}$ mS, $b = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ mA. MATLAB returns $V = \begin{bmatrix} 7.1429 \\ 5.7143 \end{bmatrix}$ V with residual 10^{-16} mA. Which check reveals the error most directly?

- A) Recompute the residual with more digits.
- B) Re-solve with $\text{inv}(A)*b$ to confirm the answer.
- C) Check each diagonal entry against the conductances at that node: a_{11} should be $G_1 + G_2 + G_3 = 1.7$ mS, not 1.5 — the G_2 term (R_2) is missing.
- D) Check that A is symmetric.

Your choice: A ☐ B ☐ C ☐ D ☐ **Why (one sentence):**

10. A student forgets the supply and ground connections and models two nodes joined only by R_3 : $A = \begin{bmatrix} G_3 & -G_3 \\ -G_3 & G_3 \end{bmatrix}$. MATLAB warns: "Matrix is singular to working precision." What does this mean?

- A) The equations are not independent: with no connection to the supply or ground, the model fixes only the difference $V_1 - V_2$, not the voltages themselves — a physical connection (reference) is missing.
- B) The resistor values are too small; scaling them to ohms will fix it.
- C) Round-off error; solving with more digits will give the answer.
- D) Use $\text{inv}(A)*b$ instead of $A\b b$; the inverse avoids the warning.

Your choice: A ☐ B ☐ C ☐ D ☐ **Why (one sentence):**

RATE YOURSELF on this group before moving on:

- I can do this independently.
- I can do this with a small AI hint.
- I need substantial AI help.
- I still need human/instructor help.

Reading your ratings. Mostly "independently" → you own Chapter 8: you are ready to predict a real circuit in Lab-8 and to model five systems in Homework 8. "Small AI hint" clusters → redo just those questions tomorrow without the hint. "Substantial AI help" → rework the matching Chapter 8 lecture part (Couple it · Build it · Solve it · Change it · Question it), then retry. Any "human/instructor help" → bring that exact question to your instructor or ask an expert — that is what they are for. If you can answer these, you can answer the chapter's question: does the matrix describe the system — and how would you know? AI assists. Humans verify. Engineers decide.