

FUNDAMENTALS OF AI-AUGMENTED ENGINEERING: A CONVERSATIONAL APPROACH

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The engineering workflow you will use throughout this book

CHAPTER - 9

PART 3 · ≈ 35 MIN

When Do Iterations Converge — and When Do They Fail?

An iterative solver always produces numbers, and eventually it stops. Part 3 asks the central question of the chapter: how do we decide whether those numbers are a trustworthy engineering answer?

3

Problem

›

Model

›

Compute

›

Analyze

›

Validate

›

Improve



CONTENTS OF CHAPTER 9:

Chapter 8: a few coupled unknowns, solved directly. **Chapter 9:** far too many — solved by repeated correction, and checked.

1

Why do we need another way to solve equations?

From two unknowns to 100,000 · mostly-empty matrices · the 80 GB surprise · direct vs. iterative



2

How does Jacobi actually work?

Repeated correction by hand · using only the previous guess · pseudocode and compact MATLAB



3

When do iterations converge — and when do they fail? · you are here

Convergence and divergence · equation order · change vs. residual vs. error · stopping safely



4

Is Jacobi the only way? What happens at engineering scale?

Gauss-Seidel · a growing sparse network · LU in concept · what industrial solvers do



5

Can we trust the solver — and the AI?

Auditing AI-written solver code · ACCEPT / MODIFY / REJECT · a verification checklist



Course spiral: Ch 4–5 — one unknown, improved step by step until a stopping test is met (bisection, Newton) · Ch 8 — several coupled unknowns, solved directly · Ch 9 — the same kind of system, solved by repeated correction, and judged by evidence.



It Runs. But Does It Converge?

PART 2 · WHAT WE SAW

- Jacobi on the familiar circuit, from (0, 0) V.
- The loop ended at **k = 10** (change 0.643 mV < 1 mV).
- The reference (2, 1) V confirmed it: error 0.257 mV.
- **Will it always go this well?**

A LOOP CAN END IN THREE WAYS

break

change < tol

k = max_iter

ran out of iterations

error(...)

Inf or NaN appeared

Only the first one even *claims* success — and a claim is not proof.

THE CENTRAL QUESTION OF CHAPTER 9

An iterative solver produces numbers and eventually stops. How do we decide whether those numbers are a trustworthy engineering answer?

RUNS ≠ CORRECT. A loop that ends has only proven that it ended.

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Three Possible Stories

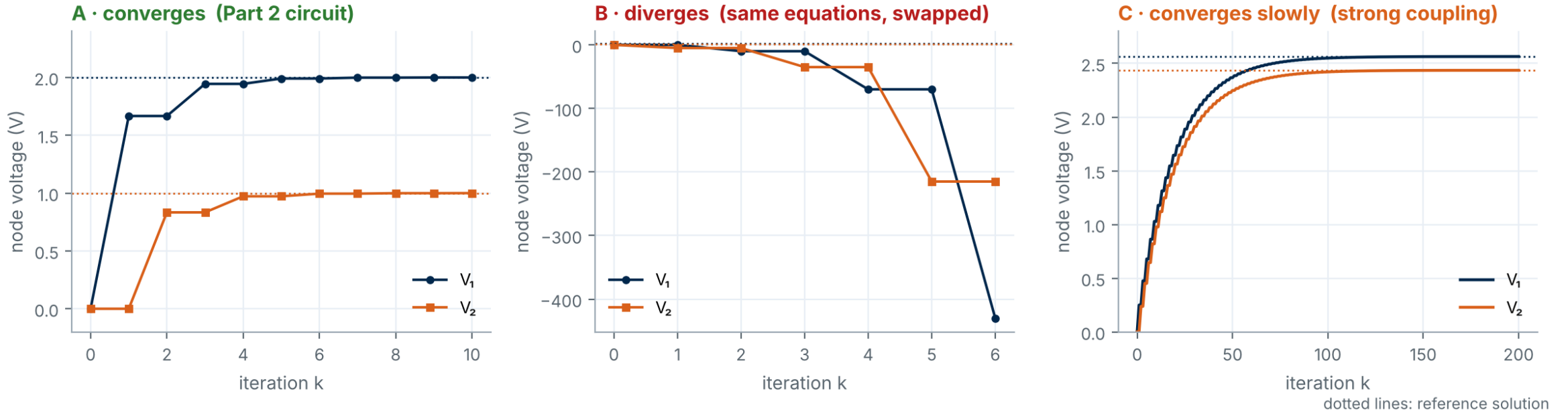


Figure 1. Computed Jacobi iterates from $(0, 0)$ V, tolerance not applied. A: Part 2 circuit. B: the same two equations in swapped order. C: a strongly coupled two-node circuit (slide 10). Dotted lines: reference solutions.

CONVERGES Iterates settle onto the reference.

DIVERGES Iterates run away — A stopping rule cannot turn a divergent iteration into a convergent

CONVERGES SLOWLY Correct direction, tiny steps — a trap for the stopping rule.

Same method, three outcomes. Jacobi is not "working" or "broken" — its behaviour depends on the system it is given.

Problem



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What Made Our First Circuit Behave?

EXPERIMENT A

3	-1
-1	2

mS

maize = diagonal

THE ROW TEST

Strictly diagonally dominant (by rows): in every row, $|\text{diagonal}|$ is **larger than** the sum of the other $|\text{entries}|$.

Row 1: $|3| > |-1|$ ✓

Row 2: $|2| > |-1|$ ✓

IN CIRCUIT TERMS

- Diagonal: **total** conductance at the node.
- Off-diagonal: conductance to a neighbour node.
- Dominant: each node is tied to ground or a source, not only to its neighbours.

THE GUARANTEE

If the matrix is strictly diagonally dominant by rows, Jacobi **converges from any starting guess**. Part 2's success was no accident.

WHAT IT DOES NOT SAY

- Not dominant $\Rightarrow$ fails? **No (slide 9)**.
- Dominant $\Rightarrow$ fast? **No (slide 10)**.

A sufficient condition: when every row passes the test, Jacobi convergence is guaranteed.

Problem



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Same Circuit. Same Answer. Different Behavior.

SWAP ROWS 1 AND 2

original

$$3V_1 - V_2 = 5$$

$$-V_1 + 2V_2 = 0$$

swapped (same equations, new order)

$$-V_1 + 2V_2 = 0$$

$$3V_1 - V_2 = 5$$

$$\begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix} \begin{matrix} A_{\text{swap}} \text{ (mS)} \\ b_{\text{swap}} = [0; 5] \text{ mA} \end{matrix}$$

Columns stay V_1, V_2 : rearranging equations is **not** renaming unknowns.

NEW JACOBI UPDATES

Row 1 solved for V_1 (diagonal -1):

$$V_1^{(k+1)} = 2V_2^{(k)}$$

Row 2 solved for V_2 (diagonal -1):

$$V_2^{(k+1)} = 3V_1^{(k)} - 5$$

Same solution? $2(1) = 2 \checkmark$ $3(2) - 5 = 1 \checkmark$

Row test: $|-1| < |2|$, $|-1| < |3|$ — **fails**

PREDICT · FROM (0, 0) V

A converges to (2, 1) V

B converges somewhere else

C runs away

D cannot tell without trying

One letter, one reason. Compute $x^{(1)}$ and $x^{(2)}$ if you want evidence.

Same system, same answer — different iteration. Which equation updates which unknown is part of the method.

Problem



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Watch the Iterations Run Away

k	V ₁ (V)	V ₂ (V)	max error (V)
0	0	0	2
1	0	-5	6
2	-10	-5	12
3	-10	-35	36
4	-70	-35	72
5	-70	-215	216
6	-430	-215	432

Table 1. Swapped-order Jacobi from (0, 0) V, computed; error = $\max |x^{(k)} - (2, 1)|$.

Start closer, at (1.9, 0.9) V? Still runs away: error 130 V by k = 8.

Not a bug — a different iteration. The code ran perfectly; the iteration it carried out does not converge.

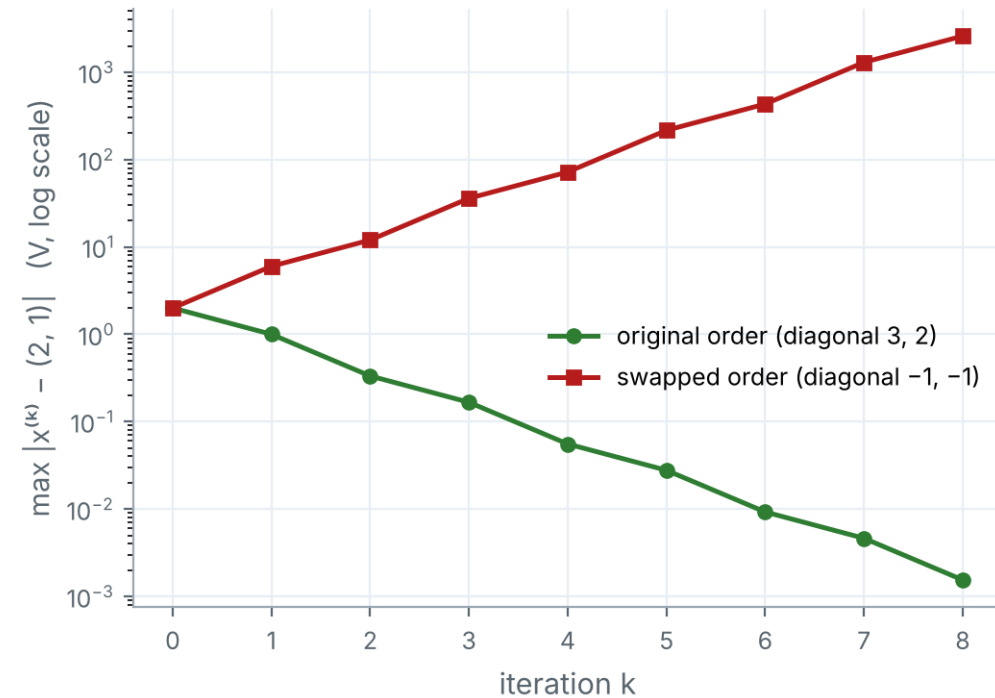


Figure 2. Computed reference error, original vs. swapped order (log scale).

Problem



Model



Compute

**Analyze**

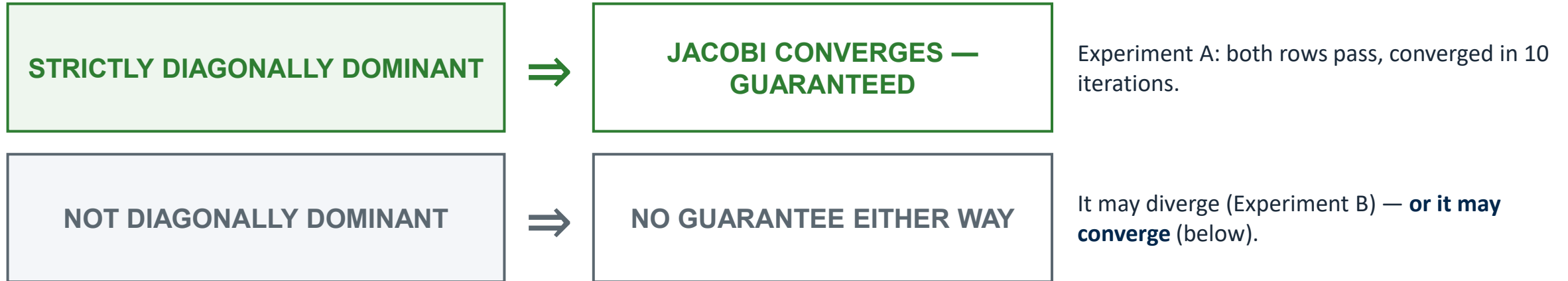
Validate



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Does Diagonal Dominance Guarantee Success?



NOT DOMINANT — STILL CONVERGES

$\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$
Row 1: $|2| < |-3|$ **fails**
 $b = [-1; 3]$, solution (1, 1).
 Jacobi from (0, 0): change $< 10^{-3}$ at **k = 15**.

READ THE ARROW ONE WAY

- **Sufficient:** passing the test is enough for convergence.
- **Not necessary:** failing the test proves nothing.
- **Not about speed:** a dominant matrix can still converge slowly.

Dominance is a guarantee, not a requirement. When the test fails, you need evidence from the iterates themselves.

Problem



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A Solver Can Converge Painfully Slowly

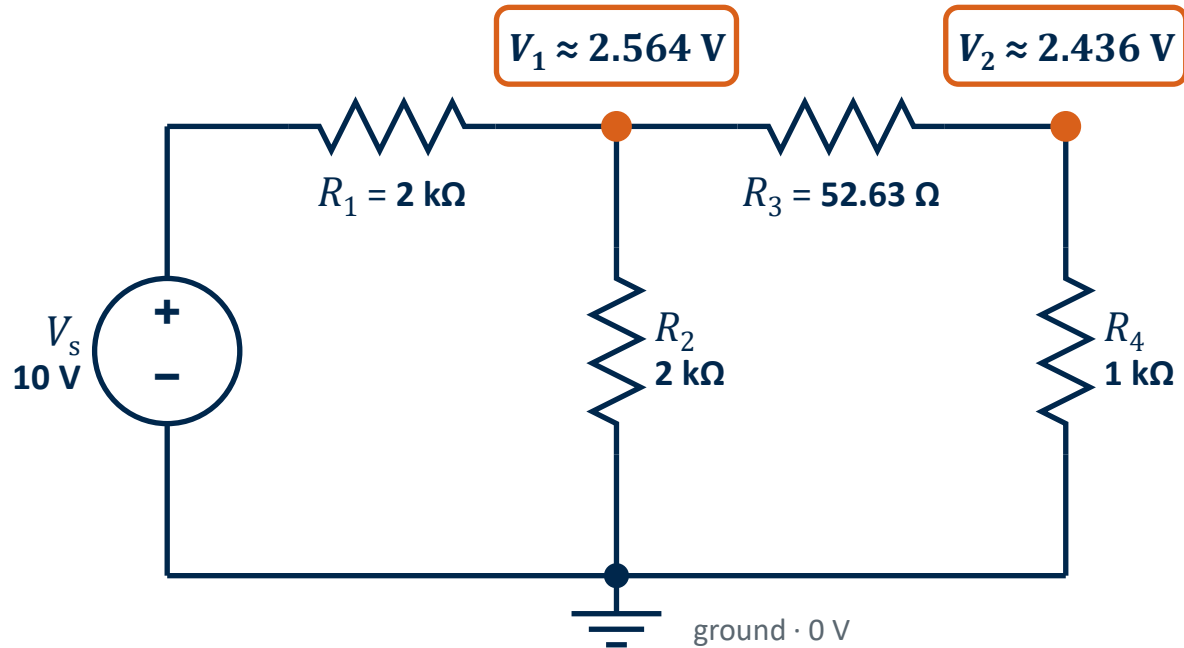


Figure 3. Strongly coupled circuit: $R_3 = 52.6 \, \Omega$ (19 mS) ties the nodes together; $R_1 = R_2 = 2 \, \text{k}\Omega$ (0.5 mS), $R_4 = 1 \, \text{k}\Omega$ (1 mS), $V_s = 10 \, \text{V}$.

THE MODEL · KCL

$$\begin{aligned} 20V_1 - 19V_2 &= 5 \\ -19V_1 + 20V_2 &= 0 \end{aligned}$$

A in mS, b in mA

Row test: $20 > 19$ ✓

Reference: $x^* = (100/39, 95/39) \, \text{V}$
 $\approx (2.56410256, 2.43589744) \, \text{V}$

SAME CODE · TOLERANCE 1 mV

10

iterations · Part 2 circuit

109

iterations · this circuit

Dominant — but only just. $20 > 19$ guarantees convergence. It says nothing about speed.

Problem



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The 1 mV Trap

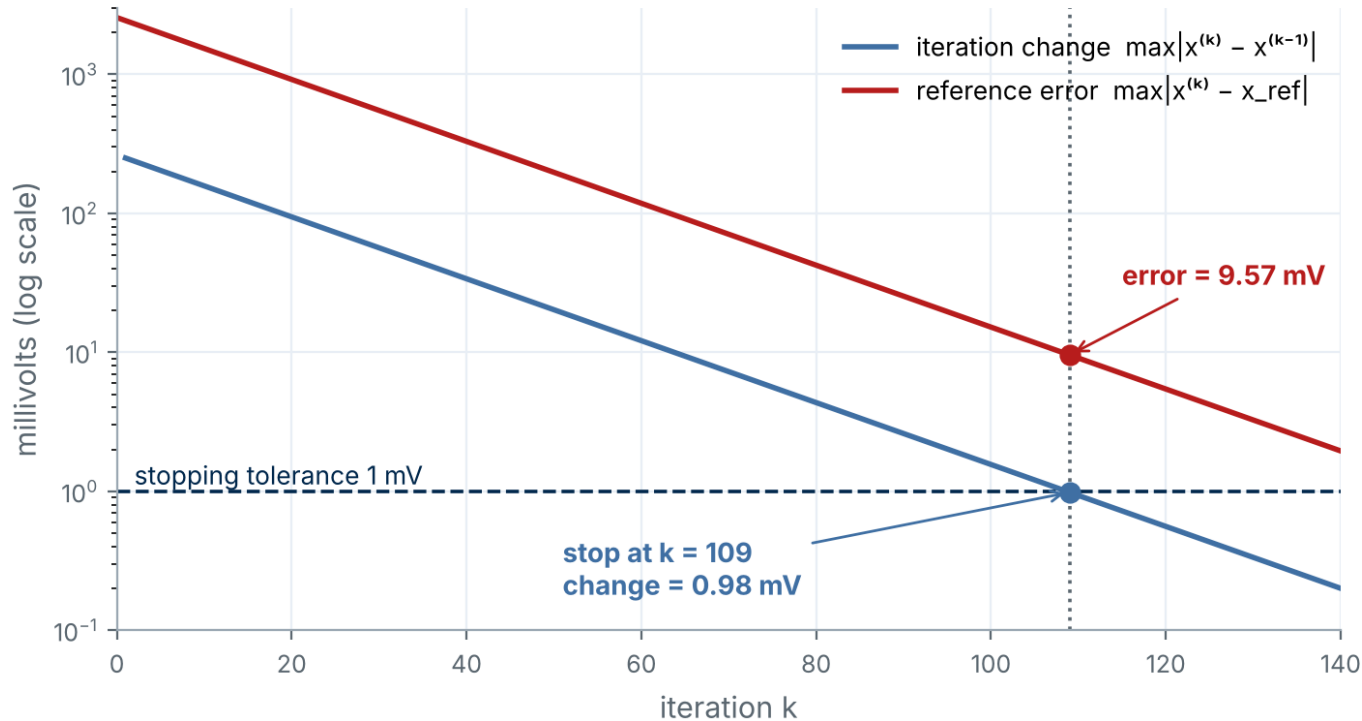


Figure 4. Computed iteration change and reference error for Experiment C (log scale, mV). Dashed: the 1 mV stopping tolerance.

STOPPING CRITERION

0.98 mV < 1 mV ✓

largest change between iterations 108 and 109

REFERENCE ERROR

9.57 mV $\approx 10 \times$ tolerance

distance from the trusted solution (2.5641, 2.4359) V

The solver **did exactly what it was told**. The mistake is reading "1 mV tolerance" as "1 mV accuracy".

Small steps are not small errors. A stopping rule decides when to stop — it is not an accuracy guarantee.

Problem



Model



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Three Measures. Three Different Questions.

ITERATION CHANGE

$$\delta = \max(\text{abs}(\mathbf{x} - \mathbf{x}_{\text{old}}))$$

How much did the estimate change?

0.98 mV

$9.82 \times 10^{-4} \text{ V}$

units: V · always available

RESIDUAL

$$\max(\text{abs}(\mathbf{A} \cdot \mathbf{x} - \mathbf{b}))$$

How well does the estimate satisfy the equations?

0.0187 mA

$\mathbf{r} = (0.0000, -0.0187) \text{ mA}$

units: mA · always available

REFERENCE ERROR

$$\max(\text{abs}(\mathbf{x} - \mathbf{x}_{\text{ref}}))$$

How far is the estimate from a trusted reference?

9.57 mV

$9.57 \times 10^{-3} \text{ V}$

units: V · only for test problems

Experiment C, $k = 109$ (the moment the solver stopped). Values from `ch9p3_audit.m`.

- Never compare a residual in **mA** with a tolerance in **V**.
- A small residual or a small change does **not** always mean a small error.
- Real problems have no reference: the error is **not observable**.
- Matching a numerical reference does not prove the **physical model** is right.

Three numbers, three questions: changing? satisfying the equations? close to a trusted answer?

Problem



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What Does a Responsible Stopping Rule Look Like?

PART 2 LOOP + FOUR ADDITIONS

```

IF any A(i,i) == 0: status = "invalid"; STOP
converged = false; status = "iteration limit"
FOR k = 1 TO max_iter
  x_old = x
  x = Jacobi update using x_old only
  IF any x is NaN or Inf:
    status = "numerical failure"; leave loop
  change = max |x - x_old|
  IF change < tol:
    converged = true; status = "converged"
    leave loop
END
r = A*x - b
REPORT status, converged, k, change (V), max|r| (mA)

```

THREE DIFFERENT ENDINGS

- converged**
criterion met — even on the last allowed iteration
- iteration limit**
max_iter reached, criterion not met
- numerical failure**
zero diagonal, NaN or Inf

READ THE FLAG CORRECTLY

- **converged = true**: the stopping rule was met — **not** "x is accurate".
- Experiment C with max_iter = 109: **true**; with 100: **false**.

A responsible solver reports how it ended — and the evidence (change, residual) that goes with its answer.

Problem



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YOUR TURN

Would You Accept This Answer?

SOLVER REPORT · EXPERIMENT C

```
method ..... Jacobi, x(0) = (0, 0) V
status ..... CONVERGED
iterations ..... 109 of 1000 allowed
tolerance ..... 1 mV (iteration change)
final change ..... 0.98 mV
max |residual| ... 0.0187 mA
reference error .. 9.57 mV
(test case: trusted reference known)
```

ENGINEERING REQUIREMENT node voltages within **2 mV**

The solver reports convergence. Would you approve its answer for a decision that needs 2 mV accuracy?

ACCEPT

REJECT

INVESTIGATE

Pick one. Justify it with a number from the report.

CONVERGED ≠ ACCURATE ENOUGH. Decide with the evidence and the requirement — not with the status line.

Problem



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Convergence Is Evidence, Not an Assumption

DOES IT CONVERGE?**HOW FAST?****WHY DID IT STOP?****ACCURATE ENOUGH?**

WE KNOW

Strict diagonal dominance guarantees Jacobi convergence — but it is not required, and it does not promise speed. Equation order matters.

WE MEASURE

Iteration change (V), residual (mA) and, for test problems, reference error (V) — three different questions.

WE DECIDE

Accept an answer only when the evidence meets the engineering requirement — not because the status says converged.

NEXT — PART 4 · IS JACOBI THE ONLY WAY? WHAT HAPPENS AT ENGINEERING SCALE?

Experiment C needed 109 iterations. Could a different way of updating — or a different kind of solver — do better on large networks?

Convergence is evidence, not an assumption. Show it with numbers — and judge it against the requirement.

Problem



Model



Compute



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**Validate**

Improve